STELLAR STRUCTURE AND EVOLUTION

1. OBSERVABLE PROPERTIES OF STARS

Basic large-scale observable properties:

Luminosity
Surface temperature
Radius
Mass

Further observable:

Spectrum ... yields information about surface chemical composition and gravity

Evidence from:

- Individual stars
- Binary systems
- Star clusters....these reveal how stars evolve with time
- Nuclear physics...energy source, synthesis of heavy elements

No direct information about physical conditions in stellar interiors (except from helioseismology and solar neutrinos)

No direct evidence for stellar evolution......typical timescale $10^6 - 10^9$ years.......(except for a few very unusual stars and supernovae)

1.1 LUMINOSITY (ZG: 11; CO: 3.1)

('power', [J/s=W])

$$\boxed{L_s = \int_0^\infty L_\lambda d\lambda = 4\pi R_s^2 \int_0^\infty F_\lambda \, d\lambda}$$

where F_λ is the radiative flux at wavelength λ at the stellar surface, R_s the stellar radius. Energy may also be lost in the form of neutrinos or by direct mass loss (generally unobservable).

Astronomers measure:

$$f_\lambda = (R_s/D)^2 \, F_\lambda \qquad \text{at Earth's surface}$$

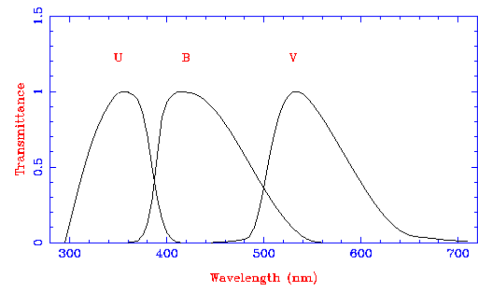
- To obtain L_λ we must know the star's distance D and correct for:
  - ▷ absorption in the Earth's atmosphere (standard methods)
  - ▷ absorption in interstellar space (negligible for nearby stars)
- Measurements from the Hipparcos satellite (1989–1993) have yielded parallaxes accurate to 0.002 arcsec for about 100,000 stars. The largest stellar parallax (Proxima Centauri) is 0.765 arcsec.

1.2 STELLAR MAGNITUDES (ZG: 11; CO: 3.2, 3.6)

- measure stellar flux (i.e. $f = L/4\pi D^2$, L: luminosity, D: distance)
  - ▷ for Sun: $L_\odot = 3.86 \times 10^{26}\,\mathrm{W}$, $f = 1.360 \times 10^3\,\mathrm{W\,m^{-2}}$ (solar constant)
  - ▷ luminosity measurement requires distance determination ($1\mathrm{A.U.} = 1.50 \times 10^{11}\,\mathrm{m}$)
- define apparent magnitudes of two stars, m_1, m_2, by $m_1 - m_2 = 2.5 \log f_2/f_1$
- zero point: Vega (historical) $\to$ $m_\odot = -26.82$
- to measure luminosity define absolute magnitude M to be the apparent magnitude of the object if it were at a distance 10 pc ($1\,\mathrm{pc} = 3.26$ light years $= 3.09 \times 10^{16}\,\mathrm{m}$)
- define bolometric magnitude as the absolute magnitude corresponding to the luminosity integrated over all wavebands; for the Sun $M_\odot^{\mathrm{bol}} = 4.72$
- in practice, the total luminosity is difficult to measure because of atmospheric absorption and limited detector response
- define magnitudes over limited wavelength bands

THE UBV SYSTEM

- the UBV system (ultraviolet, blue, visual) which can be extended into the red, infrared (RI)

