The Formation of a Blast Wave by a Very Intense Explosion. II. The Atomic Explosion of 1945
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The formation of a blast wave by a very intense explosion. II. The atomic explosion of 1945

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[Plates 7 to 9]

Photographs by J. E. Mack of the first atomic explosion in New Mexico were measured, and the radius, R, of the luminous globe or 'ball of fire' which spread out from the centre was determined for a large range of values of t, the time measured from the start of the explosion. The relationship predicted in part I, namely, that $R^{\frac{5}{2}}$ would be proportional to t, is surprisingly accurately verified over a range from $R=20$ to 185 m. The value of $R^{\frac{5}{2}}t^{-1}$ so found was used in conjunction with the formulae of part I to estimate the energy E which was generated in the explosion. The amount of this estimate depends on what value is assumed for γ, the ratio of the specific heats of air.

Two estimates are given in terms of the number of tons of the chemical explosive T.N.T. which would release the same energy. The first is probably the more accurate and is 16,800 tons. The second, which is 23,700 tons, probably overestimates the energy, but is included to show the amount of error which might be expected if the effect of radiation were neglected and that of high temperature on the specific heat of air were taken into account. Reasons are given for believing that these two effects neutralize one another.

After the explosion a hemispherical volume of very hot gas is left behind and Mack's photographs were used to measure the velocity of rise of the glowing centre of the heated volume. This velocity was found to be 35 m./sec.

Until the hot air suffers turbulent mixing with the surrounding cold air it may be expected to rise like a large bubble in water. The radius of the 'equivalent bubble' is calculated and found to be 293 m. The vertical velocity of a bubble of this radius is $\frac{2}{3}\sqrt{(g\,29300)}$ or 35·7 m./sec. The agreement with the measured value, 35 m./sec., is better than the nature of the measurements permits one to expect.

Comparison with photographic records of the first atomic explosion

Two years ago some motion picture records by Mack (1947) of the first atomic explosion in New Mexico were declassified. These pictures show not only the shape of the luminous globe which rapidly spread out from the detonation centre, but also gave the time, t, of each exposure after the instant of initiation. On each series of photographs a scale is also marked so that the rate of expansion of the globe, or 'ball of fire', can be found. Two series of declassified photographs are shown in figure 6, plate 7.

These photographs show that the ball of fire assumes at first the form of a rough sphere, but that its surface rapidly becomes smooth. The atomic explosive was fired at a height of 100 ft. above the ground and the bottom of the ball of fire reached the ground in less than 1 msec. The impact on the ground does not appear to have disturbed the conditions in the upper half of the globe which continued to expand as a nearly perfect luminous hemisphere bounded by a sharp edge which must be taken as a shock wave. This stage of the expansion is shown in figure 7, plate 8 which corresponds with $t = 15$ msec. When the radius R of the ball of fire reached about 130 m., the intensity of the light was less at the outer surface than in the interior. At later times the luminosity spread more slowly and became less sharply defined, but a sharp-edged dark sphere can be seen moving ahead of the luminosity. This must be regarded as showing the position of the shock wave when it ceases to be luminous. This stage is shown in figure 8, plate 9, taken at $t = 127$ msec. It will be seen that the edge of the luminous area is no longer sharp.

The measurements given in column 3 of table 1 were made partly from photographs in Mack (1947), partly from some clearer glossy prints of the same photographs kindly sent to me by Dr N. E. Bradbury, Director of Los Alamos Laboratory and partly from some declassified photographs lent me by the Ministry of Supply. The times given in column 2 of table 1 are taken directly from the photographs.

Table 1. Radius R of blast wave at time t after the explosion

authority	t (msec.)	R (m.)	$\log_{10} t$	$\log_{10} R$	$\frac{5}{2}\log_{10} R$
strip of small images MDDC 221	0·10	11·1	$\bar{4}$·0	3·045	7·613
	0·24	19·9	$\bar{4}$·380	3·298	8·244
	0·38	25·4	$\bar{4}$·580	3·405	8·512
	0·52	28·8	$\bar{4}$·716	3·458	8·646
	0·66	31·9	$\bar{4}$·820	3·504	8·759
	0·80	34·2	$\bar{4}$·903	3·535	8·836
	0·94	36·3	$\bar{4}$·973	3·560	8·901
strip of declassified photographs lent by Ministry of Supply	1·08	38·9	$\bar{3}$·033	3·590	8·976
	1·22	41·0	$\bar{3}$·086	3·613	9·032
	1·36	42·8	$\bar{3}$·134	3·631	9·079
	1·50	44·4	$\bar{3}$·176	3·647	9·119
	1·65	46·0	$\bar{3}$·217	3·663	9·157
	1·79	46·9	$\bar{3}$·257	3·672	9·179
	1·93	48·7	$\bar{3}$·286	3·688	9·220
strip of small images from MDDC 221	3·26	59·0	$\bar{3}$·513	3·771	9·427
	3·53	61·1	$\bar{3}$·548	3·786	9·466
	3·80	62·9	$\bar{3}$·580	3·798	9·496
	4·07	64·3	$\bar{3}$·610	3·809	9·521
	4·34	65·6	$\bar{3}$·637	3·817	9·543
	4·61	67·3	$\bar{3}$·688	3·828	9·570
large single photographs MDDC 221	15·0	106·5	$\bar{2}$·176	4·027	10·068
	25·0	130·0	$\bar{2}$·398	4·114	10·285
	34·0	145·0	$\bar{2}$·531	4·161	10·403
	53·0	175·0	$\bar{2}$·724	4·243	10·607
	62·0	185·0	$\bar{2}$·792	4·267	10·668

To compare these measurements with the analysis given in part I of this paper, equation (38) was used. It will be seen that if the ball of fire grows in the way contemplated in my theoretical analysis, $R^{\frac{5}{2}}$ will be found to be proportional to t. To find out how far this prediction was verified, the logarithmic plot of $\frac{5}{2}\log R$ against $\log t$ shown in figure 1 was made. The values from which the points were plotted are given in table 1. It will be seen that the points lie close to the 45° line which is drawn in figure 1. This line represents the relation

$$\tfrac{5}{2}\log_{10} R - \log_{10} t = 11{\cdot}915. \tag{1}$$

The ball of fire did therefore expand very closely in accordance with the theoretical prediction made more than four years before the explosion took place. This is surprising, because in those calculations it was assumed that air behaves as though γ, the ratio of the specific heats, is constant at all temperatures, an assumption which is certainly not true.

