The Formation of a Blast Wave by a Very Intense Explosion. I. Theoretical Discussion
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The formation of a blast wave by a very intense explosion
I. Theoretical discussion

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Summary and introduction

This paper was written early in 1941 and circulated to the Civil Defence Research Committee of the Ministry of Home Security in June of that year. The present writer had been told that it might be possible to produce a bomb in which a very large amount of energy would be released by nuclear fission—the name atomic bomb had not then been used—and the work here described represents his first attempt to form an idea of what mechanical effects might be expected if such an explosion could occur. In the then common explosive bomb mechanical effects were produced by the sudden generation of a large amount of gas at a high temperature in a confined space. The practical question which required an answer was: Would similar effects be produced if energy could be released in a highly concentrated form unaccompanied by the generation of gas? This paper has now been declassified, and though it has been superseded by more complete calculations, it seems appropriate to publish it as it was first written, without alteration, except for the omission of a few lines, the addition of this summary, and a comparison with some more recent experimental work, so that the writings of later workers in this field may be appreciated.

An ideal problem is here discussed. A finite amount of energy is suddenly released in an infinitely concentrated form. The motion and pressure of the surrounding air is calculated. It is found that a spherical shock wave is propagated outwards whose radius R is related to the time t since the explosion started by the equation

$$R = S(\gamma)\, t^{\frac{2}{5}} E^{\frac{1}{5}} \rho_0^{-\frac{1}{5}},$$

where ρ_0 is the atmospheric density, E is the energy released and $S(\gamma)$ a calculated function of γ, the ratio of the specific heats of air.

The effect of the explosion is to force most of the air within the shock front into a thin shell just inside that front. As the front expands, the maximum pressure decreases till, at about 10 atm., the analysis ceases to be accurate. At 20 atm. 45 % of the energy has been degraded into heat which is not available for doing work and used up in expanding against atmospheric pressure. This leads to the prediction that an atomic bomb would be only half as efficient, as a blast-producer, as a high explosive releasing the same amount of energy.

In the ideal problem the maximum pressure is proportional to R^{-3}, and comparison with the measured pressures near high explosives, in the range of radii where the two might be expected to be comparable, shows that these conclusions are borne out by experiment.

Similarity assumption

The propagation and decay of a blast wave in air has been studied for the case when the maximum excess over atmospheric pressure does not exceed 2 atm. At great distances R from the explosion centre the pressure excess decays as in a sound wave proportionally to R^{-1}. At points nearer to the centre it decays more rapidly than R^{-1}. When the excess pressure is 0·5 atm., for instance, a logarithmic plot shows that it varies as $R^{-1\cdot 9}$. When the excess pressure is 1·5 atm. the decay is proportional to $R^{-2\cdot 8}$. It is difficult to analyze blast waves in air at points near the explosion centre because the initial shock wave raises the entropy of the air it traverses by an amount which depends on the intensity of the shock wave. The passage of a spherical shock wave, therefore, leaves the air in a state in which the entropy decreases radially so that after its passage, when the air has returned to atmospheric pressure, the air temperature decreases with increasing distance from the site of the explosion. For this reason the density is not a single-valued function of the pressure in a blast wave. After the passage of the blast wave, the relationship between pressure and density for any given particle of air is simply the adiabatic one corresponding with the entropy with which that particle was endowed by the shock wave during its passage past it. For this reason it is in general necessary to use a form of analysis in which the initial position of each particle is retained as one of the variables. This introduces great complexity and, in general, solutions can only be derived by using step-by-step numerical integration. On the other hand, the great simplicity which has been introduced into two analogous problems, namely, the spherical detonation wave (Taylor 1950) and the air wave surrounding a uniformly expanding sphere (Taylor 1946), by assuming that the disturbance is similar at all times, merely increasing its linear dimensions with increasing time from initiation, gives encouragement to an attempt to apply similar principles to the blast wave produced by a very intense explosion in a very small volume.

It is clear that the type of similarity which proved to be possible in the two above-mentioned problems cannot apply to a blast wave because in the latter case the intensity must decrease with increasing distance while the total energy remains constant. In the former the energy associated with the motion increased proportionally to the cube of the radius while the pressure and velocity at corresponding points was independent of time.

The appropriate similarity assumptions for an expanding blast wave of constant total energy are

$$\text{pressure, } p/p_0 = y = R^{-3} f_1, \tag{1}$$

$$\text{density, } \rho/\rho_0 = \psi, \tag{2}$$

$$\text{radial velocity, } u = R^{-\frac{3}{2}} \phi_1, \tag{3}$$

where R is the radius of the shock wave forming the outer edge of the disturbance, p_0 and ρ_0 are the pressure and density of the undisturbed atmosphere. If r is the radial co-ordinate, $\eta = r/R$ and f_1, ϕ_1 and ψ are functions of η. It is found that these assumptions are consistent with the equations of motion and continuity and with the equation of state of a perfect gas.

The equation of motion is
$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}=-\frac{p_0}{\rho}\frac{\partial y}{\partial r}. \tag{4}$$

Substituting from (1), (2) and (3) in (4) and writing f_1', ϕ_1' for $\frac{\partial}{\partial\eta}f_1, \frac{\partial}{\partial\eta}\phi_1$,
$$-(\tfrac{3}{2}\phi_1+\eta\phi_1')\,R^{-\frac{5}{2}}\frac{dR}{dt}+R^{-4}\left(\phi_1\phi_1'+\frac{p_0}{\rho_0}\frac{f_1'}{\psi}\right)=0. \tag{5}$$

This can be satisfied if
$$\frac{dR}{dt}=AR^{-\frac{3}{2}}, \tag{6}$$
where A is a constant, and
$$-A(\tfrac{3}{2}\phi_1+\eta\phi_1')+\phi_1\phi_1'+\frac{p_0}{\rho_0}\frac{f_1'}{\psi}=0. \tag{7}$$

The equation of continuity is
$$\frac{\partial\rho}{\partial t}+u\frac{\partial\rho}{\partial r}+\rho\left(\frac{\partial u}{\partial r}+\frac{2u}{r}\right)=0. \tag{8}$$

Substituting from (1), (2), (3) and (6), (8) becomes
$$-A\eta\psi'+\psi'\phi_1+\psi\left(\phi_1'+\frac{2}{\eta}\phi_1\right)=0. \tag{9}$$

The equation of state for a perfect gas is
$$\left(\frac{\partial}{\partial t}+u\frac{\partial}{\partial r}\right)(p\rho^{-\gamma})=0. \tag{10}$$
where γ is the ratio of specific heats.