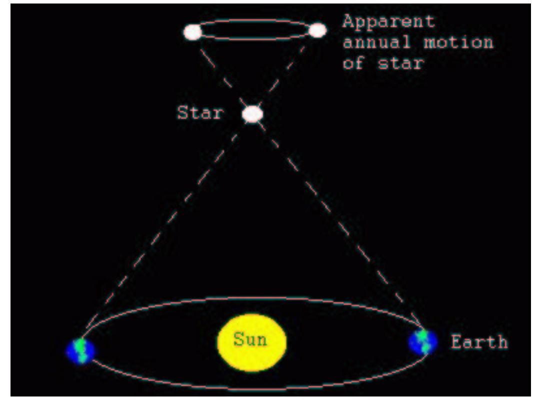


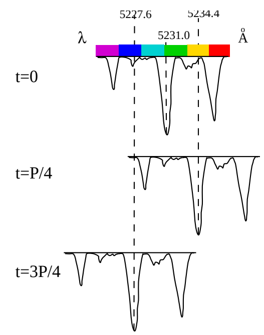
approximate region	notation for magnitudes apparent	absolute	solar value
ultraviolet	U or m_U	M_U	5.61
blue	B or m_B	M_B	5.48
visual (near yellow)	V or m_V	M_V	4.83

- colours (colour indices): relative magnitudes in different wavelength bands, most commonly used: $B - V$, $U - B$
- define bolometric correction: $\mathrm{B.C.} = M_{\mathrm{bol}} - M_V$ (usually tabulated as a function of $B - V$ colour)
- visual extinction A_V: absorption of visual star light due to extinction by interstellar gas/dust (can vary from ~ 0 to 30 magnitudes [Galactic centre])
- distance modulus: $(m - M)_V = 5 \times \log D/10\mathrm{pc}$
- summary: $M_V = \underbrace{-2.5 \log L/L_\odot + 4.72}_{M_{\mathrm{bol}}} - \mathrm{B.C.} + A_V$

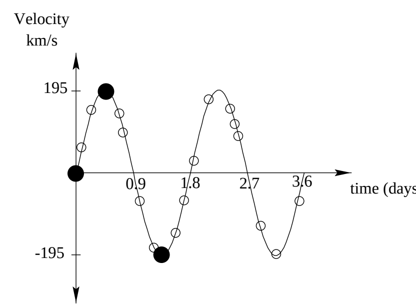
Nearby Stars to the Sun (from Norton 2000)

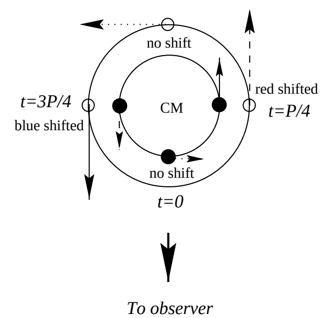
Common Name (Scientific Name)	Distance (light year)	Magnitudes apparent	absolute	spectral type
Sun		-26.8	4.8	G2V
Proxima Centauri (V645 Cen)	4.2	11.05 (var)	15.5	M5.5V
Rigel Kentaurus (Alpha Cen A)	4.3	-0.01	4.4	G2V
(Alpha Cen B)	4.3	1.33	5.7	K1V
Barnard's Star	6.0	9.54	13.2	M3.8V
Wolf 359 (CN Leo)	7.7	13.53 (var)	16.7	M5.8V
(BD +36 2147)	8.2	7.50	10.5	M2.1V
Luyten 726-8A (UV Cet A)	8.4	12.52 (var)	15.5	M5.6V
Luyten 726-8B (UV Cet B)	8.4	13.02 (var)	16.0	M5.6V
Sirius A (Alpha CMa A)	8.6	-1.46	1.4	A1V
Sirius B (Alpha CMa B)	8.6	8.3	11.2	DA
Ross 154	9.4	10.45	13.1	M4.9V





Doppler shifted absorption line spectra





- Accurate information about relative luminosities has been obtained from measuring relative apparent brightnesses of stars within clusters.
- Some wavelengths outside the visible region are completely absorbed by the Earth's atmosphere. Hence we must use theory to estimate contributions to L_s from obscured spectral regions until satellite measurements become available.
- Observations of clusters show that optical luminosities of stars cover an enormous range:

$$10^{-4}\ L_\odot < L_s < 10^6\ L_\odot$$

- By direct measurement:

$$L_\odot = (3.826 \pm 0.008) \times 10^{26}\ \mathrm{W}.$$

- The luminosity function for nearby stars shows the overwhelming preponderance of intrinsically faint stars in the solar neighbourhood. Highly luminous stars are very rare: the majority of nearby stars are far less luminous than the Sun.
- Initial mass function (IMF): distribution of stellar masses (in mass interval dM)

$$f(M)\,dM \propto M^{-\gamma}\,dM \qquad \gamma \simeq 2.35\ \text{[Salpeter] to } 2.5$$