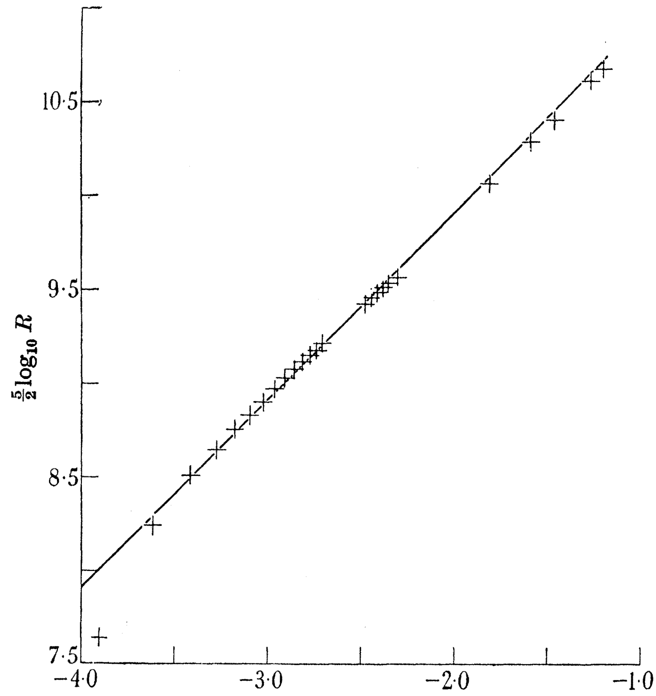

FIGURE 1. Logarithmic plot showing that $R^{\frac{5}{2}}$ is proportional to t.

At room temperatures $\gamma = 1{\cdot}40$ in air, but at high temperatures γ is reduced owing to the absorption of energy in the form of vibrations which increases C_v. At very high temperatures γ may be increased owing to dissociation. On the other hand, the existence of very intense radiation from the centre and absorption in the outer regions may be expected to raise the apparent value of γ. The fact that the observed value of R^5t^{-2} is so nearly constant through the whole range of radii covered by the photographs of the ball of fire suggests that these effects may neutralize one another, leaving the whole system to behave as though γ has an effective value identical with that which it has when none of them are important, namely, 1·40.

CALCULATION OF THE ENERGY RELEASED BY THE EXPLOSION

The straight line in figure 1 corresponds with

$$R^5t^{-2} = 6{\cdot}67 \times 10^2\,(\text{cm.})^5\,(\text{sec.})^{-2}. \tag{2}$$

The energy, E, is then from equation (18) of part I

$$E = \rho_0 A^2 \left\{ 2\pi I_1 + \frac{4\pi}{\gamma(\gamma-1)} I_2 \right\}, \tag{3}$$

where $$I_1 = \int_0^1 \psi\phi^2\eta^2\,d\eta \tag{4}$$

and $$I_2 = \int_0^1 f\eta^2\,d\eta, \tag{5}$$

where f, ϕ and ψ are non-dimensional quantities proportional to pressure, velocity and density which are defined in part I.

A is found by integrating equation (6) of part I, so that

$$A = \tfrac{2}{5}R^{\frac{5}{2}}t^{-1}. \tag{6}$$

Writing $$K = \frac{4}{25}\left\{2\pi I_1 + \frac{4\pi}{\gamma(\gamma-1)}I_2\right\}, \tag{7}$$

$$E = K\rho_0 R^5 t^{-2}. \tag{8}$$

It was shown in part I that I_1 and I_2 are functions of γ only. For $\gamma = 1{\cdot}40$, their values were found to be $I_1 = 0{\cdot}185$, $I_2 = 0{\cdot}187$, using step-by-step methods for integrating the equations connecting f, ϕ and ψ. Using the approximate formulae (22) to (30) of part I, values of f, ϕ and ψ were calculated in part I for $\gamma = 1{\cdot}667$ and are there given in table 2 of part I. Further calculations have now been made for $\gamma = 1{\cdot}20$ and $\gamma = 1{\cdot}30$ using the approximate formulae. The results for $\gamma = 1{\cdot}30$ are given in table 2 and are shown in figure 2. The corresponding values of I_1, I_2 and K are given in table 3; K is shown as a function of γ in figure 3.

TABLE 2. CALCULATION FOR $\gamma = 1{\cdot}30$ USING FORMULAE (22) to (30) OF PART I

η	f	ϕ	ψ	T/T_1
1·0	1·130	0·869	7·667	1·00
0·98	0·896	0·833	4·534	1·34
0·96	0·772	0·801	2·989	1·75
0·94	0·666	0·772	2·067	2·19
0·92	0·606	0·745	1·454	2·73
0·90	0·563	0·721	1·058	3·61
0·85	0·499	0·669	0·509	6·76
0·80	0·468	0·623	0·255	12·5
0·75	0·453	0·580	0·128	24·0
0·70	0·445	0·540	0·063	48·0
0·65	0·441	0·501	0·029	103
0·60	0·439	0·462	0·013	229
0·55	0·438	0·423	0·006	—
0·50	0·438	0·386	0·002	—
0·45	0·438	0·347	0·001	—
0·40	0·438	0·309	0·000	—

ENERGY OF THE FIRST ATOMIC EXPLOSION IN NEW MEXICO

Having determined R^5t^{-2} and assuming that ρ_0 may be taken as $1{\cdot}25 \times 10^{-3}$ g./cm.^{3}, the figures in table 3 were used to determine E from equations (8) and (2). Different values are found for different assumed values of γ. These are given, expressed in ergs, in line 5 of table 3. It has become customary to describe large explosions by stating the weight of T.N.T. which would liberate the same amount of energy. Taking 1 g.

