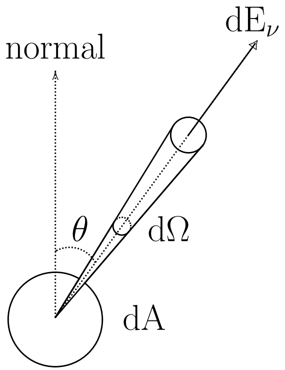
Radiative Transfer Introduction

Relative abundances of chemical elements are adjusted until synthetic spectra agree with observation; requirements are:

- Accurate laboratory determinations of atomic and molecular data, primarily oscillator strengths.
- $T_{\rm eff}$ and $\log g$ from some combination of energy distributions, Balmer line profiles, ionisation ratios and molecular association ratios.
- Temperature, gas pressure and electron pressure dependence on geometric or optical depth in the stellar atmosphere.
- Broadening theory for lines used in abundance determinations.
- Radiation transfer theory to calculate the wavelength-dependent emergent flux

Dependent on radiative transfer; the subject of this lecture.

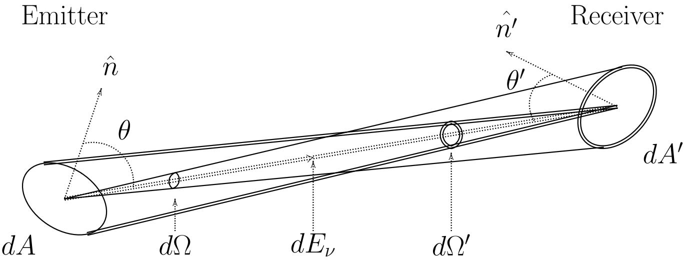
Specific Intensity



$$dE_\nu = I_\nu \cos\theta \, dA \cos\theta \, d\Omega \, d\nu \, dt$$

$$I_\lambda = \frac{c}{\lambda^2} I_\nu$$

Specific Intensity Invariance



$$
\begin{aligned}
dE_\nu &= I_\nu(\cos\theta)\; dA(\cos\theta)\; d\Omega\; d\nu\; dt \\
&= I_\nu{}'(\cos\theta')\; dA'(\cos\theta')\; d\Omega'\; d\nu\; dt
\end{aligned}
$$

$$
d\Omega = \frac{\text{projected area}}{\text{distance}^2} = \frac{dA'(\cos\theta')}{d^2},\; d\Omega' = \frac{dA(\cos\theta)}{d^2}
$$

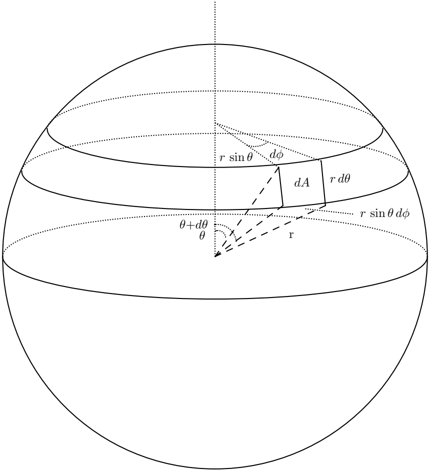
$$
I_\nu = I_\nu{}'
$$

Spherical Coordinates

$$d\Omega = \frac{dA}{r^2}$$

$$dA = (r\,d\theta)(r\sin\theta\,d\phi)$$

$$d\Omega = \sin\theta\,d\theta\,d\phi$$



$$\text{Let } \mu = \cos\theta$$

$$d\mu = -\sin\theta\,d\theta$$

$$d\Omega = -\,d\mu\,d\phi$$

Radiative Flux

Energy flowing through element of area dA in unit time, per unit frequency interval, per unit area is the monochromatic physical flux F_ν:

$$
\begin{aligned}
F_\nu &= \pi \, {F_\nu}^a \\
&= \oint I_\nu(\cos\theta)\cos\theta \, d\Omega \\
&= \int_0^{2\pi}\int_0^{\pi} I_\nu(\cos\theta)\cos\theta \, \sin\theta \, d\theta \, d\phi \\
&= \int_0^{2\pi}\int_0^{\pi/2} I_\nu(\cos\theta)\cos\theta \, \sin\theta \, d\theta \, d\phi + \int_0^{2\pi}\int_{\pi/2}^{\pi} I_\nu(\cos\theta)\cos\theta \, \sin\theta \, d\theta \, d\phi \\
&= {F_\nu}^+ + {F_\nu}^-
\end{aligned}
$$

Stellar Luminosity

Energy flowing through element of area dA in unit time, per unit area is the total physical flux $F_{\rm rad}$:

$$F_{\rm rad} = \int_0^\infty F_\nu \, d\nu$$

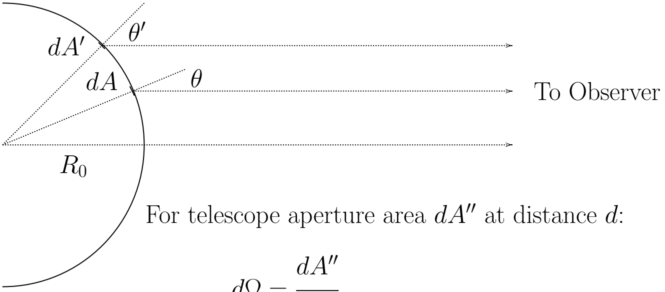
In general, no incident flux at outer boundary of atmosphere ($r = R_0$):

$$\begin{aligned} F_{\rm rad} &= \int_0^\infty {F_\nu}^+ \, d\nu \\ &= \int_0^\infty \int_0^{2\pi} \int_0^{\pi/2} I_\nu(\cos\theta) \cos\theta \sin\theta \, d\theta \, d\phi \, d\nu \end{aligned}$$

Stellar luminosity is energy per unit time from the entire stellar surface:

$$L = 4\pi \, {R_0}^2 \, F_{\rm rad}$$

Observed Flux - I



For telescope aperture area dA'' at distance d:

$$d\Omega = \frac{dA''}{d^2}$$

Contribution of dA to energy received by observer at frequency ν in interval $d\nu$, in time dt (note $dA = {R_0}^2 \sin\theta \, d\theta \, d\phi$):

$$dE_\nu = I_\nu(\cos\theta) \, \cos\theta \, dA \, d\Omega \, d\nu \, dt$$

Observed Flux - II

$$dE_\nu = I_\nu(\cos\theta)\cos\theta\, dA\, d\Omega\, d\nu\, dt$$
$$= \frac{dA''}{d^2} {R_0}^2 I_\nu(\cos\theta)\cos\theta\ \sin\theta\, d\theta\, d\phi\, d\nu\, dt$$

Integrating over the half-sphere facing the observer:

$$E_\nu = \frac{dA''}{d^2} {R_0}^2\, d\nu\, dt \int_0^{2\pi}\int_0^{\pi/2} I_\nu(\cos\theta)\cos\theta\ \sin\theta\, d\theta\, d\phi$$
$$= \frac{dA''}{d^2} {R_0}^2\, d\nu\, dt\, {F_\nu}^+$$

Monochromatic flux received by observer in unit time, per unit area per unit frequency:

$$f_\nu = \frac{{R_0}^2}{d^2} {F_\nu}^+$$

Specific Intensity Moments

$$J_\nu = \frac{1}{4\pi} \int I_\nu \, d\Omega = \frac{1}{4\pi} \int_0^{2\pi} d\phi \int_{-1}^{+1} I_\nu \, d\mu = \frac{1}{2} \int_{-1}^{+1} I_\nu \, d\mu$$

$$H_\nu = \frac{1}{4\pi} \int I_\nu \, \cos\theta \, d\Omega = \frac{1}{2} \int_{-1}^{+1} I_\nu \, \mu \, d\mu = \frac{{F_\nu}^a}{4}$$

$$K_\nu = \frac{1}{4\pi} \int I_\nu \cos^2\theta \, d\Omega = \frac{1}{2} \int_{-1}^{+1} I_\nu \, \mu^2 \, d\mu$$