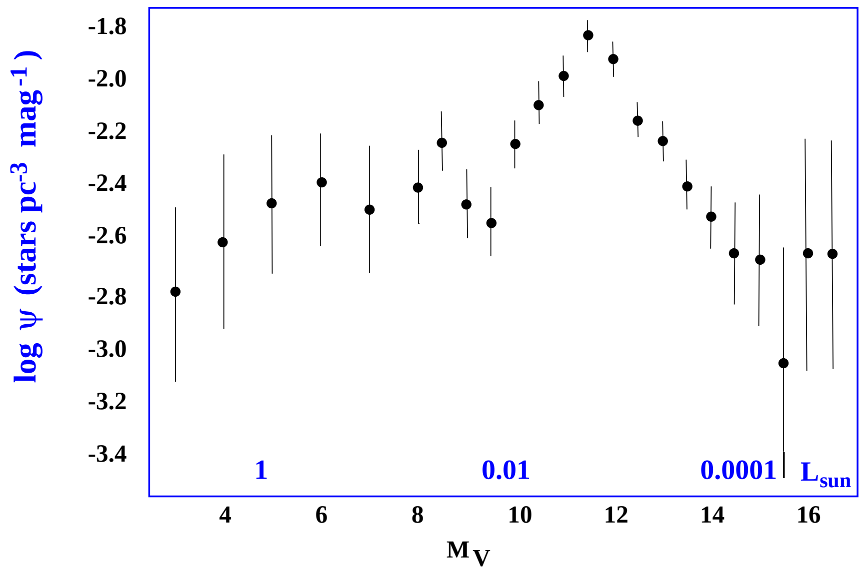
(good for stars more massive than $\gtrsim 0.5\,M_\odot$).

$\rightarrow$ most of the mass in stars is locked up in low-mass stars (brown dwarfs?)

$\triangleright$ but most of the luminosity comes from massive stars.

Luminosity Function

(after Kroupa)



1.3 STELLAR SURFACE TEMPERATURES
(ZG: 8-6; CO: 3.4)

Various methods for ascribing a temperature to the stellar photosphere:

1. **Effective temperature, T_{eff}** (equivalent black-body temperature):

$$\boxed{L_s = 4\pi R_s^2 \int F_\lambda d_\lambda = 4\pi R_s^2 \sigma T_{eff}^4}$$

Direct determination of T_{eff} not generally possible because R_s is not measurable except in a few cases. T_{eff} can be derived indirectly using model atmospheres.

2. **Colour temperature**

▷ Match shape of observed continuous spectrum to that of a black body,

$$\Phi(\lambda) = \frac{2hc^2}{\lambda^5} \frac{1}{\exp(hc/\lambda kT) - 1}.$$

▷ An empirical relationship between colour temperature and B-V has been constructed (B and V are magnitudes at λ_B and λ_V respectively).

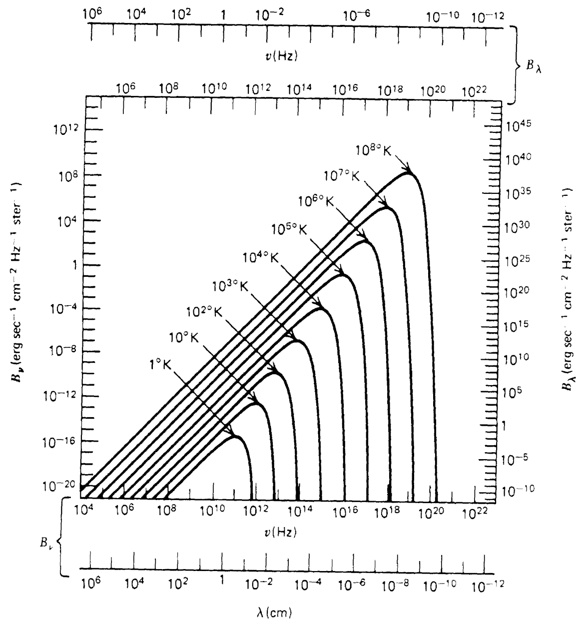


Figure 1.11 Spectrum of blackbody radiation at various temperatures (taken from Kraus, J. D. 1966, Radio Astronomy, McGraw-Hill Book Company)