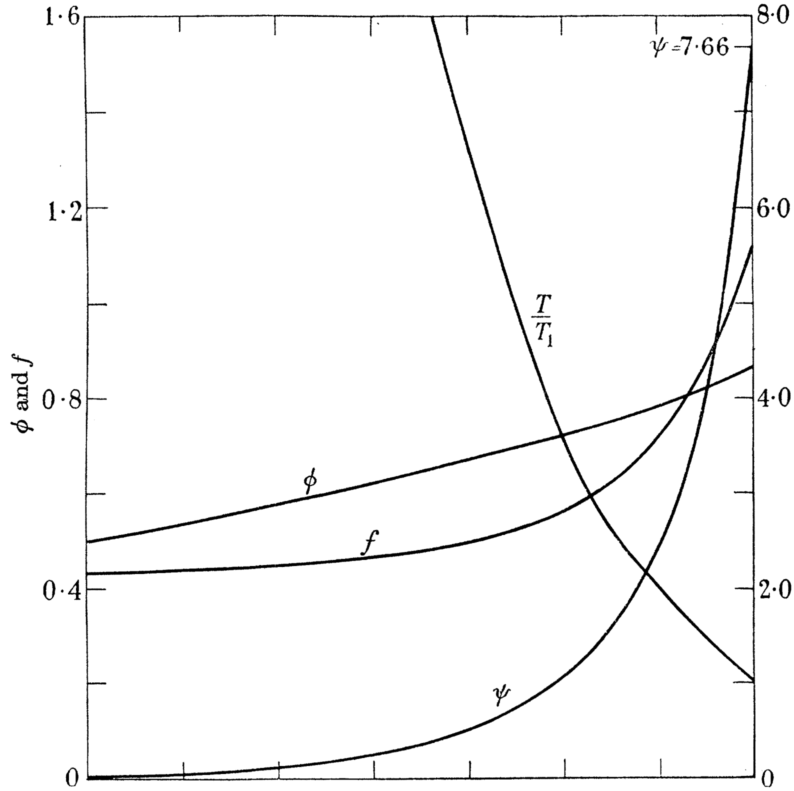

FIGURE 2. Distribution of radial velocity ϕ, pressure f, density ψ and temperature T/T_1 for $\gamma = 1{\cdot}30$ expressed in non-dimensional form.

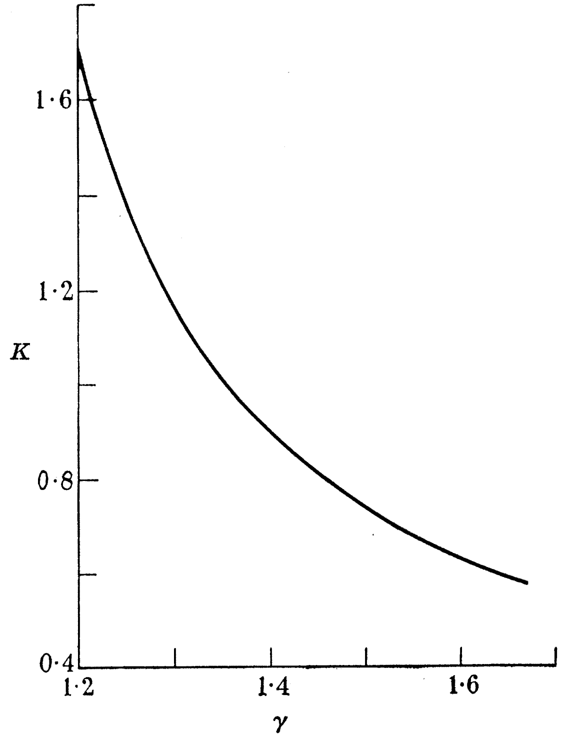

FIGURE 3. Variation of K with γ.

of T.N.T. as liberating 1000 calories or $4{\cdot}18\times10^{10}$ ergs, 1 ton will liberate $4{\cdot}25\times10^{16}$ ergs. The T.N.T. equivalents found by dividing the figures given in line 5 of table 3 by $4{\cdot}25\times10^{16}$ are given in line 6.

TABLE 3. CALCULATED CONSTANTS USED IN DETERMINING THE ENERGY OF THE EXPLOSION WITH A RANGE OF ASSUMED VALUES FOR γ

γ	1·20	1·30	1·40	1·667
I_1	0·259	0·221	0·185	0·123
I_2	0·175	0·183	0·187	0·201
K	1·727	1·167	0·856	0·487
$E\times10^{-20}$ erg	14·4	9·74	7·14	4·06
T.N.T. equivalent (tons)	34,000	22,900	16,800	9,500

It will be seen that if $\gamma = 1{\cdot}40$, the T.N.T. equivalent of the energy of the New Mexico explosion, or more strictly that part of the energy which was not radiated outside the ball of fire, was *16,800 tons*.

AN ALTERNATIVE POSSIBILITY

If the effect of radiation, which cannot be estimated, is disregarded, a mean value of γ might be taken which is appropriate to the temperature calculated to correspond with the mean value of R in the range over which $R^{\frac{5}{2}}t^{-1}$ is nearly constant. The least value of R which lies near the line in figure 1 is $R = 20$ m. and the greatest is $R = 185$ m. The mean value is therefore approximately 100 m. It will be found that the value of γ appropriate to the temperature behind the shock wave at 100 m. is about 1·3. The pressure at any point is from equations (1), (6) and (12) of part I

$$p = p_0R^{-3}f_1 = p_0R^{-3}\frac{A^2}{a^2}f = \frac{4}{25}\frac{\rho_0}{\gamma}R^{-3}f(R^5f^{-2}).$$

For $\gamma = 1{\cdot}3$, the value of f behind the detonation front is 1·13, so that

$$p = 9{\cdot}3\times10^{22}\rho_0R^{-3}.$$

The temperature T_1 at that point is given by

$$\frac{T_1}{T_0} = \frac{p}{\rho}\frac{\rho_0}{p_0},$$

where T_0 is the undisturbed atmospheric temperature. When

$\gamma = 1{\cdot}3$, $\rho_0/\rho = 1/\psi = 1/7{\cdot}667$, and if $\rho_0 = 0{\cdot}00125$, $p/p_0 = 7{\cdot}43\times10^{13}R^{-3}$.