Energy Density & Radiation Pressure

Energy Density:

$$u_\nu = \frac{\text{radiation energy}}{\text{volume}}$$

$$= \frac{1}{c} \int_0^{4\pi} I_\nu \, d\Omega$$

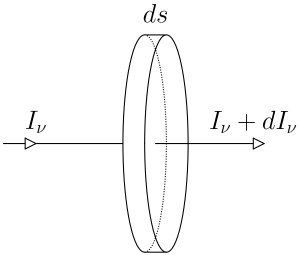
Radiation Pressure:

$$p_\nu = \frac{\text{force}}{\text{area}}$$

$$= \frac{d\,\text{momentum (= E/c)}}{dt} \frac{1}{\text{area}}$$

$$= \frac{1}{c} \int_0^{4\pi} I_\nu \cos^2\theta \, d\Omega$$

Absorption of Radiation



$$dI_\nu = -\kappa_\nu \, I_\nu \, ds$$

κ_ν : absorption coefficient $\left[\mathrm{cm}^{-1}\right]$

Microscopically: $\kappa_\nu = n \, \sigma_\nu$

Over a distance s:

$$\int_0^s \frac{dI_\nu}{I_\nu} = -\int_0^s \kappa_\nu \, ds = -\tau_\nu, \qquad d\tau_\nu = \kappa_\nu \, ds$$

$$I_\nu(s) = I_\nu(0) \exp(-\tau_\nu)$$

By convention, $\tau_\nu = 0$ at top of atmosphere and increases inwards.

Optical Depth

Optical thickness of a layer determines the specific intensity fraction passing through it.

- If $\tau_\nu = 1, \quad I_\nu(s) = I_\nu(0)/e \simeq 0.37 I_\nu(0)$.
- We can see through an atmosphere to the point where $\tau_\nu \sim 1$.
- Optically thick(thin) medium: $\tau_\nu > (<) 1$.
- $\tau_\nu = 1$ has a geometrical interpretation in terms of the mean free path of photons $\bar{s}$.

$$\tau_\nu = 1 = \int_0^{\bar{s}} \kappa_\nu \, ds$$

- Photons travel on average for a distance $\bar{s}$ before absorption.

Radiative Acceleration

Infinitesimal energy absorbed :

$$dE_\nu{}^a = dI_\nu{}^a \cos\theta \, dA \, d\Omega \, dt \, d\nu$$

$$= \kappa_\nu \, I_\nu \cos\theta \, dA \, d\Omega \, dt \, d\nu \, ds$$

If κ_ν is isotropic, total energy absorbed is:

$$E^a = \int_0^\infty \kappa_\nu \oint I_\nu \cos\theta \, d\Omega \, d\nu \, dA \, ds \, dt$$

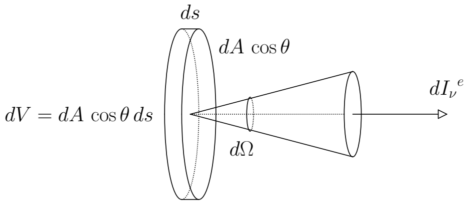
$$= \int_0^\infty \kappa_\nu \, F_\nu \, d\nu \, dA \, ds \, dt$$

Consequent rate of change of photon momentum $(E/c)/dt$ leads to the radiative acceleration ($g_{\rm rad}$):

$$\frac{1}{c} \frac{\int_0^\infty \kappa_\nu \, F_\nu \, d\nu}{dt} \, dA \, dt \, ds = g_{\rm rad} \, dm$$

$$g_{\rm rad} = \frac{1}{c\rho} \int_0^\infty \kappa_\nu \, F_\nu \, d\nu \qquad dm = \rho \, dA \, ds$$