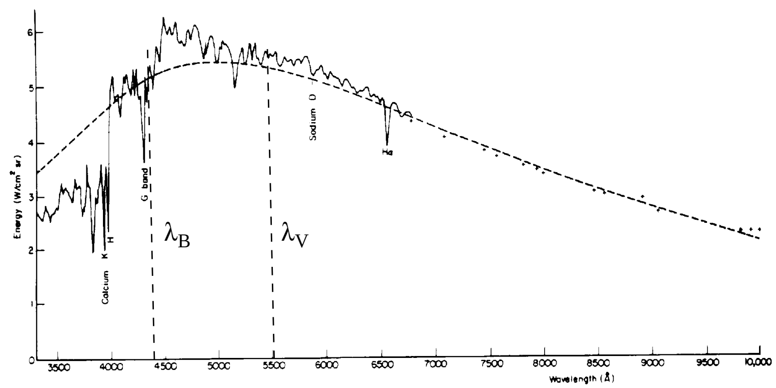
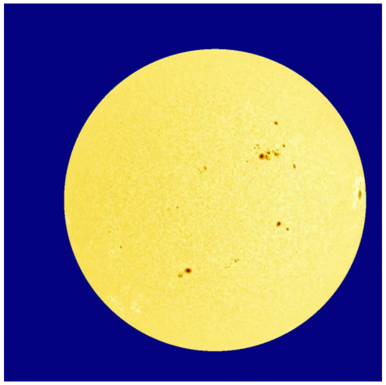
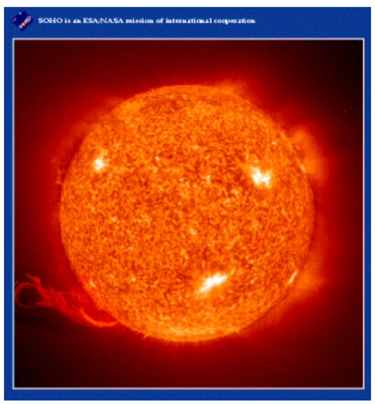
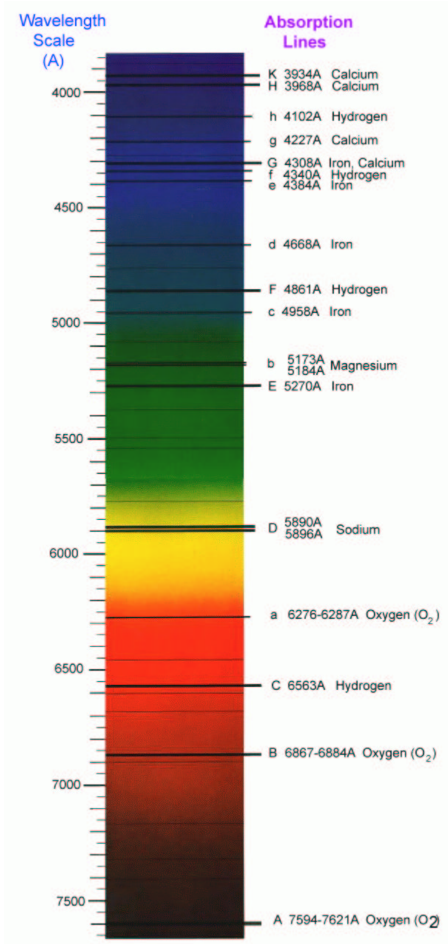


Figure 9.5 The spectrum of the Sun. The dashed line is the curve of an ideal blackbody having the Sun's effective temperature. (Figure from Aller, *Atoms, Stars, and Nebulae*. Revised Edition, Harvard University Press, Cambridge, MA, 1971.)