Thus
$$\frac{T_1}{T_0} = 0{\cdot}97\times10^{13}R^{-3}. \tag{9}$$

At $R = 100$ m., $T_1/T_0 = 9{\cdot}7$, so that if $T_0 = 15^\circ$ C $= 288^\circ$ K, $T_1 = 2800^\circ$ K.

Values of C_p at temperatures up to 5000° K have been calculated for nitrogen by Johnston & Davis (1934) and for oxygen by Johnston & Walker (1935). At 2800° K, C_p is given for nitrogen as 8·82 and for oxygen as 9·43, so that for air $C_p = 8{\cdot}92$. Since there is little dissociation at that temperature, it seems that $C_v = C_p - R = 6{\cdot}92$

and $\gamma = \dfrac{8\cdot92}{6\cdot92} = 1\cdot29$. Thus the use of $\gamma = 1\cdot30$ for calculating the temperature at $R = 100\,\text{m.}$ is justified when effects of radiation are neglected.

Using the curve, figure 3, it seems that the value of K appropriate to $\gamma = 1\cdot29$ is $1\cdot21$. Using $\rho_0 = 0\cdot00125$ and $R^5t^{-2} = 6\cdot67 \times 10^{23}$, equation (8) gives $E = 1\cdot01 \times 10^{21}$ ergs and the T.N.T. equivalent is *23,700 tons*.

Of these two alternative estimates, it seems that the first, namely, 16,800 tons, is the more likely to be accurate.

Some dynamical features of the atomic explosion

It will be seen in figure 1 of part I that if $\gamma = 1\cdot4$ the air density is a maximum at the shock wave front where it reaches six times atmospheric density. Within the shock wave, the density falls rapidly till at a radius of about $0\cdot6R$ it is nearly zero. Within the radius $0\cdot6R$ the gas has a radial velocity which is proportional to the distance from the centre, a very high temperature, and a uniform pressure about $0\cdot43$ time the maximum pressure.

The maximum pressure at the shock front is found by inserting the value of E from line 5 of table 3 in the formula (35) of part I. The pressure—expressed in atmospheres—is

$$\frac{p}{p_0} = 0\cdot155R^{-3}(7\cdot14 \times 10^{20}) \times 10^{-6} = 1\cdot11 \times 10^{14}R^{-3}. \tag{10}$$

At $R = 30\,\text{m.}$ this is 4100 atm., or 27 tons/sq.in. At $R = 100\,\text{m.}$ the pressure would be 1 ton/sq.in. These pressures are much less than would act on rigid bodies exposed to such blasts, but the pressures on obstacles depend on their shape so that no general statement can be made on this subject.

The temperature rises rapidly as the centre is approached, in fact the ratio T/T_1, T_1 being the temperature just inside the shockwave, is equal to

$$\frac{p_1}{p}\left(\frac{\rho_1}{\rho}\right) = \frac{p}{p_0}\frac{p_0}{p_1}\frac{\rho_1}{\rho_0}\frac{\rho_0}{\rho} = \left(\frac{f}{f_{\eta=1}}\right)\left(\frac{\psi_{\eta=1}}{\psi}\right). \tag{11}$$

The values of T/T_1 for $\gamma = 1\cdot3$, namely $\dfrac{7\cdot66\,f}{1\cdot13\,\psi}$, are given in the fifth column of table 2, and are shown in figure 2.

The initial rate of rise of air from the seat of the explosion

When the shock wave had passed away from the ball of fire, it left a cloud of very hot air which then rapidly rose. Mack (1947, p. 37) gives a rough picture of the process in a series of diagrams representing the outlines of the boundary of the heated region, so far as his photographs could define them, at successive times from $t = 0\cdot1$ to $t = 15\cdot0\,\text{sec.}$ It is not possible to know exactly what these outlines represent, though in the later numbers of the series they seem to show the limits of the region to which dust thrown off from the ground and sucked into the ascending column of air has penetrated. This dust rapidly expands into a roughly spherical shape owing to turbulent diffusion or convection currents in the central region. The radius of the outer edge of the glowing region is not the same in all photographs taken nearly simultaneously, but the height of its apparent centre seems to be consistent when photographs taken simultaneously from different places are compared.

The heights, h, of the top of the illuminated column and their radii, b, were determined, so far as was possible, from Mack's published photographs. These rough measurements are given in table 4. The height of the centre of the glowing area is taken as $h-b$, and points corresponding with those given in table 4 are plotted in figure 4. It will be seen that the centre of the glowing volume seems to rise at a regular rate. The line drawn in figure 4 corresponds with a vertical velocity of

$$U = 35\,\text{m./sec.} \tag{12}$$

TABLE 4. HEIGHT, h, AND RADIUS, b, OF THE GLOWING REGION FROM $3\frac{1}{2}$ TO 15 SEC. AFTER THE EXPLOSION

authority	t (sec.)	b (radius) (m.)	h (height of top)	$h-b$ (height of centre)
Mack MDDC 221, sketches on p. 37	3·5	160	375	215
	8·0	240	688	448
	10·0	300	810	510
	15·0	360	1060	700
photograph on p. 38	14·8	550	1200	650

