Stellar Evolution

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Fundamental Stellar Parameters

Radiative Transfer

Stellar Atmospheres

Equations of Stellar Structure

Nuclear Reactions in Stellar Interiors

Basic Requirements

Understanding the evolution of a star, its previous and future evolution requires knowledge of:

- Mass (M),
- Radius (R) and
- Luminosity (L).

These are usually expressed relative to the solar mass $M_\odot$, the solar radius $R_\odot$ and solar luminosity $L_\odot$ respectively.

In addition, relative abundances of all chemical elements in the photosphere are needed.

Effective Temperature and Angular Radius

$$\sigma {T_{\text{eff}}}^4 = \int_0^\infty F_\lambda \, d\lambda$$

$$L = 4\pi R^2 \int_0^\infty F_\lambda \, d\lambda = 4\pi d^2 \int_0^\infty f_\lambda \, d\lambda$$

$$\alpha^2 = \frac{R^2}{d^2} = \frac{f_\lambda}{F_\lambda}$$

$$L = 4\pi R^2 \sigma {T_{\text{eff}}}^4$$

Effective Temperature and Surface Gravity Scaling

$$\frac{L}{L_\odot} = \left(\frac{R}{R_\odot}\right)^2 \left[\frac{T_{\rm eff}}{(T_{\rm eff})_\odot}\right]^4$$

$$\frac{g}{g_\odot} = \frac{M}{M_\odot}\left(\frac{R}{R_\odot}\right)^{-2}$$

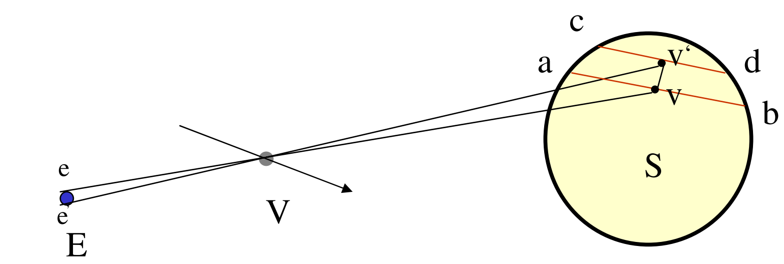
Kepler's Third Law

van den Bos WH, 1961 MNASSA 20 138

Planet	a	P	a^3	P^2
Mercury	0.388	0.241	0.0584	0.0581
Venus	0.724	0.615	0.3795	0.3785
Earth	1	1	1	1
Mars	1.5235	1.881	3.536	3.538
Jupiter	5.1965	11.857	140.3	140.6
Saturn	9.510	29.425	860.1	865.8