Emission of Radiation



Energy radiated into $d\Omega$ by $dA \cos\theta$ due to emission processes in dV:

$$
\begin{aligned}
dE_\nu{}^e &= dI_\nu{}^e\, dA\ \cos\theta\, d\Omega\, d\nu\, dt \\
&= \epsilon_\nu\, dA\, d\Omega\ \cos\theta\, d\nu\, dt\, ds \\
&= \epsilon_\nu\, dV\ d\Omega\, d\nu\, dt
\end{aligned}
$$

ϵ_ν is defined as the emission coefficient and has dimensions $\left[\mathrm{erg\,cm^{-3}\,sr^{-1}\,Hz^{-1}\,s^{-1}}\right]$

Equation of Radiative Transfer

Combine emission and absorption

$$dE_\nu{}^a = dI_\nu{}^a \, dA \, \cos\theta \, d\Omega \, dt \, d\nu = -\kappa_\nu \, I_\nu \, \cos\theta \, dA \, d\Omega \, dt \, d\nu \, ds$$

$$dE_\nu{}^e = dI_\nu{}^e \, dA \, \cos\theta \, d\Omega \, dt \, d\nu = \epsilon_\nu \, \cos\theta \, dA \, d\Omega \, dt \, d\nu \, ds$$

$$\begin{aligned} dE_\nu{}^a + dE_\nu{}^e &= (dI_\nu{}^a + dI_\nu{}^e) \, dA \, \cos\theta \, d\Omega \, d\nu \, dt \\ &= (-\kappa_\nu \, I_\nu + \epsilon_\nu) \, dA \, \cos\theta \, d\Omega \, d\nu \, dt \, ds \end{aligned}$$

Writing

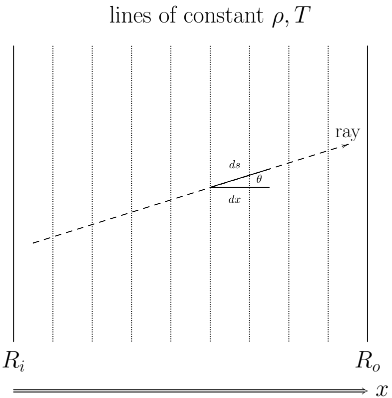
$$dI_\nu = (dI_\nu{}^a + dI_\nu{}^e)$$

gives the differential equation (the equation of radiative transfer)

$$\frac{dI_\nu}{ds} = -\kappa_\nu \, I_\nu + \epsilon_\nu$$

describing the flow of radiation through matter.

Equation of Radiative Transfer - Plane Parallel

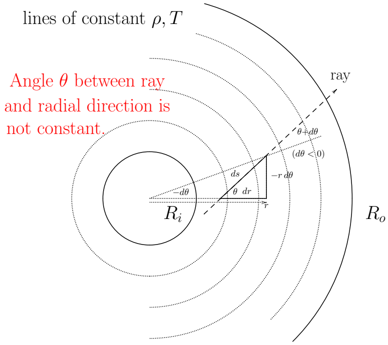


$$dx = \cos\theta \, ds = \mu \, ds$$

$$\frac{d}{ds} = \mu \, \frac{d}{dx}$$

$$\mu \, \frac{dI_\nu(\mu, x)}{dx} = -\kappa_\nu \, I_\nu(\mu, x) + \epsilon_\nu$$

Equation of Radiative Transfer - Spherical



$$\frac{d}{ds} = \frac{dr}{ds}\frac{\partial}{\partial r} + \frac{d\theta}{ds}\frac{\partial}{\partial \theta}$$

$$dr = \cos\theta\, ds = \mu\, ds \qquad \frac{dr}{ds} = \cos\theta$$

$$-r\, d\theta = \sin\theta\, ds \qquad \frac{d\theta}{ds} = \frac{-\sin\theta}{r}$$

$$\frac{\partial}{\partial\theta} = \frac{\partial\mu}{\partial\theta}\frac{\partial}{\partial\mu} = -\sin\theta\frac{\partial}{\partial\mu}$$