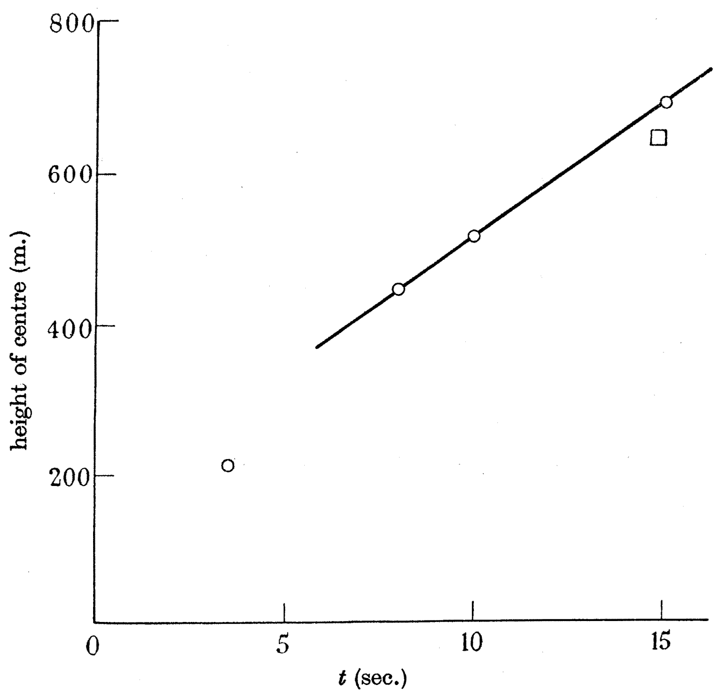

FIGURE 4. Height of centre of glowing region from $3\frac{1}{2}$ to 15 sec. after the explosion. ○ from diagram p. 37, □ from photograph p. 38 of MDDC 221.

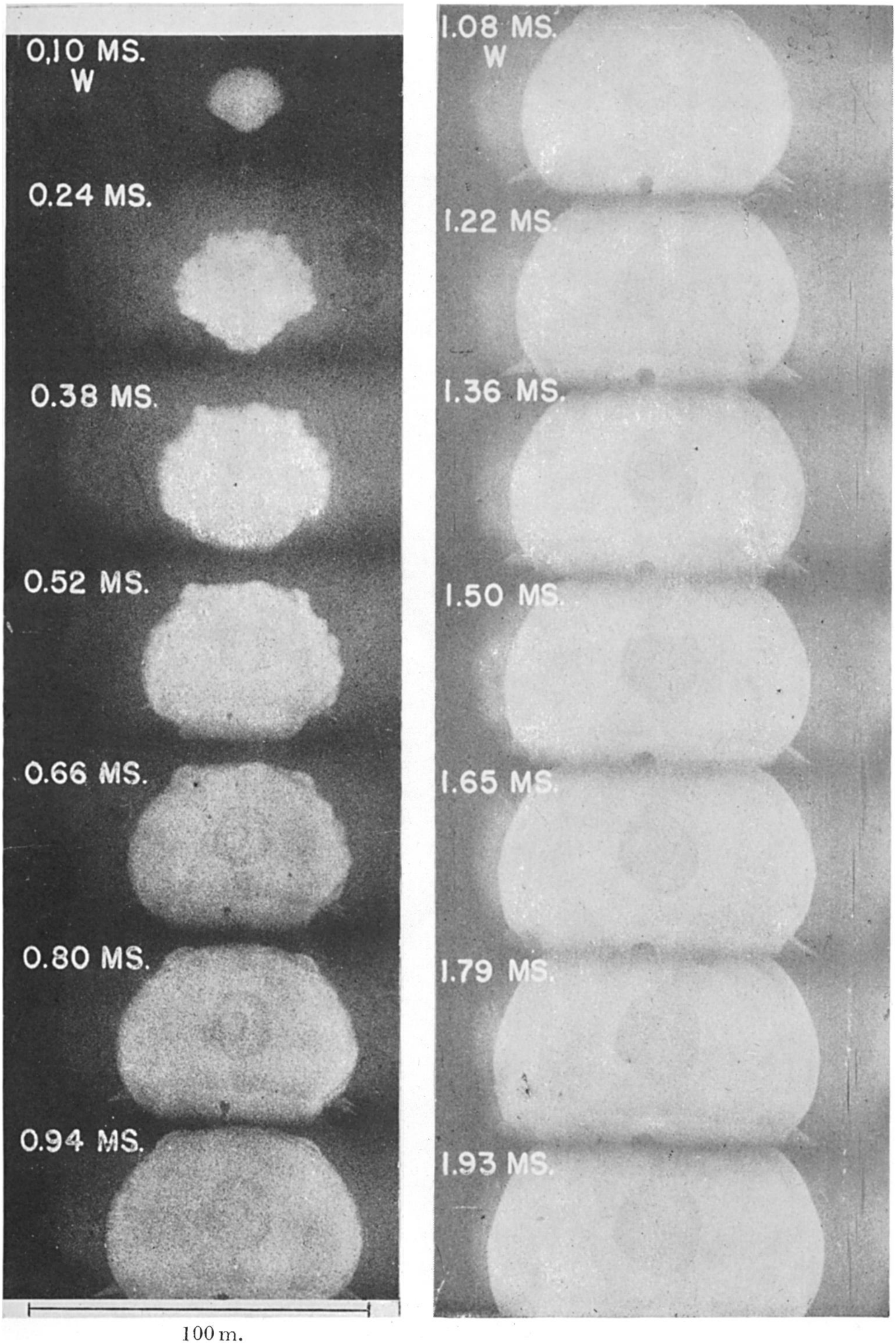

FIGURE 6. Succession of photographs of the 'ball of fire' from $t = 0{\cdot}10$ msec. to $1{\cdot}93$ msec.