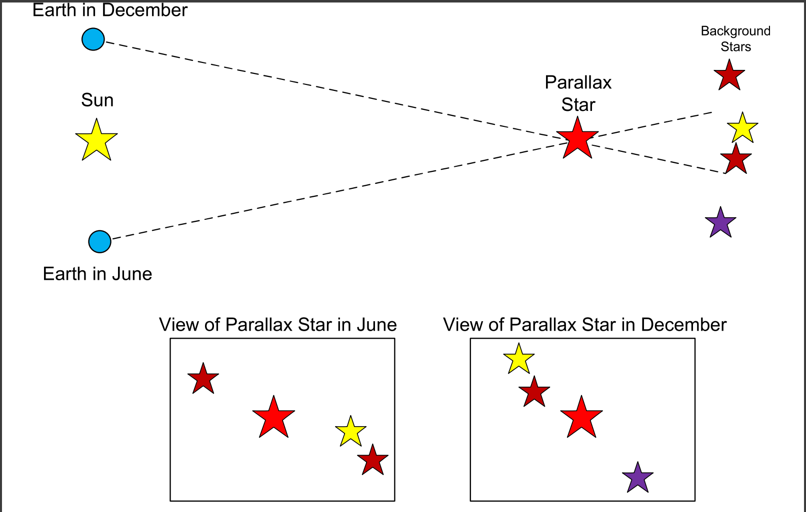
Earth-Sun Distance Measurement

Metz D, 2009 Science & Education 18 581

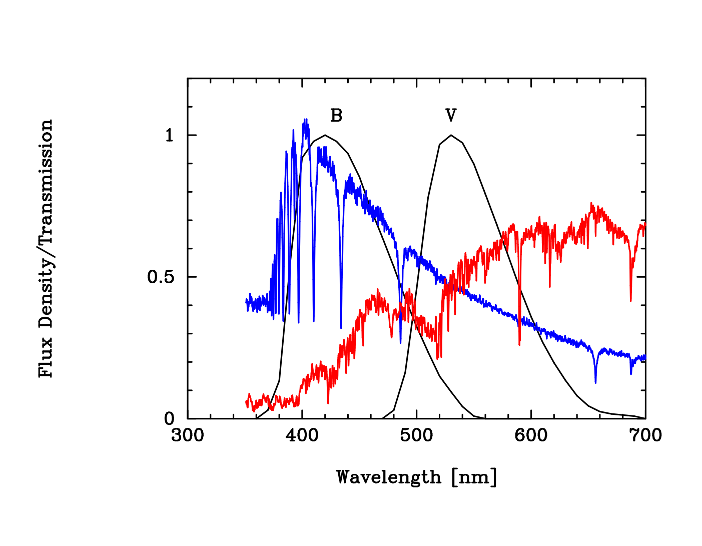


Parallax Stellar Distance Measurement

van Belle GT, 2009 New Astronomy Reviews 53 336



Stellar Magnitudes and Colours - I



Stellar Magnitudes and Colours - II

$$m_2 - m_1 = -2.5 \log_{10}(F_2/F_1)$$

$$V = V_0 - 2.5 \log_{10} \left[\frac{\int_0^\infty F_\lambda S_\lambda(V)\, d\lambda}{\int_0^\infty S_\lambda(V)\, d\lambda} \right]$$

$$\lambda_{\rm eff} = \left[\frac{\int_0^\infty \lambda F_\lambda S_\lambda(V)\, d\lambda}{\int_0^\infty F_\lambda S_\lambda(V)\, d\lambda} \right]$$

Absolute Magnitude

$$f_\lambda(V) = \left(\frac{\mathcal{D}}{d}\right)^2 \mathcal{F}_\lambda(V)$$

$$\begin{aligned} m_V - \mathcal{M}_V &= 2.5\log_{10}\left(\frac{\mathcal{F}_\lambda(V)}{f_\lambda(V)}\right) \\ &= 2.5\log_{10}\left(\frac{d}{\mathcal{D}}\right)^2 \\ &= 5\log_{10} d - 5 \end{aligned}$$

(when $\mathcal{D} = 10\text{pc}$).