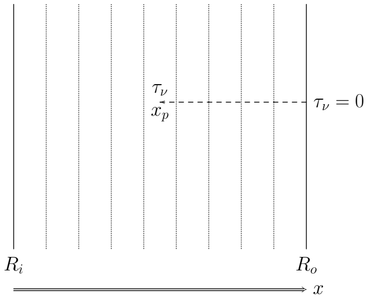
$$\frac{d}{ds} = \mu\frac{\partial}{\partial r} + \frac{\sin^2\theta}{r}\frac{\partial}{\partial\mu}$$

$$= \mu\frac{\partial}{\partial r} + \frac{1-\mu^2}{r}\frac{\partial}{\partial\mu}$$

$$\mu\frac{\partial I_\nu(\mu, r)}{\partial r} + \frac{1-\mu^2}{r}\frac{\partial I_\nu(\mu, r)}{\partial\mu} = -\kappa_\nu\, I_\nu(\mu, r) + \epsilon_\nu$$

Optical Depth & Source Function

Optical depth increasing towards interior.



$$-\kappa_\nu \, dx = d\tau_\nu \qquad \tau_\nu = -\int_{R_o}^{x_p} \kappa_\nu \, dx$$

For a plane-parallel atmosphere

$$\mu \frac{dI_\nu(\mu, x)}{dx} = -\kappa_\nu(x) \, I_\nu(\mu, x) + \epsilon_\nu$$

gives

$$\mu \frac{dI_\nu(\mu, \tau_\nu)}{d\tau_\nu} = I_\nu(\mu, \tau_\nu) - S_\nu(\tau_\nu)$$

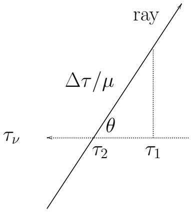
where

$S_\nu = \epsilon_\nu / \kappa_\nu$ is the SourceFunction.

$$\kappa_\nu = \frac{d\tau_\nu}{ds} \simeq \frac{\Delta\tau_\nu}{\Delta s} \simeq \frac{1}{\bar{s}} \quad S_\nu = \epsilon_\nu / \kappa_\nu \simeq \epsilon_\nu \, \bar{s}$$

Since photon mean free path is $\Delta\tau_\nu = 1$, S_ν corresponds to intensity emitted over this distance.

Radiative Transfer Equation – Formal Solution



For a plane-parallel atmosphere:

$$\mu \frac{dI_\nu}{d\tau_\nu} = I_\nu - S_\nu$$

$$e^{-\tau_\nu/\mu}\, \mu \frac{dI_\nu}{d\tau_\nu} = I_\nu\, e^{-\tau_\nu/\mu} - S_\nu\, e^{-\tau_\nu/\mu}$$

$$\frac{d}{d\tau_\nu}(I_\nu\, e^{-\tau_\nu/\mu}) = -\frac{S_\nu\, e^{-\tau_\nu/\mu}}{\mu}$$

Integrate between τ_1 (outside) to τ_2 (inside, $\tau_1 < \tau_2$):

$$\left[I_\nu\, e^{-\tau_\nu/\mu}\right]_{\tau_1}^{\tau_2} = -\int_{\tau_1}^{\tau_2} S_\nu\, e^{-\tau_\nu/\mu} \frac{d\tau_\nu}{\mu}$$

$$I_\nu(\tau_1, \mu) = I_\nu(\tau_2, \mu)\, e^{-(\tau_2-\tau_1)/\mu} + \int_{\tau_1}^{\tau_2} S_\nu(t)\, e^{-(t-\tau_1)/\mu} \frac{dt}{\mu}$$

In general, S_ν depends on I_ν and an actual solution is challenging.