(*Facing p.* 182)

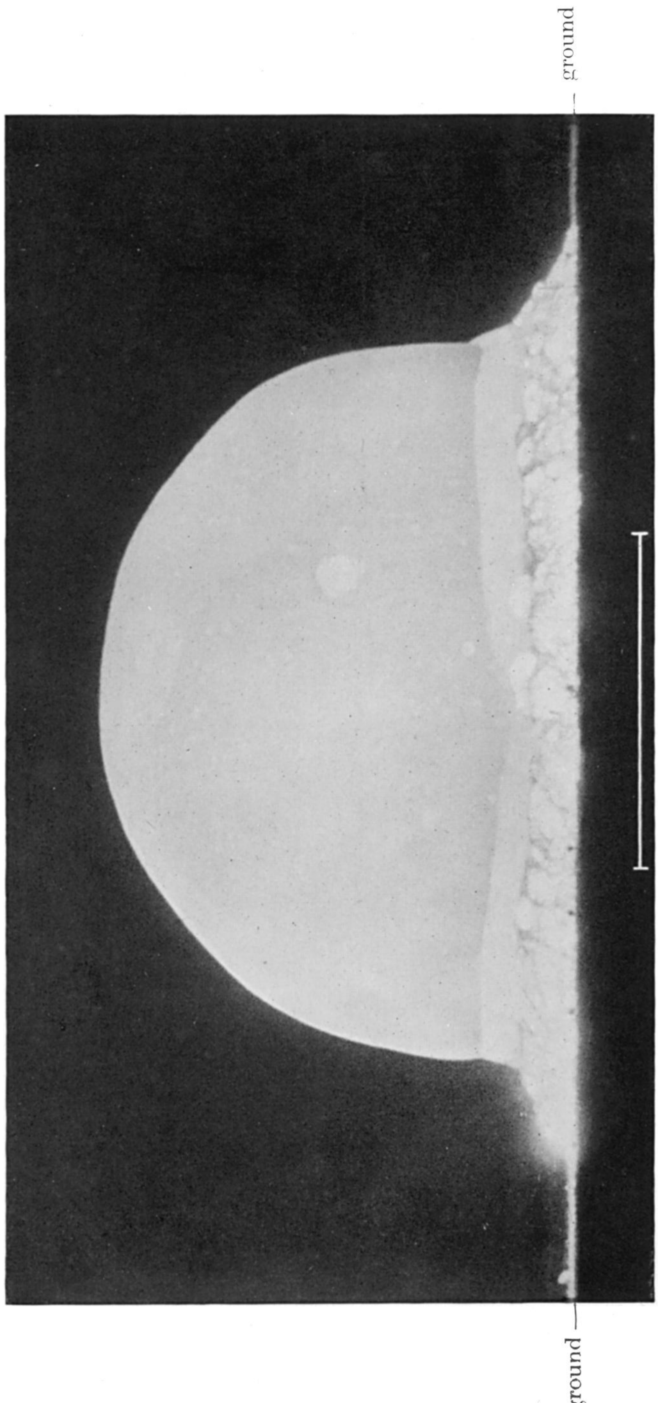

FIGURE 7. The ball of fire at $t = 15$ msec., showing the sharpness of its edge.