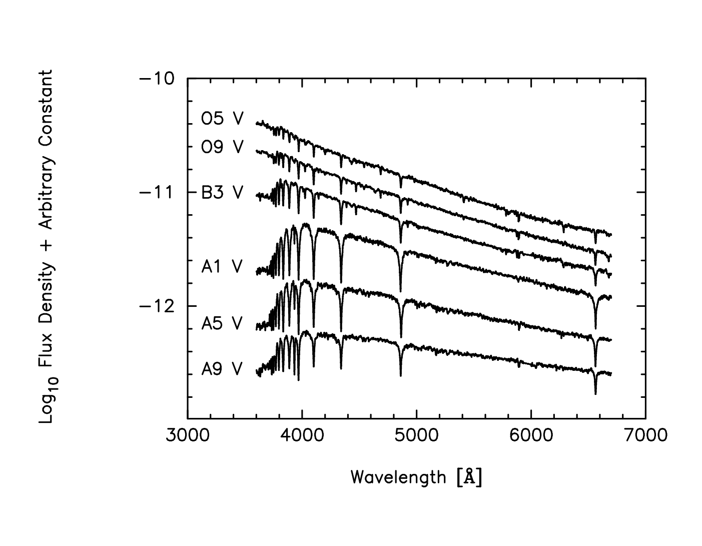
Bolometric Correction

$$\ell = \int_0^\infty f_\lambda \, d\lambda$$

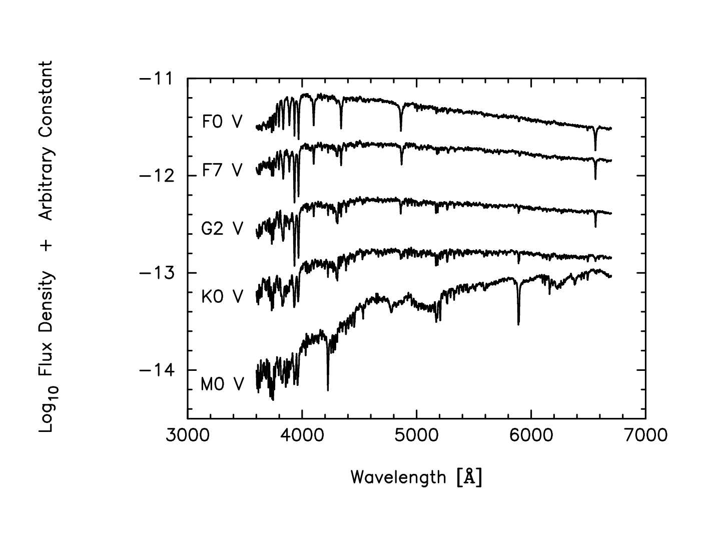
$$m_{\rm bol} - m_V = 2.5 \log_{10} \frac{\ell(V)}{\ell}$$

$$= 2.5 \log_{10} \left[\frac{\displaystyle\int_0^\infty f_\lambda(V) S_\lambda(V) d\lambda}{\displaystyle\int_0^\infty f_\lambda \, d\lambda} \right]$$

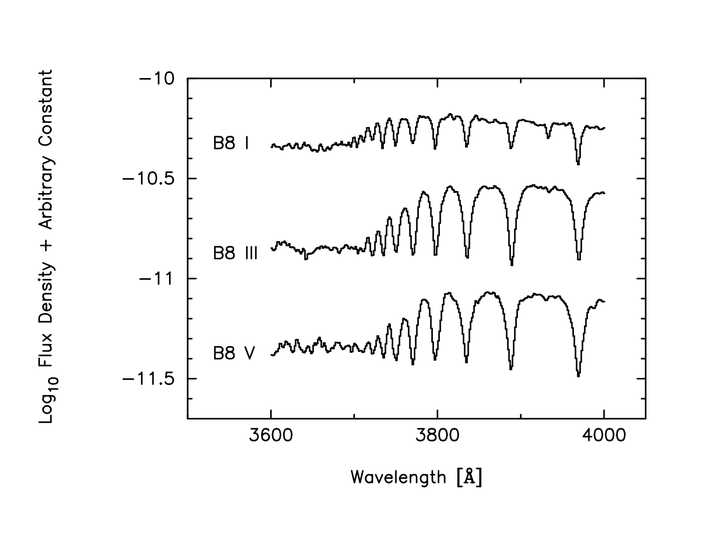
Stellar Type - Temperature Classification - I



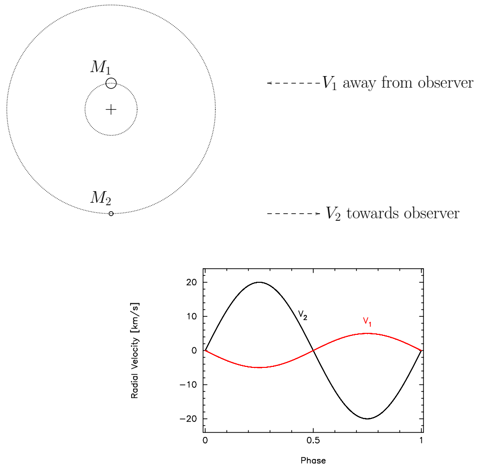
Stellar Type - Temperature Classification - II



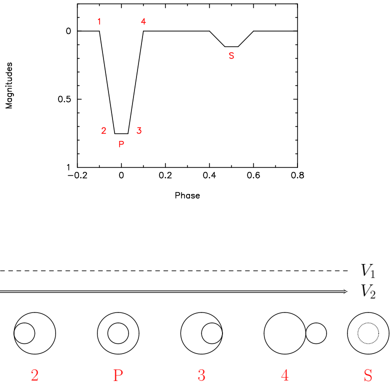
Spectral Type - Luminosity Classification



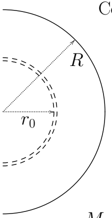
Eclipsing Binaries - I



Eclipsing Binaries - II



Radial Pulsation - I



Consider spherical shell of thickness dr_0 with equlbrium distance r_0 from centre of star of radius R.

P_0 - equilibrium pressure at r_0.

ρ_0 - equilibrium density at r_0.