Substituting from (1), (2), (3) and (6), (10) becomes
$$A(3f_1+\eta f_1')+\frac{\gamma f_1}{\psi}\psi'(-A\eta+\phi_1)-\phi_1 f_1'=0. \tag{11}$$

The equations (7), (9) and (11) may be reduced to a non-dimensional form by substituting
$$f=f_1a^2/A^2, \tag{12}$$
$$\phi=\phi_1/A, \tag{13}$$
where a is the velocity of sound in air so that $a^2=\gamma p_0/\rho_0$. The resulting equations which contain only one parameter, namely, γ, are
$$\phi'(\eta-\phi)=\frac{1}{\gamma}\frac{f'}{\psi}-\tfrac{3}{2}\phi, \tag{7a}$$
$$\frac{\psi'}{\psi}=\frac{\phi'+2\phi/\eta}{\eta-\phi}, \tag{9a}$$
$$3f+\eta f'+\frac{\gamma\psi'}{\psi}f(-\eta+\phi)-\phi f'=0. \tag{11a}$$

Eliminating ψ' from (11a) by means of (7a) and (9a) the equation for calculating f' when f, ϕ, ψ and η are given is
$$f'\{(\eta-\phi)^2-f/\psi\}=f\{-3\eta+\phi(3+\tfrac{1}{2}\gamma)-2\gamma\phi^2/\eta\}. \tag{14}$$

When f' has been found from (14), ϕ' can be calculated from (7a) and hence ψ' from (9a). Thus if for any value of η, f, ϕ and ψ are known their values can be computed step-by-step for other values of η.

Shock-wave conditions

The conditions at the shock wave $\eta = 1$ are given by the Rankine-Hugoniot relations which may be reduced to the form

$$\frac{\rho_1}{\rho_0} = \frac{\gamma-1+(\gamma+1)y_1}{\gamma+1+(\gamma-1)y_1}, \tag{15}$$

$$\frac{U^2}{a^2} = \frac{1}{2\gamma}\{\gamma-1+(\gamma+1)y_1\}, \tag{16}$$

$$\frac{u_1}{U} = \frac{2(y_1-1)}{\gamma-1+(\gamma+1)y_1}, \tag{17}$$

where ρ_1, u_1 and y_1 represent the values of ρ, u and y immediately behind the shock wave and $U = dR/dT$ is the radial velocity of the shock wave.

These conditions cannot be satisfied consistently with the similarity assumptions represented by (1), (2) and (3). On the other hand, when y_1 is large so that the pressure is high compared with atmospheric pressure, (15), (16) and (17) assume the approximate asymptotic forms

$$\frac{\rho_1}{\rho_0} = \frac{\gamma+1}{\gamma-1}, \tag{15a}$$

$$\frac{U^2}{a^2} = \frac{2\gamma}{\gamma+1}y_1, \tag{16a}$$

$$\frac{u_1}{U} = \frac{2}{\gamma+1}. \tag{17a}$$

These approximate boundary conditions are consistent with (1), (2), (3) and (6); in fact (15a) yields, for the conditions at $\eta = 1$,

$$\psi = \frac{\gamma+1}{\gamma-1}, \tag{15b}$$

(16a) yields

$$f = \frac{2\gamma}{\gamma+1}, \tag{16b}$$

and (17a) yields

$$\phi = \frac{2}{\gamma+1}. \tag{17b}$$

Energy

The total energy E of the disturbance may be regarded as consisting of two parts, the kinetic energy

$$K.E. = 4\pi\int_0^R \tfrac{1}{2}\rho u^2 r^2\,dr,$$

and the heat energy

$$H.E. = 4\pi\int_0^R \frac{pr^2}{\gamma-1}\,dr.$$

In terms of the variables f, ϕ, ψ and η

$$E = 4\pi A^2\left\{\tfrac{1}{2}\rho_0\int_0^1 \psi\phi^2\eta^2\,d\eta + \left(\frac{p_0}{a^2(\gamma-1)}\int_0^1 f\eta^2\,d\eta\right)\right\},$$

or since $p_0 = a^2\rho_0/\gamma$, $E = B\rho_0 A^2$, where B is a function of γ only whose value is

$$B = 2\pi\int_0^1 \psi\phi^2\eta^2\,d\eta + \frac{4\pi}{\gamma(\gamma-1)}\int_0^1 f\eta^2\,d\eta. \tag{18}$$

Since the two integrals in (18) are both functions of γ only it seems that for a given value of γ, A^2 is simply proportional to E/ρ_0.

Numerical solution for $\gamma = 1{\cdot}4$

When $\gamma = 1{\cdot}4$ the boundary values of f, ϕ and ψ at $\eta = 1$ are from (15*a*), (16*a*). (17*a*), $\frac{7}{6}$, $\frac{5}{6}$ and 6. Values of f, ϕ and ψ were calculated from $\eta = 1{\cdot}0$ to $\eta_1 = 0{\cdot}5$, using intervals of 0·02 in η. Starting each step with values of f', ϕ', ψ', f, ϕ and ψ found in previous steps, values of f', ϕ' and ψ' at the end of the interval were predicted by assuming that the previous two values form a geometrical progression with the predicted one; thus the $(s+1)$th term, f'_{s+1} in a series of values of f' was taken as $f'_{s+1} = (f'_s)^2/f'_{s-1}$. With this assumed value the mean value of f' in the sth interval was taken as $\frac{1}{2}(f'_{s+1}+f'_s)$ and the increment in f was taken as $(0{\cdot}02)(\frac{1}{2})(f'_{s+1}+f'_s)$. The values of f'_{s+1}, ϕ'_{s+1} and ψ'_{s+1} were then calculated from formulae (14), (7*a*) and (9*a*). If they differed appreciably from the predicted values a second approximation was worked out, replacing the estimated values of f'_{s+1} by this new calculated value. In the early stages of the calculation near $\eta = 1$ two or three approximations were made, but in the later stages the estimated value was so close to the calculated one that the value of f' calculated in this first approximation was used directly in the next stage.