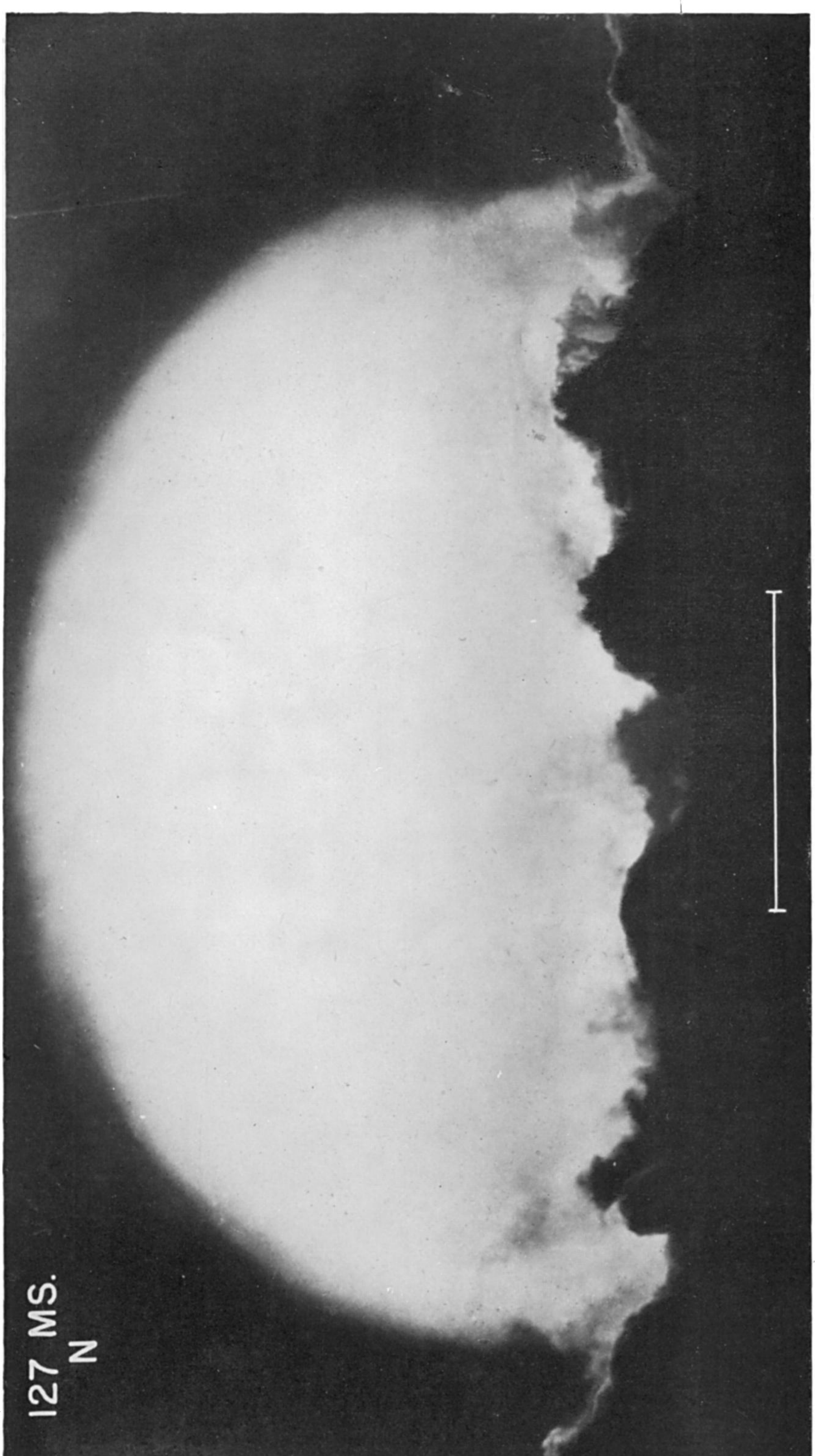

FIGURE 8. The glowing volume $t = 127$ msec. showing an indefinite edge and hemispherical shape.

Distribution of air density after the explosion

To give a dynamical description of the rise of hot air from the seat of the explosion it is first necessary to know the distribution of density immediately after the shock wave has passed away and left hot air at atmospheric pressure. Formulae are given in part I for the temperature ultimately attained by air which passed through the shock wave when its pressure was $y_1 p_0$, but the position at which this air comes to rest when atmospheric pressure was attained was not discussed. If the ground had not obstructed the blast wave the distribution of temperature would evidently be spherical. It has been pointed out, however, that the shape of the upper half of the blast wave has not been affected by the presence of the ground. The same thing seems to be approximately true of the temperature distribution, for Mack publishes a photograph showing the luminous volume at $t = 0{\cdot}127$ sec. when the shock wave had moved well away from the very hot area. This is reproduced in figure 8, plate 9. It will be seen that the glowing air occupies a nearly hemispherical volume, the bottom half of the sphere being below the ground. It seems that it may be justifiable to assume that most of the energy associated with the part of the blast wave which strikes the ground is absorbed there. In that case we may neglect the effect of shock waves reflected from the ground and consider the temperature distribution as being that calculated in part I for an unobstructed wave. With this assumption the distribution of density will be calculated.

The following symbols will be used:

T_0, p_0, ρ_0, the temperature, pressure and density in the undisturbed air,

T, $y_1 p_0$, ρ, R, the temperature, pressure, density and radius at the shock wave,

T_1, ρ_1, r, the temperature, density and radius after the pressure has become atmospheric.

From (43), part I,
$$\frac{T}{T_1} = y_1^{(\gamma-1)/\gamma} = \left(\frac{A^2 f R^{-3}}{a^2}\right)^{(\gamma-1)/\gamma} \tag{13}$$

and
$$\frac{T}{T_0} = \frac{f}{\psi}\frac{A^2}{a^2}R^{-3}, \tag{14}$$

so that
$$\frac{T_1}{T_0} = \frac{T}{T_1}\frac{T_0}{T} = \frac{1}{\psi}\left(\frac{A^2 f R^{-3}}{a^2}\right), \tag{15}$$

and since
$$\frac{A^2}{a^2} = \frac{4E}{25K\gamma p_0}, \tag{16}$$

$$\frac{T_1}{T_0} = \beta\left(\frac{R}{r_0}\right)^{-3/\gamma},$$

where
$$\beta = \frac{1}{\psi}\left(\frac{4f}{25K\gamma}\right)^{1/\gamma}, \tag{17}$$

and r_0 is the length defined by
$$r_0^3 = E/p_0. \tag{18}$$

r_0 is introduced in order to make the equations non-dimensional, ψ and f have their values at the shock wave front. When $\gamma = 1{\cdot}40, f = 1{\cdot}167, \psi = 6{\cdot}0; K = 0{\cdot}856$ (see table 3), so that
$$\beta = 0{\cdot}044, \tag{19}$$

(16) may be written

$$\left(\frac{R}{r_0}\right)^3 = \beta^\gamma\left(\frac{T_0}{T_1}\right)^\gamma = 0\cdot1265\left(\frac{T_0}{T_1}\right)^\gamma \quad \text{when} \quad \gamma = 1\cdot40. \tag{20}$$

To find the values of r and T_1/T_0 corresponding with a given value of y_1 the equation of continuity must be used. This is

$$\rho_0 R^2 dR = \rho_1 r^2 dr, \tag{21}$$

and since $T_0/T_1 = \rho_1/\rho_0$, (21) can be integrated after substitution from (20), thus

$$\left(\frac{r}{r_0}\right)^3 = \beta^\gamma\left(\frac{\gamma}{\gamma-1}\right)\left(\frac{T_0}{T_1}\right)^{\gamma-1}, \tag{22}$$

and from (35) of part I and (20)

$$\left(\frac{R}{r_0}\right)^3 = \frac{0\cdot155}{y_1} = \beta^\gamma\left(\frac{T_0}{T_1}\right)^\gamma. \tag{23}$$

Eliminating T_0/T_1 from (22) and (23) and using values appropriate to $\gamma = 1\cdot40$

$$\left(\frac{r}{r_0}\right)^3 = 0\cdot0906 y_1^{-0\cdot2857} \tag{24}$$

and

$$\frac{\rho_0}{\rho_1} = \frac{T_1}{T_0} = 0\cdot167 y_1^{0\cdot7143}. \tag{25}$$

Values of R/r_0, ρ_1/ρ_0, r/r_0 calculated for a range of values of y, are given in table 5.