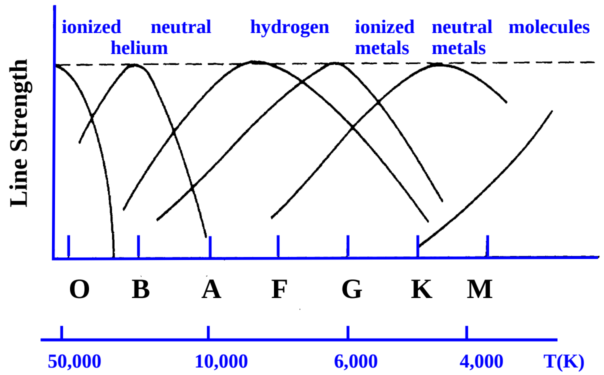




1.4 SPECTRAL CLASSIFICATION (ZG: 13-2/3; CO: 5.1, 8.1, 8.3)

- Strengths of spectral lines are related to excitation temperature and ionization temperature of photosphere through Boltzmann and Saha equations.
- An empirical relation between spectral class and surface temperature has been constructed (e.g. Sun: G2 $\rightarrow$ 5,800 K).
- Different properties yield different temperatures. Only a full model atmosphere calculation can describe all spectral features with a unique $T_{\rm eff}$: not available for most stars. Normally astronomers measure V and $B-V$ and use an empirical relation based on model atmosphere analysis of a limited number of stars to convert V to L_s and $B-V$ to $T_{\rm eff}$.
- L_s and $T_{\rm eff}$ are the key quantities output by stellar structure model calculations.
- Range of $T_{\rm eff}$: $2000\,{\rm K} < T_{\rm eff} < 100,000\,{\rm K}$

Spectral Classification



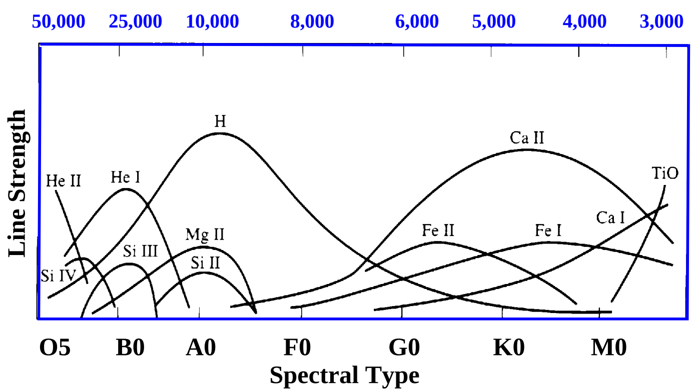


Figure 8.9 The dependence of spectral line strengths on temperature.

Luminosity Classes

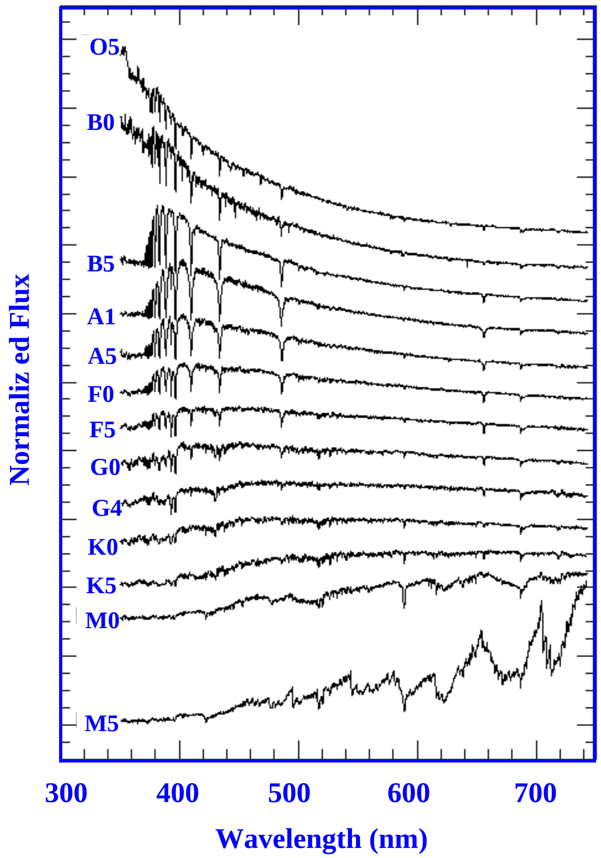
Class	Type of Star
Ia	Luminous supergiants
Ib	Less Luminous supergiants
II	Bright giants
III	Normal giants
IV	Subgiants
V	Main-sequence stars (Dwarfs)
VI, sd	Subdwarfs
D	White Dwarfs

