Explain how Newton-Raphson iteration can be used to solve systems of non-linear equations, given an approximate solution. Two interstellar species A and B react to form the molecules A_2, A_2B and AB by the reaction network

$$\begin{aligned} A + A &\rightarrow A_2, \\ A_2 + B &\rightarrow A_2B, \\ A_2B &\rightarrow AB + A \quad \text{and} \\ AB &\rightarrow A + B \end{aligned}$$

for which the the rate coefficients are k_1, k_2, k_3 and k_4 respectively. Write down the conservation and rate equations assuming the total number densities of A and B are respectively $n^o(A)$ and $n^o(B)$ and that the equilibrium number densities of A, B, A_2, A_2B and AB are respectively $n(A)$, $n(B)$, $n(A_2)$ $n(A_2B)$ and $n(AB)$.

Use Newton-Raphson iteration to solve the resulting system of non-linear equations for $n(A)$, $n(B)$, $n(A_2)$, $n(A_2B)$ and $n(AB)$ assuming equilibrium and $k_1 = 10^{-11}$ cm^3s^{-1}, $k_2 = 5 \times 10^{-12}$ cm^3s^{-1}, $k_3 = 10^{-11}$ cm^3s^{-1}, $k_4 = 10^{-11}$ cm^3s^{-1}, $n^o(A) = 1$ cm^{-3} and $n^o(B) = 1$ cm^{-3}. It is suggested that you tackle the problem by writing a short computer program to call the "Numerical Recipes" routine MNEWT, using your own version of USRFUN which supplies the appropriate vector of function values and Jacobian matrix for a trial solution. You would also need to use the routines LUDCMP and LUBKSB to perform LU-decomposition and back-subsitution in a derived system of linear equations.