TABLE 5. DENSITY ρ_1 AT RADIUS r EXPRESSED AS A PROPORTION OF ρ_0, THE UNDISTURBED DENSITY

y_1 (atm.)	r/r_0	ρ_1/ρ_0 uncorrected	C	ρ_1/ρ_0 corrected	R/r_0
10	(0·360)	1·155	0·753	0·873	0·249
20	0·337	0·704	0·858	0·605	0·198
30	0·325	0·527	0·900	0·475	0·173
40	0·316	0·430	0·923	0·397	0·157
60	0·304	0·321	0·947	0·305	0·137
100	0·289	0·223	0·968	0·216	0·116
500	0·247	0·071	—	0·071	0·068
5000	0·200	0·013	—	0·013	0·031

It has been pointed out that T_1/T_0 contains two factors, T/T_0 and T_1/T. T/T_0 represents the temperature change through the shock wave and equal to $\rho_0 y_1/\rho$. In calculating this the approximate expression $\dfrac{\rho_0}{\rho} = \dfrac{\gamma-1}{\gamma+1}$ was used instead of the true value

$$\frac{\rho_0}{\rho} = \frac{\gamma+1+(\gamma-1)y_1}{\gamma-1+(\gamma+1)y_1}. \tag{26}$$

The proportional error in calculating T/T_0 for a given values of y_1 is therefore equal to the proportional error in using $(\gamma-1)/(\gamma+1)$ instead of the correct expression for ρ_0/ρ. The second factor, T_1/T, which represents the reduction in temperature for a given expansion ratio is correct, so that correct values of ρ/ρ_0 for a given value of y_1 can be found by multiplying the figures given in column 3 of table 5 by a correcting factor

$$C = \frac{\gamma - 1 + (\gamma + 1)\, y_1}{\gamma + 1 + (\gamma - 1)\, y_1} \cdot \frac{\gamma - 1}{\gamma + 1}. \tag{27}$$

Values of C are given in column 4, table 5, and the corrected values of ρ_1/ρ_0 in column 5. It must be pointed out that though the figures in column 5 are correct, the values obtained for r/r_0 when y_1 is less than about 40 are subject to an appreciable error owing to using approximate values of f, ϕ and ψ at the shock front.

The variation of ρ_1/ρ_0 with r/r_0 is shown in figure 5. It will be seen that the density is very small when $r/r_0 < 0{\cdot}2$, that it begins to rise steeply at about $r/r_0 = 0{\cdot}28$, and that it has nearly attained atmospheric density when $r/r_0 = 0{\cdot}36$.

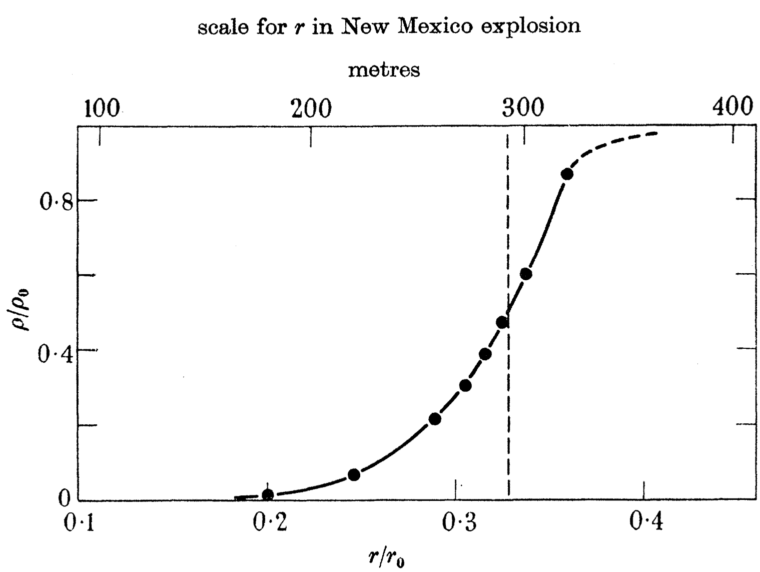

FIGURE 5. Distribution of density after the explosion. The upper scale corresponds with calculations using the value of E given in table 3 for $\gamma = 1{\cdot}40$.

Calculation of the rate of rise of the heated air

Though it would be difficult to calculate the effect of buoyancy on a fluid with the density distribution shown in figure 5, the rise of bubbles in water, when the change from very light to heavy fluid is discontinuous, has been studied. It has been shown both experimentally and theoretically (Davies & Taylor 1950) that the vertical velocity U of a large bubble is related to a, the radius of curvature of the top of the bubble, by the formula

$$U = \tfrac{2}{3}\sqrt{(ga)}. \tag{28}$$

It seems worth while to compare the observed rate of rise of the air heated by the New Mexico explosion with that of a large bubble in water, and for that purpose it is necessary to decide on the radius of a sphere of zero density which might be expected to be comparable with air having the density distribution of figure 5. The simplest guess is to take the radius as the value of r for which $\rho/\rho_0 = \frac{1}{2}$, and in figure 5 this corresponds with the broken line for which

$$r/r_0 = 0{\cdot}328. \tag{29}$$

In the New Mexico explosion the best estimate of E was that corresponding with the measured rate of expansion of the ball of fire, assuming $\gamma = 1{\cdot}40$. This is given in table 3, namely $E = 7{\cdot}14 \times 10^{20}$ ergs.

Using this value and $p_0 = 10^6$ dynes/sq.cm.

$$r_0 = (7{\cdot}14 \times 10^{14})^{\frac{1}{3}} = 8{\cdot}9 \times 10^4 = 894\,\text{m}. \tag{30}$$

The radius chosen for comparison with a bubble rising in water is therefore

$$r = 0{\cdot}328 \times 894 = 293\,\text{m}. \tag{31}$$

The predicted velocity of rise is therefore

$$U = \tfrac{2}{3}\sqrt{(981 \times 2{\cdot}93 \times 10^4)} = 35{\cdot}7\,\text{m./sec.} \tag{32}$$

Comparing this with the observed value of the vertical velocity of the centre of the glowing volume, namely, 35 m./sec., it will be seen that the agreement is better than the nature of the measurements would justify one in expecting. A far less good agreement would justify a belief that the foregoing dynamical picture of the course of events after the atomic explosion is essentially correct.

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