Astrophysics Graduate Course
General Relativity
(Ph. Podsiadlowski, Oxford, MT08/HT09)

Please do either all of the problems in Part A or *one* of the literature review problems in Part B.

Textbooks

- Black Holes, White Dwarfs and Neutron Stars, The Physics of Compact Objects, S. L. Shapiro & S. A. Teukolsky (classical textbook, but a bit out of date now)
- Compact Objects in Astrophysics, White Dwarfs, Neutron Stars and Black Holes, M. Camenzind (modern version of Shapiro and Teukolsky, quite demanding in places)

Part A: Problems (Taken from Shapiro & Teukolsky)

(1) The Principle of Equivalence

Consider two particles of mass m at distances r and $r + h$ ($h \ll r$) on the same vertical line from the centre of the Earth. The particles fall freely from rest at time $t = 0$ towards the Earth's surface. Show that an observer falling with one particle will see the separation between the particles gradually increase. Translate this into a quantitative statement about the observer's local inertial frame. In particular, determine the time at which the effects of spacetime curvature will become apparent if measurements can be made to a precision of $\Delta h_{\rm min}$.

(2) The Oppenheimer-Volkoff Equation

Show that inside a uniform density star ($\rho =$ constant) of radius R and mass M

$$\frac{P}{\rho c^2} = \frac{(1-2\gamma r^2/R^2)^{1/2} - (1-2\gamma)^{1/2}}{3(1-2\gamma)^{1/2} - (1-2\gamma r^2/R^2)^{1/2}},$$

$$e^{\Phi} = \frac{3}{2}(1-2\gamma)^{1/2} - \frac{1}{2}(1-2\gamma r^2/R^2)^{1/2},$$

where

$$\gamma \equiv \frac{GM}{Rc^2},$$

and $\Phi(r)$ is the metric function (see below) (in the classical limit it becomes $Gm(r)/(rc^2)$; here set $\Phi(r=0)=0$).

Show that the condition $P_{\rm c} < \infty$ ($P_{\rm c}$ is the central pressure) implies

$$\frac{2GM}{Rc^2} < \frac{8}{9}.$$

This limit for the maximum "compaction" of a uniform density equilibrium sphere applies, in fact, to a sphere of arbitrary density profiles, as long as the density does not increase outwards.

Note: Einstein's equations of hydrostatical equilibrium for a spherical star are

$$\frac{\mathrm{d}m}{\mathrm{d}r} = 4\pi r^2 \rho,$$

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{G\rho m}{r^2}\left(1+\frac{P}{\rho c^2}\right)\left(1+\frac{4\pi r^3 P}{mc^2}\right)\left(1-\frac{2Gm}{rc^2}\right)^{-1},$$

$$\frac{\mathrm{d}\Phi}{\mathrm{d}r} = -\frac{1}{\rho c^2}\frac{\mathrm{d}P}{\mathrm{d}r}\left(1+\frac{P}{\rho c^2}\right).$$

Part B: Literature Review

(3) Testing General Relativity using Double Neutron Stars

The Taylor-Hulse pulsar or more recently the double pulsar (PSR J0737-3039) have provided some of the best tests of general relativity to date. Summarize the main relativistic effects that can be measured and how it constrains GR. Specifically, what potential does the the double pulsar provide?

[There is a detailed discussion in Shapiro & Teukolsky, for the double pulsar papers by M. Kramer.]

(4) Alternative Versions of General Relativity

There are numerous alternatives to Einstein's version of GR. Summarize some main alternative approaches without going into any technical details.

[Wikipedia: Alternatives to general relativity]