M_{r_0} - mass confined within r_0.

dM_{r_0} - mass confined within dr_0.

$$M_{r_0} = 4\pi \int_0^{r_0} \rho_0 \, {r_0}^2 \, dr_0 \qquad dM_{r_0} = 4\pi \rho_0 \, {r_0}^2 \, dr_0$$

In hydrostatic equilibrium, pressure gradient balances gravity:

$$\frac{dP_0}{dr_0} = G \frac{M_{r_0}}{{r_0}^2} \rho_0$$

Radial Pulsation - II

Compress star and release; it then oscillates (or pulsates) leading to a time-dependent radial shift of mass shells within the star:

$$\frac{\Delta r}{r_0} = x(t) \quad \text{or} \quad r(t) = r_0\left[1 + x(t)\right] \quad \text{and} \quad dr = \left[1 + x(t)\right] dr_0$$

where $x(t)$ is a time-dependent perturbation. If the shells conserve their mass ($M_r = M_{r_0}$)

$$\rho r^2 dr = \rho_0 {r_0}^2 dr_0,$$

and do not exchange energy, the pulsation is adiabatic:

$$P = P_0 \left(\frac{\rho}{\rho_0}\right)^\gamma$$

Radial Pulsation - III

Then for $x(t) \ll 1$

$$\rho = \rho_0 \left[1 + x(t)\right]^{-3} \simeq \rho_0 \left[1 - 3x(t)\right],$$

$$P = P_0 \left[1 + x(t)\right]^{-3\gamma} \simeq P_0 \left[1 - 3\gamma x(t)\right]$$

$$\text{and} \quad \frac{1}{r^2} \simeq \frac{1}{{r_0}^2}\left[1 - 2x(t)\right].$$

Hydrostatic equilibrium no longer applies; shell acceleration needs to be included in the equation of motion (EOM):

$$\frac{dP}{dr} = -G\frac{M_r}{r^2}\rho - \rho\frac{d^2 r}{dt^2}$$

Radial Pulsation - IV

Left Hand Side of EOM:

$$\frac{dP}{dr} = \frac{dP}{dr_0}\frac{dr_0}{dr} = \frac{dP_0}{dr_0}\frac{1-3\gamma x}{1+x}$$
$$\simeq \frac{dP_0}{dr_0}(1-3\gamma x)(1-x) = G\frac{M_{r_0}}{{r_0}^2}\rho_0\,(1-3\gamma x)(1-x)$$
$$\simeq G\frac{M_{r_0}}{{r_0}^2}\rho_0\,(1-x(3\gamma+1))$$

Right Hand Side of EOM:

$$G\frac{M_r}{r^2}\rho = G\frac{M_{r_0}}{{r_0}^2}(1-2x)\rho_0(1-3x) \simeq G\frac{M_{r_0}}{{r_0}^2}\rho_0(1-5x)$$

and

$$-\rho\frac{d^2r}{dt^2} = -\rho_0\,(1-3x)\,r_0\,\frac{d^2x}{dt^2}$$

Radial Pulsation - V

EOM becomes:

$$G\frac{M_{r_0}}{{r_0}^2}\rho_0\left(1-x(3\gamma+1)\right)=G\frac{M_{r_0}}{{r_0}^2}\rho_0(1-5x)-\rho_0\left(1-3x\right)r_0\,\frac{d^2x}{dt^2}$$

$$r_0\,\frac{d^2x}{dt^2}=G\,\frac{M_{r_0}}{{r_0}^2}\,\frac{(4-3\gamma)}{1-3x}\,x$$

The boundary condition is that $r=r_0(1-x(t))=R$ and $M_{r_0}=M$ at the stellar surface giving:

$$\frac{d^2x}{dt^2}+(3\gamma-4)\,\frac{3G\bar{\rho}}{4\pi}x=0,\quad\text{with}\quad\bar{\rho}=\frac{M}{(4\pi/3)\,R^3}$$

For pulsation period Π, the solution is:

$$x=x_0\,\exp(i\omega t)\quad\text{with}\quad\omega^2=4\pi^2/\Pi^2=(3\gamma-4)\,\frac{3G\bar{\rho}}{4\pi}.$$