The results are given in table 1 and are shown in the curves of figure 1. These curves and also table 1 show three striking features: (*a*) the ϕ curve rapidly settles down to a curve which is very nearly a straight line through the origin, (*b*) the density curve ψ rapidly approaches the axis $\psi = 0$, in fact at $\eta = 0{\cdot}5$ the density is only 0·007 of the density of the undisturbed atmosphere, (*c*) the pressure becomes practically constant and equal to $0{\cdot}436/1{\cdot}167 = 0{\cdot}37$ of the maximum pressure. These facts suggest that the solution tends to a limiting form as η decreases in which $\phi = c\eta$, $\phi' = c =$ constant, $f = 0{\cdot}436$, f', ψ and ψ' become small. Substituting for $\dfrac{1}{\gamma}\dfrac{f'}{\psi}$ from (7*a*), (14) becomes

$$\frac{f'}{f}(\eta-\phi)^2 = \gamma\phi'(\eta-\phi)+\tfrac{3}{2}\gamma\phi-3\eta+(3+\tfrac{1}{2}\gamma)\phi-\frac{2\gamma\phi^2}{\eta}. \tag{19}$$

Dividing by $\eta-\phi$ (19) becomes

$$\frac{f'}{f}(\eta-\phi) = \gamma\phi'-3+\frac{2\gamma\phi}{\eta}. \tag{20}$$

If the left-hand side which contains f'/f be neglected the approximate solution of (20) for which ϕ vanishes at $\eta = 0$ is

$$\phi = \eta/\gamma. \tag{21}$$

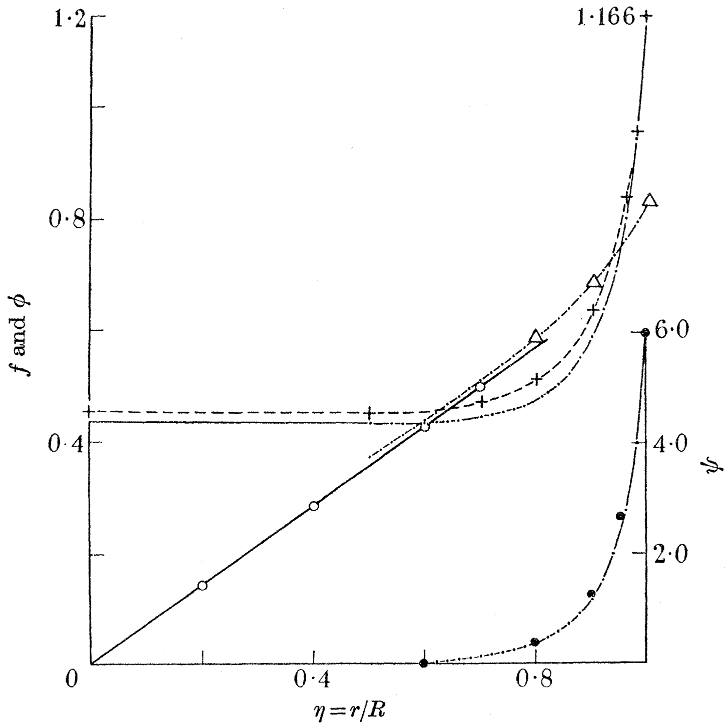

FIGURE 1. – · –, curves f and ψ (step-by-step calculation); – – + – –, curve f (approximate formulae). In the other curves the small dots represent the steps of the calculations, the larger symbols represent approximate formulae for: △, curve ϕ; ○, curve $\phi = \eta/\gamma$; •, curve ψ.

TABLE 1. STEP-BY-STEP CALCULATION FOR $\gamma = 1{\cdot}4$

η	f	ϕ	ψ
1·00	1·167	0·833	6·000
0·98	0·949	0·798	4·000
0·96	0·808	0·767	2·808
0·94	0·711	0·737	2·052
0·92	0·643	0·711	1·534
0·90	0·593	0·687	1·177
0·88	0·556	0·665	0·919
0·86	0·528	0·644	0·727
0·84	0·507	0·625	0·578
0·82	0·491	0·607	0·462
0·80	0·478	0·590	0·370
0·78	0·468	0·573	0·297
0·76	0·461	0·557	0·239
0·74	0·455	0·542	0·191
0·72	0·450	0·527	0·152
0·70	0·447	0·513	0·120
0·68	0·444	0·498	0·095
0·66	0·442	0·484	0·074
0·64	0·440	0·470	0·058
0·62	0·439	0·456	0·044
0·60	0·438	0·443	0·034
0·58	0·438	0·428	0·026
0·56	0·437	0·415	0·019
0·54	0·437	0·402	0·014
0·52	0·437	0·389	0·010
0·50	0·436	0·375	0·007

The line $\phi = \eta/\gamma$ is shown in figure 1. It will be seen that the points calculated by the step-by-step method nearly run into this line. The difference appears to be due to the accumulation of errors in calculation.

Approximate formulae

The fact that the ϕ curve seems to leave the straight line $\phi = \eta/\gamma$ rather rapidly after remaining close to it over the range $\eta = 0$ to $\eta = 0{\cdot}5$ suggests that an approximate set of formulae might be found assuming

$$\phi = \eta/\gamma + \alpha\eta^n, \tag{22}$$

where n is a positive number which may be expected to be more than, say, 3 or 4. If this formula applies at $\eta = 1$,

$$\frac{1}{\gamma} + \alpha = \frac{2}{\gamma+1} \quad \text{or} \quad \alpha = \frac{\gamma-1}{\gamma(\gamma+1)}; \tag{23}$$

inserting $\phi = \eta/\gamma + \alpha\eta^n$, $\phi' = 1/\gamma + n\alpha\eta^{n-1}$ in (20), the value of f'/f at $\eta = 1$ is $f'/f = \alpha\gamma(n+2)(\gamma+1)/(\gamma-1)$. From (14) and (15*b*), (16*b*), (17*b*) the true value of f'/f at $\eta = 1$ is $\dfrac{2\gamma^2+7\gamma-3}{\gamma-1}$. Equating these two forms,

$$n = \frac{7\gamma-1}{\gamma^2-1}. \tag{24}$$

The values of α and n have now been determined to give the correct values of f'/f; ϕ and ϕ' at $\eta = 1$, ψ' is determined by (9*a*) so that all the six correct values of f, ϕ, ψ, f', ϕ', ψ' are consistent with (22) at $\eta = 1$. Substituting for ϕ from (22) in (20),

$$\frac{f'}{f} = \frac{(n+2)\,\alpha\gamma^2\eta^{n-2}}{\gamma-1-\gamma\alpha\eta^{n-1}}. \tag{25}$$