- The luminosity class is essentially based on the width of spectral lines
- narrow lines $\longrightarrow$ low surface pressure $\longrightarrow$ low surface gravity $\longrightarrow$ big star
- supergiants have narrow lines, white dwarfs (the compact remnants of low-/intermediate-mass stars) very broad lines

L Stars/T Dwarfs

- recent extension of the spectral classification for very cool ($T_{\rm eff} < 2500\,$K) objects, mainly brown dwarfs (?) (low-mass objects many with $M < 0.08\,M_\odot$ which are not massive enough for nuclear reactions in the core)

Spectra of Dwarf Stars (Luminosity Class V)



1.5 STELLAR ATMOSPHERES (ZG: 13-1; CO: 9.1, 9.4)

- Continuum spectrum: defines effective temperature (T_{eff}) and photospheric radius (R_{ph}) through $L_{bol} = 4\pi R_{ph}^2 \sigma T_{eff}^4$
- absorption lines in the spectrum are caused by cooler material above the photosphere
- emission lines are caused by hotter material above the photosphere
- spectral lines arise from transitions between the bound states of atoms/ions/molecules in the star's atmosphere
- spectral lines contain a wealth of information about
  - ▷ the temperature in regions where the lines are produced $\rightarrow$ spectral type
  - ▷ the chemical composition $\rightarrow$ nucleosynthesis in stars
  - ▷ pressure $\rightarrow$ surface gravity $\rightarrow$ luminosity class
  - ▷ stellar rotation: in rapidly rotating stars, spectral lines are Doppler broadened by rotation
  - ▷ orbital velocities (due to periodic Doppler shifts) in binaries

1.6 STELLAR MASSES (ZG: 12-2/3; CO: 7.2, 7.3)

Only one direct method of mass determination: study dynamics of binary systems. By Kepler's third law:

$$\boxed{(M_1 + M_2)/M_\odot = a^3/P^2}$$

a = semi-major axis of apparent orbit in astronomical units; P = period in years.

a) Visual binary stars:

- ▷ Sum of masses from above
- ▷ Ratio of masses if absolute orbits are known

$$M_1/M_2 = a_2/a_1 \qquad a = a_1 + a_2$$

- ▷ Hence M_1 and M_2 but only a few reliable results.

b) Spectroscopic binary stars:

- ▷ Observed radial velocity yields $v \sin i$ (inclination i of orbit in general unknown). From both velocity curves, we can obtain M_1/M_2 and $M_1 \sin^3 i$ and $M_2 \sin^3 i$ i.e. lower limits to mass (since $\sin i < 1$).
- ▷ For spectroscopic eclipsing binaries $i \sim 90°$; hence determination of M_1 and M_2 possible. About 100 good mass determinations; all main-sequence stars.

- Summary of mass determinations:
  - ▷ Apart from main-sequence stars, reliable masses are known for 3 white dwarfs
a few giants
  - ▷ Range of masses: $0.1 M_\odot < M_s < 200 M_\odot$.