Radiative Transfer Equation – Boundary Conditions

For incoming radiation $\mu < 0$ and inward radiation from outside is usually neglected: $I_\nu(\tau_\nu = 0, \mu < 0) = 0$

$$I_\nu^{\rm in}(\tau_\nu, \mu) = \int_{\tau_\nu}^{0} S_\nu(t)\, e^{-(t-\tau_\nu)/\mu}\, \frac{dt}{\mu}$$

For outgoing radiation $\mu > 0$ and we have either

- a finite slab or shell for which $I_\nu(\tau_{\rm max}, \mu) = I_\nu^{+}(\mu)$ or
- a semi-infinite (planar or spherical) case where $\lim_{\tau_\nu \to \infty} I_\nu(\tau_\nu, \mu)\, e^{-\tau_\nu/\mu} = 0$

The second applies in the case of stellar atmospheres and so

$$I_\nu^{\rm out}(\tau_\nu, \mu) = \int_{\tau_\nu}^{\infty} S_\nu(t)\, e^{-(t-\tau_\nu)/\mu}\, \frac{dt}{\mu}$$

For the case where $\tau_\nu = 0$, the above equation gives the emergent intensity:

$$I_\nu(0, \mu) = \int_{0}^{\infty} S_\nu(t)\, e^{-t/\mu}\, \frac{dt}{\mu}$$

Source Function – Simple Cases – I

- In Local Thermodynamic Equilibrium, photons are absorbed and re-emitted at the local temperature (T) (Kirchhoff's Law)

$$S_\nu = \frac{\epsilon_\nu}{\kappa_\nu} = B_\nu(T).$$

- For coherent isotropic scattering, absorption is characterised by the scattering coefficient σ_ν, analogous to κ_ν:

$$dI_\nu = -\sigma_\nu \, I_\nu \, ds \qquad dE_\nu{}^{\mathrm{em}} = \oint_{4\pi} \epsilon_\nu{}^{\mathrm{sc}} \, d\Omega \qquad dE_\nu{}^{\mathrm{abs}} = \oint_{4\pi} \sigma_\nu{}^{\mathrm{sc}} \, I_\nu \, d\Omega.$$

At each ν, $dE_\nu{}^{\mathrm{em}} = dE_\nu{}^{\mathrm{abs}}$:

$$\oint_{4\pi} \epsilon_\nu{}^{\mathrm{sc}} \, d\Omega = \oint_{4\pi} \sigma_\nu I_\nu \, d\Omega$$

$$\epsilon_\nu{}^{\mathrm{sc}} \oint_{4\pi} d\Omega = \sigma_\nu \oint_{4\pi} I_\nu \, d\Omega$$

$$\frac{\epsilon_\nu{}^{\mathrm{sc}}}{\kappa_\nu} = \frac{1}{4\pi} \oint_{4\pi} I_\nu \, d\Omega$$

$$S_\nu = J_\nu.$$

Source function is completely dependent on radiation field and independent of T.

Source Function – Simple Cases – II

- Mixed case:

$$S_\nu = \frac{\epsilon_\nu + {\epsilon_\nu}^{\rm sc}}{\kappa_\nu + \sigma_\nu}$$

$$= \frac{\kappa_\nu}{\kappa_\nu + \sigma_\nu} \frac{\epsilon_\nu}{\kappa_\nu} + \frac{\sigma_\nu}{\kappa_\nu + \sigma_\nu} \frac{{\epsilon_\nu}^{\rm sc}}{\sigma_\nu}$$

$$= \frac{\kappa_\nu}{\kappa_\nu + \sigma_\nu} B_\nu + \frac{\sigma_\nu}{\kappa_\nu + \sigma_\nu} J_\nu$$