The integral of (25) which gives the correct value of f at $\eta = 1$ is

$$\log f = \log\frac{2\gamma}{\gamma+1} - \frac{2\gamma^2+7\gamma-3}{7-\gamma}\log\left(\frac{\gamma+1}{\gamma} - \frac{\eta^{n-1}}{\gamma}\right). \tag{26}$$

At $\eta = 0{\cdot}5$ this gives $f = 0{\cdot}457$ when $\gamma = 1{\cdot}4$. The value calculated by the step-by-step integration is $0{\cdot}436$, a difference of 5 %.

The approximate form for ψ might be found by inserting the approximate forms for ϕ and ϕ' in (9*a*). Thus

$$\log\psi = \log\frac{\gamma+1}{\gamma-1} - \int_\eta^1 \frac{3+(n+2)\,\alpha\gamma\eta^{n-1}}{(\gamma-1)\,\eta - \alpha\gamma\eta^n}\,d\eta. \tag{27}$$

Integrating this and substituting for α from (23),

$$\log\psi = \log\frac{\gamma+1}{\gamma-1} + \frac{3}{\gamma-1}\log\eta - 2\frac{(\gamma+5)}{7-\gamma}\log\left(\frac{\gamma+1-\eta^{n-1}}{\gamma}\right). \tag{28}$$

When η is small this formula gives

$$\psi = D\eta^{3/(\gamma-1)}, \tag{29}$$

where
$$\log D = \log\frac{\gamma+1}{\gamma-1} - 2\frac{(\gamma+5)}{7-\gamma}\log\left(\frac{\gamma+1}{\gamma}\right). \tag{30}$$

When $\gamma = 1{\cdot}4$ (30) gives $D = 1{\cdot}76$ so that

$$\psi = 1{\cdot}76\eta^{7{\cdot}5}. \tag{29a}$$

At $\eta = 0{\cdot}5$ this gives $\psi = 0{\cdot}0097$; the step-by-step calculation gives $\psi = 0{\cdot}0073$. At $\eta = 0{\cdot}8$ formula (28) gives $\psi = 0{\cdot}387$, while table 1 gives $\psi = 0{\cdot}370$. At $\eta = 0{\cdot}9$ formula (28) gives $\psi = 1{\cdot}24$, while the step-by-step solution gives $1{\cdot}18$. Some points calculated by the approximate formulae are shown in figure 1.

In the central region of the disturbance the density decreases proportionally to $r^{3/(\gamma-1)}$; the fact that the pressure is nearly constant there means that the temperature increases proportionally to $r^{-3/(\gamma-1)}$. At first sight it might be supposed that these very high temperatures involve a high concentration of energy near the centre. This is not the case, however, for the energy *per unit volume* of a gas is simply $p/(\gamma-1)$ so that the distribution of energy is uniform.

Values of f, ϕ and ψ for $\gamma = \frac{5}{3}$ calculated by the approximate formulae are given in table 2.

TABLE 2. APPROXIMATE CALCULATION $\gamma = 1{\cdot}666$

η	f	ϕ	ψ
1·00	1·250	0·750	4·00
0·95	0·892	0·680	2·30
0·90	0·694	0·620	1·14
0·80	0·519	0·519	0·63
0·70	0·425	0·445	0·29
0·50	0·379	0·300	0·05
0·00	0·344	0·000	0·00

BLAST WAVE EXPRESSED IN TERMS OF THE ENERGY OF THE EXPLOSION

It has been seen in (18) that $E/\rho_0 A^2$ is a function of γ only. Evaluating the integrals in (18) for the case for $\gamma = 1{\cdot}4$, and using the step-by-step calculations, it is found that
$$\int_0^1 \eta^2\phi^2\psi\, d\eta = 0{\cdot}185 \quad \text{and} \quad \int_0^1 \eta^2 f\, d\eta = 0{\cdot}187.$$

The kinetic energy of the disturbance is therefore

$$K.E. = 2\pi(0{\cdot}185)\,\rho_0 A^2 = 1{\cdot}164\rho_0 A^2, \tag{31}$$

while the heat energy is

$$H.E. = \frac{4\pi}{(1{\cdot}4)\,(0{\cdot}4)}(0{\cdot}187)\rho_0 A^2 = 4{\cdot}196\rho_0 A^2; \tag{32}$$

the total energy is therefore
$$E = 5{\cdot}36\rho_0 A^2. \tag{33}$$

Pressure

The pressure p at any point is

$$p = p_0 R^{-3} f \frac{A^2}{a^2} = R^{-3} f \frac{\rho_0 A^2}{\gamma} = 0{\cdot}133 R^{-3} E f. \tag{34}$$

The maximum pressure at any distance corresponds with $f = 1{\cdot}166$ at $r = R$. This is therefore

$$p_{\text{max.}} = 0{\cdot}155 R^{-3} E. \tag{35}$$

Velocity of air and shock wave

The velocity u of the gas at any point is

$$u = R^{-\frac{3}{2}} A \phi = R^{-\frac{3}{2}} E^{\frac{1}{2}} (B\rho_0)^{-\frac{1}{2}} \phi. \tag{36}$$

The velocity of radial expansion of the disturbance is, from (6),

$$\frac{dR}{dt} = A R^{-\frac{3}{2}} = R^{-\frac{3}{2}} E^{\frac{1}{2}} (B\rho_0)^{-\frac{1}{2}}, \tag{37}$$

so that, if t is the time since the beginning of the explosion,

$$t = \tfrac{2}{5} R^{\frac{5}{2}} (B\rho_0)^{\frac{1}{2}} E^{-\frac{1}{2}} = 0{\cdot}926 R^{\frac{5}{2}} \rho_0^{\frac{1}{2}} E^{-\frac{1}{2}}, \tag{38}$$

when $\gamma = 1{\cdot}4$.