1.7 STELLAR RADII (ZG: 12-4/5; 7.3)

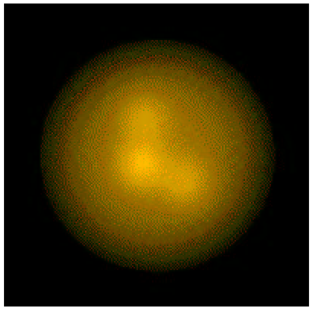
In general, stellar angular diameters are too small to be accurately measurable, even for nearby stars of known distance.

$$R_\odot = 6.96 \times 10^5 \text{ km}$$

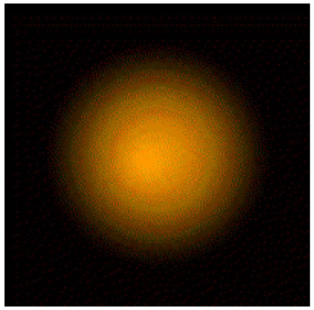
- Interferometric measurements:
  - a) Michelson stellar interferometer results for 6 stars ($R_s >> R_\odot$)
  - b) Intensity interferometer results for 32 stars (all hot, bright main-sequence stars with $R_s \sim R_\odot$)
- Eclipsing binaries:
  - ▷ Measure periodic brightness variations
  - ▷ reliable radii for a few hundred stars.
- Lunar occultations:
  - ▷ Measure diffraction pattern as lunar limb occults star
  - ▷ results for about 120 stars.

Optical Interferometry (WHT, COAST): Betelgeuse

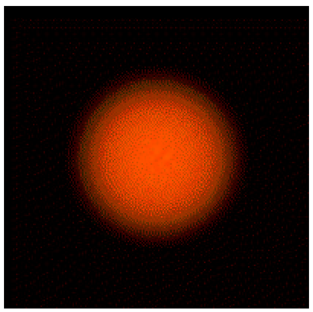
700 nm

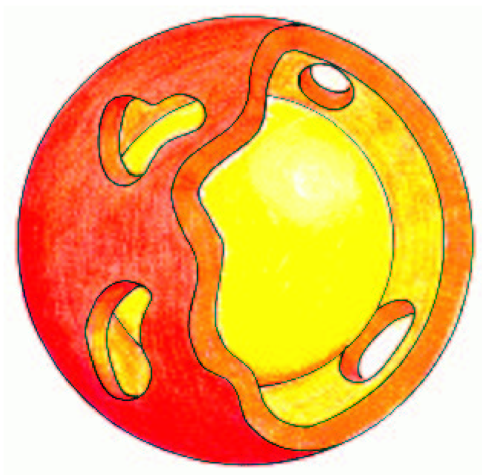


905 nm

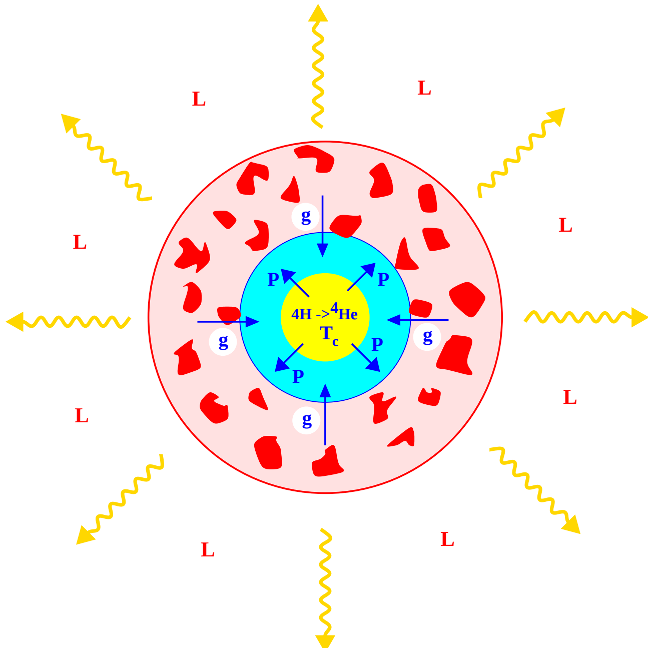


1290 nm





- Indirect methods:
  - ▷ e.g. use of $L_s = 4\pi R_s^2 \sigma T_{eff}^4$ with estimates of L_s and T_{eff}.
- Summary of measurements of radii:
  - ▷ Main-sequence stars have similar radii to the Sun; R_s increases slowly with surface temperature.
  - ▷ Some stars have much smaller radii $\sim 0.01 R_\odot$ (white dwarfs)
  - ▷ Some stars have much larger radii $> 10 R_\odot$ (giants and supergiants)
  - ▷ Range of radii: $0.01 R_\odot < R_s < 1000 R_\odot$.