Diffusion Approximation – I

At large optical depths ($\tau_\nu \gg 1$) and photons are local so that $S_\nu \rightarrow B_\nu$. Expanding as a power series about τ_ν:

$$S_\nu(t) = B_\nu(t) = \sum_{n=0}^{\infty} \frac{d^n B_\nu(\tau_\nu)}{d{\tau_\nu}^n}(t-\tau_\nu)^n/n!$$

Photons are local and therefore $(t-\tau_\nu) \sim 0$, justifying the retention of only the first order term (Diffusion Approximation):

$$B_\nu(t) = B_\nu(\tau_\nu) + \frac{dB_\nu}{d\tau_\nu}(t-\tau_\nu)$$

$$\begin{aligned} I_\nu^{\text{out}}(\tau_\nu, \mu) &= \int_{\tau_\nu}^{\infty} S_\nu(t)\, e^{-(t-\tau_\nu)/\mu}\, \frac{dt}{\mu} \\ &= \int_{\tau_\nu}^{\infty} \left[B_\nu(\tau_\nu) + \frac{dB_\nu}{d\tau_\nu}(t-\tau_\nu) \right] e^{-(t-\tau_\nu)/\mu}\, \frac{dt}{\mu} \end{aligned}$$

Diffusion Approximation – II

Let

$$u = \frac{t - \tau_\nu}{\mu} \qquad \rightarrow \qquad dt = \mu\, du$$

and since

$$\int_0^\infty u^k\, e^{-u}\, du = k!$$

$$I_\nu^{\rm out}(\tau_\nu, \mu) = \int_{\tau_\nu/\mu}^\infty \left[B_\nu(t) + \frac{dB_\nu}{d\tau_\nu}\, \mu\, u \right] e^{-u}\, du$$

$$= B_\nu(t) + \mu\, \frac{dB_\nu}{d\tau_\nu}$$

$$I_\nu^{\rm in}(\tau_\nu, \mu) = -\int_0^{\tau_\nu/\mu} \left[B_\nu(t) + \frac{dB_\nu}{d\tau_\nu}\, \mu\, u \right] e^{-u}\, du$$

Eddington-Barbier relation for observed emergent intensity obtained for $\tau_\nu = 0$; it depends linearly on μ.

Eddington Approximation

In a planar atmosphere with

$$I_\nu = B_\nu + \mu \frac{dB_\nu}{d\tau_\nu}$$

we have:

$$J_\nu = \frac{1}{2}\int_{-1}^{+1} I_\nu \, d\mu = \frac{1}{2}\left[\mu \, B_\nu + \frac{\mu^2}{2}\frac{dB_\nu}{d\tau_\nu}\right]_{-1}^{+1} = B_\nu(\tau_\nu)$$

$$H_\nu = \frac{{F_\nu}^a}{4} = \frac{1}{2}\int_{-1}^{+1} I_\nu \, \mu \, d\mu = \frac{1}{2}\left[\frac{\mu^2}{2} B_\nu + \frac{\mu^3}{3}\frac{dB_\nu}{d\tau_\nu}\right]_{-1}^{+1}$$
$$= \frac{1}{3}\frac{dB_\nu}{d\tau_\nu} = -\frac{1}{3}\frac{1}{\kappa_\nu}\frac{dB_\nu}{dx} = -\frac{1}{3}\frac{1}{\kappa_\nu}\frac{dB_\nu}{dT}\frac{dT}{dx}$$

$$K_\nu = \frac{1}{2}\int_{-1}^{+1} I_\nu \, \mu^2 \, d\mu = \frac{1}{2}\left[\frac{\mu^3}{3} B_\nu + \frac{\mu^4}{4}\frac{dB_\nu}{d\tau_\nu}\right]_{-1}^{+1} = \frac{1}{3} B_\nu(\tau_\nu)$$