The formulae (34) to (38) show some interesting features. Though the pressure wave is conveyed outwards entirely by the air the magnitude of the pressure depends only on ER^{-3} and not on the atmospheric density ρ_0. The time scale, however, is proportional to $\rho_0^{\frac{1}{2}}$. It is of interest to calculate the pressure-time relationship for a fixed point, i.e. the pressure to which a fixed object would be subjected as the blast wave passed over it. If t_0 is the time since initiation taken for the wave to reach radius R_0 the pressure at time t at radius R_0 is given by

$$\frac{p}{p_1} = \left(\frac{R_0}{R}\right)^3 \frac{f}{[f]_{\eta=1}}, \tag{39}$$

where p_1 is the pressure in the shock wave as it passed over radius R_0 at the time t_0, R is the radius of the shock wave at time t and $\eta = R_0/R$. $[f]_{\eta=1}$ is the maximum value of f, namely, $1{\cdot}166$ when $\gamma = 1{\cdot}4$. η is related to t/t_0 through (38) so that

$$\eta = (t_0/t)^{\frac{2}{5}} \quad \text{and} \quad \frac{p}{p_1} = \frac{1}{[f]_{\eta=1}} \left(\frac{t_0}{t}\right)^{\frac{6}{5}} [f]_{\eta=(t_0/t)^{\frac{2}{5}}}. \tag{40}$$

Values of p/p_1 calculated by (40) for $\gamma = 1{\cdot}4$ are shown in figure 2.

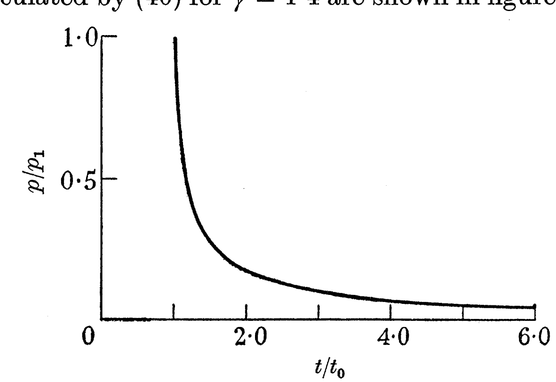

Figure 2. Pressure-time curve at a fixed point.

Temperature

The temperature T at any point is related to the pressure and density by the relationship

$$\frac{T}{T_0} = \frac{p\rho_0}{p_0\rho} = \frac{1\cdot 33 E R^{-3}}{p_0 \psi}, \quad \text{when } \gamma = 1\cdot 4. \tag{41}$$

Since f tends to a uniform value 0·436 in the central region ($r < \frac{1}{2}R$) and ψ tends to the value $\psi = 1\cdot 76\eta^{7\cdot 5}$, T tends to the value

$$\frac{T}{T_0} = \frac{ER^{-3}}{p_0}\frac{(0\cdot 133)(0\cdot 436)}{1\cdot 76}\eta^{-7\cdot 5} = 0\cdot 033\frac{T_0}{p_0}ER^{-3}\eta^{-7\cdot 5}. \tag{42}$$

Thus the temperature near the centre is very high; for instance, when the wave has expanded to such a distance that the pressure in the central region is reduced to atmospheric pressure, $p_0 = (0\cdot 133)(0\cdot 436)ER^{-3}$, then (42) gives $T/T_0 = \eta^{-7\cdot 5}/1\cdot 76$ and at $\eta = 0\cdot 5$, $\eta^{-7\cdot 5} = 181$ so that $T/T_0 = 103$. If $T_0 = 273°$, $T = 27{,}000°$. The temperature left behind by the blast wave is therefore very high, but the energy density is not high because the density of the gas is correspondingly low.

Heat energy left in the air after it has returned to atmospheric pressure

The energy available for doing mechanical work is less than the total heat energy of the air. The heated air left behind by the shock wave can in fact only do mechanical work by expanding down to atmospheric pressure, whereas to convert the whole of the heat energy into mechanical work by adiabatic expansion the air would have to be expanded to an infinite extent till the pressure was zero. After the blast wave has been propagated away and the air has returned to atmospheric pressure it is left at a temperature T_1, which is greater than T_0, the atmospheric temperature. The energy required to raise the temperature of air from T_0 to T_1 is therefore left in the atmosphere in a form in which it is not available for doing mechanical work directly on the surrounding atmosphere. This energy, the total amount of which will be denoted by E_1, is wasted as a blast-wave producer.

The energy so wasted at any stage of the disturbance can be calculated by finding the temperature T_1 to which each element of the blast wave would be reduced if it were expanded adiabatically to atmospheric pressure. If T is the temperature of an element of the blast wave

$$\frac{T}{T_1} = \left(\frac{p}{p_0}\right)^{(\gamma-1)/\gamma} = \left(\frac{A^2}{a^2}fR^{-3}\right)^{(\gamma-1)/\gamma}.$$

Also

$$\frac{T}{T_0} = \frac{p\rho_0}{p_0\rho} = \frac{f}{\psi}\frac{A^2}{a^2}R^{-3},$$

hence

$$\frac{T_1}{T_0} = \frac{f^{1/\gamma}}{\psi}\left(\frac{A^2R^{-3}}{a^2}\right)^{1/\gamma}. \tag{43}$$

The total heat energy per unit mass of air at temperature T_1 is

$$\frac{T_1}{\gamma-1}\times(\text{gas constant}) = \frac{T_1 p_0}{(\gamma-1)\rho_0 T_0}.$$