Radial Pulsation - Period-Luminosity Relation

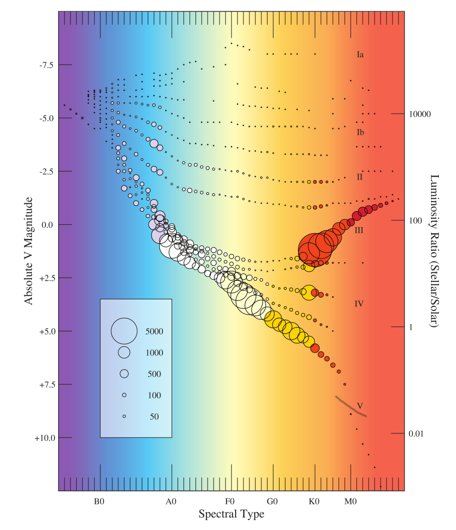
- Period – Mean Density Relation: $\Pi \sim \sqrt{1/\bar{\rho}} \sim R^{3/2} M^{-1/2}$
- Pulsation period Π has weak dependence on stellar mass (M) but strong dependence on stellar radius R.
- Absolute Magnitudes $M_{V,I,J,H,K} \sim 5 \log_{10} R$
- Predicted Period – Luminosity Relation $M_{V,I,J,H,K} \sim (10/3) \log_{10} \Pi$ which is in good agreement with observation.
- Absolute magnitudes of radial pulsators (Cepheids and RR Lyrae stars) are directly measureable from their periods which then yield distances.

Stellar Masses, Radii & Luminosities

Spectrum	$\log_{10}(M/M_\odot)$			$\log_{10}(R/R_\odot)$			$\log_{10}(L/L_\odot)$		
	I	III	V	I	III	V	I	III	V
B0	+1.70		+1.23	+1.30	+1.20	+0.88	+5.50		+4.10
A0	+1.20		+0.55	+1.60	+0.80	+0.42	+4.40		+1.90
F0	+1.10		+0.25	+1.80		+0.13	+3.90		+0.80
G0	+1.00	+0.40	+0.03	+2.00	+0.80	+0.02	+3.80	+1.50	+0.10
K0	+1.10	+0.60	−0.09	+2.30	+1.20	−0.07	+4.00	+2.00	−0.40
M0	+1.20	+0.80	−0.32	+2.70		−0.20	+4.50	+2.60	−1.20

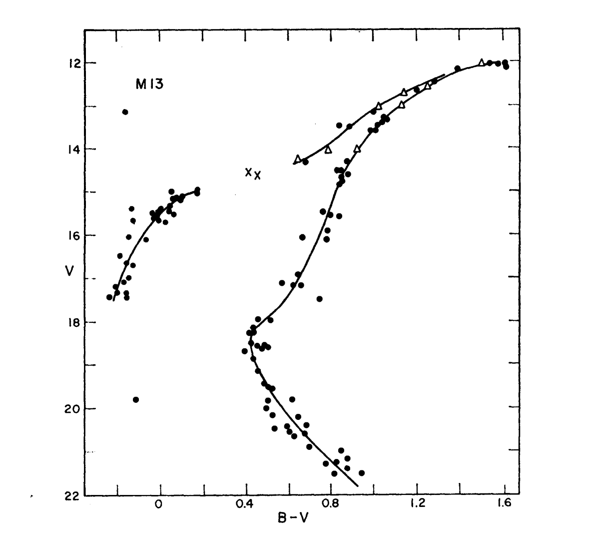
Hertzsprung-Russell Diagram

Sowell JR *et al.* 2007 AJ 134 1089



Colour-Magnitude Diagram for the Globular Cluster M13

Sandage A 1970 ApJ 162 841



Lecture 1: Summary

Essential points covered in first lecture:

- Distances of nearby stars may be measured by parallax once scale of the Solar System has been established.
- Stellar radii and masses may be determined through the study of eclipsing binary stars.
- Stars may be classfied spectroscopically; those with the same spectra have the same masses, radii and luminosities and this may be used to extend the distance scale.
- Radially pulsating stars such as Cepheids and RR Lyraes serve as distance indicators, once their pulsation periods are known, through the period-luminosity-relation.
- Distributions of stars in colour-magnitude diagrams needs to be explained by stellar evolution theory and models.

Stellar evolution depends on initial photospheric abundances and their determination from spectra are to be discussed in the next two lectures.

Acknowledgement

Material presented in this lecture on radial pulsation and the period-luminosity relation is based on slides prepared by R.-P. Kudritzki (University of Hawaii).