Schwarzschild-Milne Equations – I

$$J_\nu = \frac{1}{2}\int_{-1}^{+1} I_\nu \, d\mu = \frac{1}{2}\int_{0}^{+1} {I_\nu}^{\text{out}} \, d\mu + \frac{1}{2}\int_{-1}^{0} {I_\nu}^{\text{in}} \, d\mu$$

$$= \frac{1}{2}\left[\int_0^1 \int_{\tau_\nu}^{\infty} S_\nu(t)\, e^{-(t-\tau_\nu)/\mu} \frac{dt}{\mu}\, d\mu \; - \int_{-1}^{0}\int_0^{\tau_\nu} S_\nu(t)\, e^{-(t-\tau_\nu)/\mu} \frac{dt}{\mu}\, d\mu\right]$$

$$= \frac{1}{2}\left[\int_1^{\infty}\int_{\tau_\nu}^{\infty} S_\nu(t)\, e^{-(t-\tau_\nu)w}\, dt\, \frac{dw}{w} \; + \int_1^{\infty}\int_0^{\tau_\nu} S_\nu(t)\, e^{-(\tau_\nu-t)w}\, dt\, \frac{dw}{w}\right]$$

$$= \frac{1}{2}\left[\int_{\tau_\nu}^{\infty} S_\nu(t)\int_1^{\infty} e^{-(t-\tau_\nu)w} \frac{dw}{w} dt \; + \int_0^{\tau_\nu} S_\nu(t)\int_1^{\infty} e^{-(\tau_\nu-t)w} \frac{dw}{w} dt\right]$$

Where $w = 1/\mu$ and $w = -1/\mu$ for left and right double integrals respectively; in both cases $dw/w = -d\mu/\mu$. Since both exponents are greater than zero:

$$J_\nu = \frac{1}{2}\int_0^{\infty} S_\nu(t)\int_1^{\infty} e^{-w|t-\tau_\nu|} \frac{dw}{w} dt$$

$$= \frac{1}{2}\int_0^{\infty} S_\nu(t)\, E_1(|t-\tau_\nu|)\, dt$$

Schwarzschild-Milne Equations – II

Introducing the Λ-Operator:

$$\Lambda_{\tau_\nu} = \frac{1}{2}\int_0^\infty f(t)\, E_1(|t-\tau_\nu|)\, dt$$

$$J_\nu(\tau_\nu) = \Lambda_{\tau_\nu}\,[S_\nu(t)]$$

Similarly for the other two specific intensity moments:

$$H_\nu(\tau_\nu) = \frac{1}{2}\int_{\tau_\nu}^\infty S_\nu(t)\, E_2(|t-\tau_\nu|)\, dt \;-\frac{1}{2}\int_0^{\tau_\nu} S_\nu(t)\, E_2(|\tau_\nu - t|)\, dt$$

$$= \Phi_{\tau_\nu}\,[S_\nu(t)]$$

$$K_\nu(\tau_\nu) = \frac{1}{2}\int_0^\infty S_\nu(t)\, E_3(|t-\tau_\nu|)\, dt$$

$$= X_{\tau_\nu}\,[S_\nu(t)]$$

J_ν, H_ν and K_ν are depth-weighted means of S_ν, the largest contribution being when $t - \tau_\nu = 0$.

E_1, E_2 and E_3 are the first, second and third exponential integrals.

Lecture 2: Summary

Essential points covered in second lecture:

- Specific intensity defined and its invariance, in the absence of absorption, verified.
- It was shown how specific intensity is related to radiative flux, luminosity and observed flux.
- Energy density, radiation pressure and the absorption of radiation were discussed.
- The equation of radiative transfer, optical depth and source function were introduced. Simple special case solutions of the transfer equation were presented with formal solution and boundary conditions.
- Diffusion approximation needed for stellar structure and evolution calculations was derived.
- Schwarzschild-Milne equations were also derived as these are needed for stellar atmosphere and synthetic spectrum calculations.

Stellar evolution depends on initial photospheric abundances and their determination from spectra are to be discussed in the next lecture.

Acknowledgement

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