The increase in heat energy per unit mass over that which the air contained before the passage of the disturbance is therefore $\frac{p_0}{(\gamma-1)(\rho_0)}\left(\frac{T_1}{T_0}-1\right)$. The increase per unit volume of gas within the disturbed sphere is therefore $\frac{p_0\psi}{\gamma-1}\left(\frac{T_1}{T_0}-1\right)$. Hence from (43) the total energy wasted when the sphere has expanded to radius R is

$$E_1 = 4\pi R^3 \frac{p_0}{\gamma-1}\int_0^1 \left\{f^{1/\gamma}\left(\frac{A^2R^{-3}}{a^2}\right)^{1/\gamma} - \psi\right\}\eta^2\,d\eta. \tag{44}$$

This expression may conveniently be reduced to non-dimensional form by dividing by the total energy E of the explosion which is related to A by the formula (18). After inserting a^2/γ for p_0/ρ_0, this gives

$$\frac{E_1}{E} = \frac{4\pi}{B(\gamma-1)\gamma}\left(\frac{a^2}{A^2R^{-3}}\right)\left[\left(\frac{A^2R^{-3}}{a^2}\right)^{1/\gamma}\int_0^1 f^{1/\gamma}\eta^2\,d\eta - \int_0^1 \psi\eta^2\,d\eta\right]. \tag{45}$$

$\frac{4}{3}\pi\rho_0 R^3$ is the total mass of air in the sphere of radius R. This is also $4\pi R^3\rho_0\int_0^1 \psi\eta^2\,d\eta$, so that

$$\int_0^1 \psi\eta^2\,d\eta = \tfrac{1}{3}. \tag{46}$$

The quantity A^2R^{-3}/a^2 is related to the maximum pressure p_1 at the shock wave by the equation

$$y_1 = \frac{p_1}{p_0} = \frac{A^2R^{-3}}{a^2}[f]_{n=1},$$

where y_1 is the pressure in the shock wave expressed in atmospheres. (46) therefore reduces to

$$\frac{E_1}{E} = \frac{4\pi}{B\gamma(\gamma-1)\,y_1}[f]_{\eta=1}\left[y_1^{1/\gamma}\int_0^1\left[\frac{f}{f_{\eta=1}}\right]^{1/\gamma}\eta^2\,d\eta - \tfrac{1}{3}\right]. \tag{47}$$

For $\gamma = 1{\cdot}4$ numerical integration gives

$$B = 5{\cdot}36 \text{ (see (18))},\quad f_{n=1} = 1{\cdot}166,\quad \int_0^1 f^{1/\gamma}\eta^2\,d\eta = 0{\cdot}219.$$

(47) reduces therefore to

$$\frac{E_1}{E} = \frac{1}{y_1}[0{\cdot}958\,y_1^{1/1{\cdot}4} - 1{\cdot}63]. \tag{48}$$

Some values of E_1/E are given in the second line of table 3.

It is clear that E_1/E must continually increase as R increases, and y_1 decreases because the contribution to E_1 due to the air enclosed in the shock-wave surface when its radius is R_2, say, remains unchanged when this air subsequently expands. A further positive contribution to E_1 is made by each subsequent layer of air included

Table 3

y_1 (atm. at shock wave)	10,000	1,000	100	50	20	10*	5*
E_1/E (proportion of energy wasted)	0·069	0·132	0·240	0·281	0·325	0·333*	0·28*
$(E_1+E_2)/E$	0·096	0·189	0·337	0·393	0·455	—	—

* Formulae inaccurate when $y_1 < 10$.

within the disturbance. The fact that formula (48) gives a value of E_1/E which increases till y_1 is reduced to 10 and then subsequently decreases is due to the inaccuracy of the approximate boundary conditions (15a), (16a) and (17a), which are used to replace the true boundary conditions (15), (16) and (17).

When $\gamma = 1{\cdot}4$ and $y_1 = 10$ the true value of ρ_1/ρ_0 is 3·8 instead of 6·0 as is assumed, the true value of U^2/a^2 is 8·7 instead of 8·6 and the true value of u_1/U is 0·74 instead of 0·83.

When $y_1 = 5$ the errors are much larger, namely, ρ_1/ρ_0 is 2·8 instead of 6·0, U^2/a^2 is 4·4 instead of 4·3, and u_1/U is 0·64 instead of 0·83. The proportion of the energy wasted, namely, E_1/E, is shown as a function of y_1 in figure 3, y_1 being plotted on a logarithmic scale.

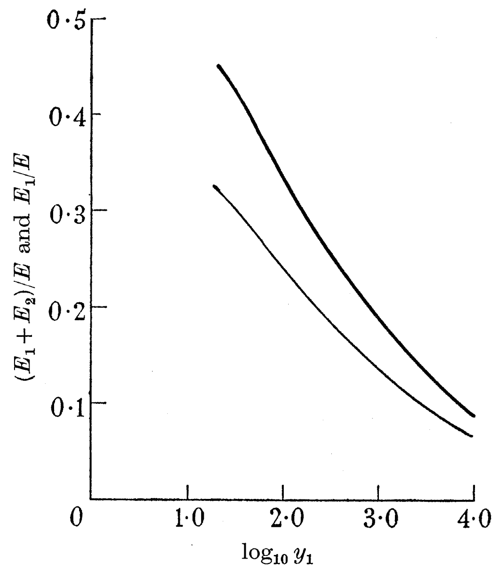

FIGURE 3. Heavy line, $(E_1+E_2)/E$; thin line E_1/E. $(E_1+E_2)/E$ is the proportion of the initial energy which is no longer available for doing work in propagation. E_2/E is the work done by heated air expanding against atmospheric pressure (see note added October 1949 (p. 172)).

It will be seen that the limiting value of E_1/E is certainly greater than 0·32, its value for $y_1 = 20$. It is not possible to find out how much greater without tracing the development of the blast wave using laborious step-by-step methods for values of η, less than, say, 10 or 20.