Stellar Structure and Stellar Evolution

- physical laws that determine the equilibrium structure of a star
- stellar birth in protostellar clouds $\rightarrow$ planet formation in circumstellar discs, binarity, brown dwarfs
- stellar evolution driven by successive phases of nuclear burning, $\rightarrow$ giants, supergiants
- final stages of stars:
  - ▷ white dwarfs and planetary nebula ejection ($M \lesssim 8\,M_\odot$)
  - ▷ supernova explosions for massive stars ($M \gtrsim 8\,M_\odot$), leaving neutron star (pulsar), black-hole remnants

Stellar Atmospheres

- basic physics that determines the structure of stellar atmospheres, line formation
- modelling spectral lines to determine atmospheric properties, chemical composition

Selected Properties of Main-Sequence Stars

Sp	M_V	$B-V$	B.C.	M_{bol}	$\log T_{eff}$ (K)	$\log R$ ($R_\odot$)	$\log M$ ($M_\odot$)
O5	-5.6	-0.32	-4.15	-9.8	4.626	1.17	1.81
O7	-5.2	-0.32	-3.65	-8.8	4.568	1.08	1.59
B0	-4.0	-0.30	-2.95	-7.0	4.498	0.86	1.30
B3	-1.7	-0.20	-1.85	-3.6	4.286	0.61	0.84
B7	-0.2	-0.12	-0.80	-1.0	4.107	0.45	0.53
A0	0.8	+0.00	-0.25	0.7	3.982	0.36	0.35
A5	1.9	+0.14	0.02	1.9	3.924	0.23	0.26
F0	2.8	+0.31	0.02	2.9	3.863	0.15	0.16
F5	3.6	+0.43	-0.02	3.6	3.813	0.11	0.08
G0	4.4	+0.59	-0.05	4.4	3.774	0.03	0.02
G2	4.7	+0.63	-0.07	4.6	3.763	0.01	0.00
G8	5.6	+0.74	-0.13	5.5	3.720	-0.08	-0.04
K0	6.0	+0.82	-0.19	5.8	3.703	-0.11	-0.07
K5	7.3	+1.15	-0.62	6.7	3.643	-0.17	-0.19
M0	8.9	+1.41	-1.17	7.5	3.591	-0.22	-0.26
M5	13.5	+1.61	-2.55	11.0	3.491	-0.72	-0.82

Exercise 1.1: The V magnitudes of two main-sequence stars are both observed to be 7.5, but their blue magnitudes are $B_1 = 7.2$ and $B_2 = 8.65$. (a) What are the colour indices of the two stars. (b) Which star is the bluer and by what factor is it brighter at blue wavelength. (c) Making reasonable assumptions, deduce as many of the physical properties of the stars as possible e.g. temperature, luminosity, distance, mass, radius [assume $A_V = 0$].

Summary I

Concepts:

- relation between astronomical observables (flux, spectrum, parallax, radial velocities) and physical properties (luminosity, temperature, radius, mass, composition)
- the stellar magnitude system (apparent and absolute magnitudes, bolometric magnitude, bolometric correction, distance modulus), the UBV system and stellar colours
- the black-body spectrum, effective temperature
- spectral classification: spectral type and luminosity classes and its implications
- measuring masses and radii

Important equations:

- *distance modulus:* $(m - M)_V = 5 \log D/10\text{pc}$
- *absolute V magnitude:* $M_V = -2.5 \log L/L_\odot + 4.72 + \text{B.C.} + A_V$
- *Salpeter initial mass function (IMF):* $f(M)\, dM \propto M^{-2.35}\, dM$
- *black-body relation:* $L = 4\pi R_s^2\, \sigma T_{eff}^4$
- *Kepler's law:* $a^3 \left(\frac{2\pi}{P}\right)^2 = G(M_1 + M_2)$