COMPARISON WITH HIGH EXPLOSIVES

The range within which any comparison between the foregoing theory and the blast waves close to actual high explosives can be made is severely limited. In the first place the condition that the initial disturbance is so concentrated that the mass of the material in which the energy is originally concentrated is small compared with the mass of the air involved in the disturbance at any time limits the comparable condition during a real explosion to one in which the whole mass of air involved is several times that of the explosive. In the second place the modified form of the shock-wave condition used in the analysis is only nearly correct when the rise in pressure at the shock-wave front is several—say at least 5 or 10—atmospheres. In a real explosive this limits the range of radii of shock wave over which comparison could be made to narrow limits. Thus with 10 lb. of C.E.* the radius R at which the weight of explosive is equal to that of the air in the blast wave is 3 ft., while at 3·8 ft. the air is only double the weight of the explosive. The pressure in the blast wave at a radius of 6 ft. was found to be 9 atm., while at 8 ft. it was about 5 atm. It seems, therefore, that in this case the range in which approximate agreement with the present theory could be expected only extends from 3·8 to 6 ft. from the 10 lb. charge.

Taking the energy released on exploding C.E. to be 0·95 kcal./g. the energy released when 10 lb. is exploded is $1{\cdot}8\times10^{14}$ ergs. If this energy had been released instantaneously at a point as in the foregoing calculations the maximum pressure at distance R given by (35) is

$$p_{\text{max.}}R^3 = (0{\cdot}155)\,(1{\cdot}80\times10^{14}) = 2{\cdot}79\times10^{13}\,\text{ergs}. \tag{49}$$

Expressed in terms of atmospheres $p_{\text{max.}}$ is identical with y_1. If R is expressed in feet, (49) becomes

$$y_1R^3 = \frac{2{\cdot}79\times10^{13}}{(30{\cdot}45)^3\times10^6} = 9{\cdot}9\times10^2. \tag{50}$$

The line representing this relationship on a logarithmic scale is shown in figure 4.

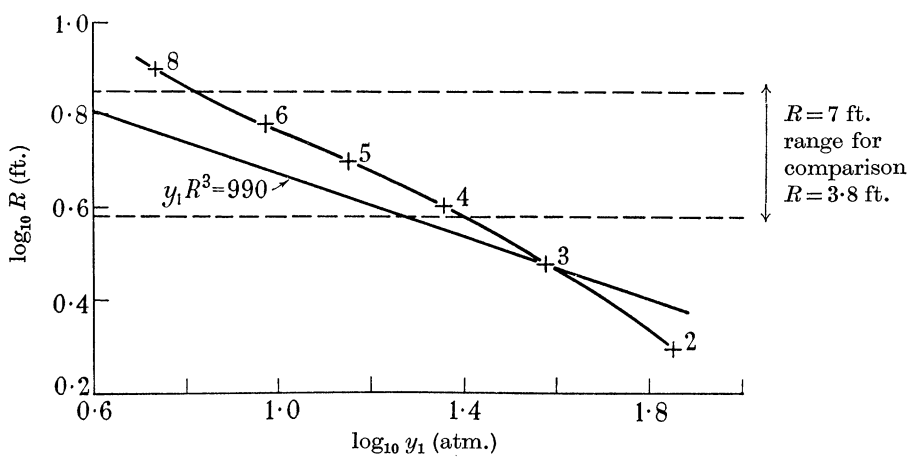

FIGURE 4. Blast pressures near 10 lb. charge of C.E. compared with calculated blast pressures due to instantaneous release of energy of 10 lb. C.E. at a point. The numbers against the points on the curve give distances in feet.

Though no suitable pressure measurements have been made, the maximum pressure in the blast from 10 lb. of C.E. has been found indirectly by observing the velocity of expansion of the luminous zone and, at greater radii, the blast-wave front. These values taken from a curve given in a report on some experiments made by the Road Research Laboratory are given in table 4. The observed values of U in ft./sec. given in column 2 of this table and the values of y_1 (in atmospheres) found from the

* C.E. is the name by which a certain high explosive used in many experiments by the Ministry of Home Security was known.

shock-wave formulae are given in column 3, where they are described as observed values though they were not observed directly. The 'observed' values are shown in figure 4. The values of y_1 calculated from (50) are given in column 4.

TABLE 4

R (ft.)	U (ft./sec.)	y_1 (atm.) observed with C.E.	y_1 calculated for concentrated explosion by (50)
8	2350	6·2	—
6	3100	9·3	4·6 } range of comparison
5	3800	14·0	7·9 } range of comparison
4	4820	22·6	15·5 } range of comparison
3	6200	37·5	—
2	8540	71·8	—

Though the observed values are higher than those calculated, it will be noticed that in the range of radii 3·8 to 6 ft., in which comparison can be made, the observed curve is nearly parallel to the theoretical line $y_1 R^3 = 990$. In this range, therefore, the intensity of the shock wave varies nearly as the inverse cube of the distance from the explosion. The fact that the observed values are about twice as great as those calculated on the assumption that the energy is emitted instantaneously at a point may perhaps be due to the fact that the measurements used in table 4 correspond with conditions on the central plane perpendicular to the axis of symmetry of the cylindrical charge used. The velocity of propagation of the luminous zone is greater on this plane and on the axis of symmetry than in other radial directions so that the pressures deduced in column 3 of table 4 are greater than the mean pressures at the corresponding radii.

On the other hand, it has been seen that by the time the maximum pressure has fallen to 20 atm., 32 % of the energy has been left behind in the neighbourhood of the concentrated explosive source, raising the air temperature there to very high values. The burnt gases of a real high explosive are at a very much lower temperature even while they are at the high pressure of the detonation wave. Their temperature is still lower when they have expanded adiabatically to atmospheric pressure, so that little heat energy is left in them. To this extent, therefore, a real high explosive may be expected to be more efficient as a blast producer than the theoretical infinitely concentrated source here considered.

Note added, October 1949. The data on which the comparison was based between the pressures deduced by theory and those observed near detonating explosives were obtained in 1940. More recent data obtained at the Road Research Laboratory using a mixture of the two explosives R.D.X. and T.N.T. have been given by Dr Marley. These are given in table 5, which shows the values of U observed for various

TABLE 5. PRESSURE $y_1 p_0$ AT DISTANCE R FROM EXPLOSION OF WEIGHT W OF T.N.T.-R.D.X. MIXTURE

$R/W^{\frac{1}{3}}$ (ft./lb.$^{\frac{1}{3}}$)	0·5	1·0	1·5	2·0	2·5	3·0	3·5
U (thousands ft./sec.)	14·3	11·0	8·4	6·6	5·1	4·0	3·3
y_1 (atm.)	198	117	68·3	42·0	25·1	15·4	10·5
$R/E^{\frac{1}{3}} \times 10^4$(cm./ergs$^{\frac{1}{3}}$)	5·39	10·8	16·2	21·5	27·0	32·4	37·8

values of $R/W^{\frac{1}{3}}$. R the distance from the explosive is expressed in feet and W its weight in pounds. The third line in table 5 shows the result of deducing y_1 from U using $\gamma = 1{\cdot}4$ in (16) and $a = 1100$ ft./sec. in (16).

For comparison with the concentrated point-source explosion, the value of $RE^{-\frac{1}{3}}$ expressed in cm. (erg.)$^{-\frac{1}{3}}$ is found by multiplying the figures in line 1, table 5, by $\frac{30{\cdot}4}{(454)^{\frac{1}{3}}} \cdot \frac{1}{(1200 \times 4{\cdot}2 \times 10^7)^{\frac{1}{3}}} = 1{\cdot}078 \times 10^{-3}$. The first factor converts ft. (lb.)$^{-\frac{1}{3}}$ to cm. (g.)$^{-\frac{1}{3}}$, and the second replaces 1 g. by the equivalent energy released by this explosive, namely, 1200 cal. The values of $RE^{-\frac{1}{3}}$ are given in line 4, table 5. In figure 5 values of $\log_{10} y_1$ are plotted against $\log_{10} RE^{-\frac{1}{3}}$, and the theoretical values for a point source of the same energy as the chemical explosive are plotted in the same diagram. Comparing figures 4 and 5 it seems that the more recent shock-wave velocity results are qualitatively similar to the older ones in their relation to the point-source theory. The range of values of y_1 for which comparison between theory and observation might be significant, is marked in figure 5.

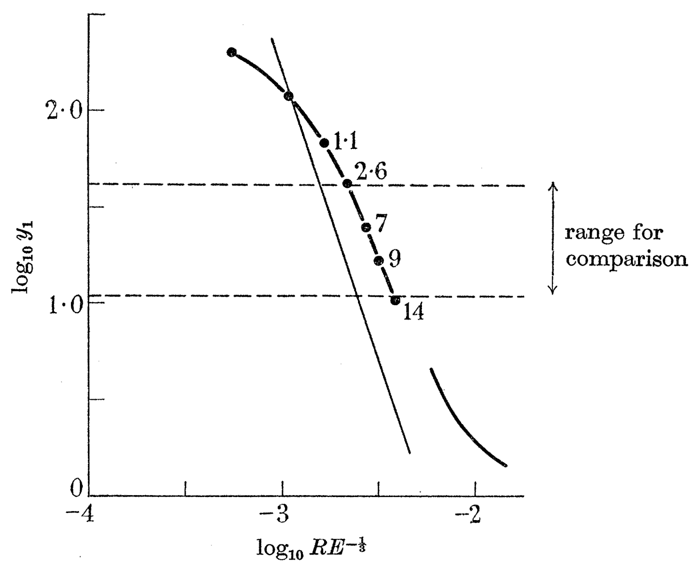

FIGURE 5. Blast pressures near a chemical explosive (R.D.X. + T.N.T.) compared with theoretical pressure for concentrated explosion with same release of energy. Heavy line (upper part) is taken from shock-wave velocity measurements. Heavy line (lower part) is from piezo-electric crystals. Thin line, $y_1 = 0{\cdot}155E/(p_0R^3)$. The figures against the points represent the ratio of the mass of the air within the shock wave to the mass of the explosive.

It will be seen that the chemical explosive is a more efficient blast producer than a point source of the same energy. The ratio of the pressures in the range of comparison is about 3 to 1. This is more than might be expected in view of the calculation of E_1/E as a function of y_1 which is given in table 3. E_1 is the heat energy which is unavailable for doing mechanical work after expanding to pressure p_0. Of the remaining energy, $E - E_1$ a part E_2 is used in doing work against atmospheric pressure during the expansion of the heated air. The remaining energy, namely, $E - E_1 - E_2$, is available for propagating the blast wave.

To find E_2, the work done by unit volume of the gas at radius ηR in expanding to atmospheric pressure is $\left(\frac{T_1 p}{T p_0} - 1\right) p_0$. From (43)

$$\frac{T_1}{T} = \left(\frac{A^2}{a^2} R^{-3} f\right)^{(1-\gamma)/\gamma} \quad \text{and} \quad \frac{p}{p_0} = f\frac{A^2 R^{-3}}{a^2},$$

hence
$$E_2 = 4\pi R^3 p_0 \int_0^1 \left\{\left(R^{-3} f \frac{A^2}{a^2}\right)^{1/\gamma} - 1\right\} \eta^2 \, d\eta, \tag{51}$$

but $\frac{A^2}{a^2 R^3} = \frac{y_1}{(f)_{\eta=1}}$, so that

$$\frac{E_2}{E} = \frac{4\pi R^3 p_0}{E}\left[\left(\frac{y_1}{f_{\eta=1}}\right)^{1/\gamma} \int_0^1 f^{1/\gamma} \eta^2 \, d\eta - \int_0^1 \eta^2 \, d\eta\right]. \tag{52}$$

The first integral has already been calculated and found to be 0·219 when $\gamma = 1{\cdot}4$ (see (47) and (48)). Substituting for $p_{\text{max.}}$ from (35),

$$\frac{E_2}{E} = 4\pi(0{\cdot}155)\left[\frac{0{\cdot}219 y_1^{(1-\gamma)/\gamma}}{(1{\cdot}166)^{1/\gamma}} - \frac{1}{3y_1}\right]. \tag{53}$$

Values of $(E_2 + E_1)/E$ have been added as a third line in table 3 and a corresponding curve to figure 3.

References

Taylor, Sir Geoffrey 1946 *Proc. Roy. Soc.* A, **186**, 273.
Taylor, Sir Geoffrey 1950 *Proc. Roy. Soc.* A, **201**, 175.