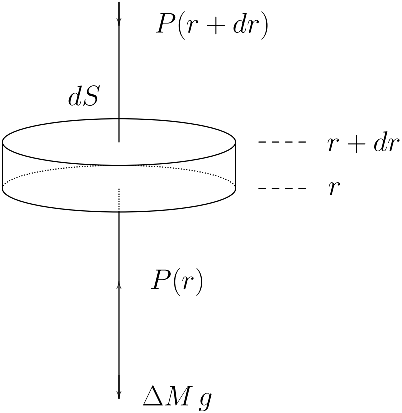
Stellar Atmospheres Introduction

Synthetic stellar spectra used for the determination of chemical element abundances were, until recently, based on one-dimensional model stellar atmospheres.

- At any optical (or geometric) depth, gravity is exactly balanced by pressure (gas, electron and radiation) – hydrostatic equilibrium
- A constant temperature is maintained at every depth; radiative energy received from below is equal to the energy radiated to layers above although frequency redistribution will have occurred – radiative equilibrium.
- Atmosphere assumed to be static and convection treated with crude mixing length theory, using an empirically determined mixing length.
- Opacities needed are dependent on abundances sought.

Model stellar atmosphere may be obtained empirically, in the case of the Sun, by direct measurement of limb-darkening; for other cases where only F_ν is observable, an iterative calculation is needed with some realistic estimate used as a starting approximation.

Hydrostatic Equilibrium – I



Consider a cylindrical element of mass ΔM and surface area dS between radii r and $r+dr$ in a star.

Mass of gas in the star at smaller radii $= m = m(r)$.

Hydrostatic Equilibrium – II

Radial forces acting on the element

- Gravity (inward)

$$F_g = -\frac{G\,m\,\Delta M}{r^2}$$

- Pressure Difference (outward – net force due to difference in pressure between upper and lower faces)

$$\begin{aligned} F_p &= P(r)\,dS - P(r+dr)\,dS \\ &= P(r)\,dS - \left[P(r) + \frac{dP}{dr}\times dr\right]\,dS \\ &= -\frac{dP}{dr}\times dr\,dS \end{aligned}$$

If ρ is gas density in element $\Delta M = \rho\,dr\,dS$.
Inward and outward forces must balance in hydrostatic equilibrium:

$$\begin{aligned} 0 &= -\frac{G\,m\,\rho\,dr\,dS}{r^2} - \frac{dP}{dr}\,dr\,dS \\ \frac{dP}{dr} &= -\frac{G\,m}{r^2}\,\rho \qquad \text{(First Equation of Stellar Structure)} \\ &= -g\,\rho \qquad\qquad \text{(in stellar atmosphere)} \end{aligned}$$

Radiative Equilibrium – I

Radiative equilibrium requires the flux at any depth to be constant.

$$F(r) = \int_0^\infty \oint_{4\pi} I_\nu \cos\theta \, d\Omega \, d\nu = 4\pi \int_0^\infty H_\nu \, d\nu$$

$$4\pi r^2 F(r) = 4\pi r^2 \, . \, 4\pi \int_0^\infty H_\nu \, d\nu = \text{constant} = L$$

For a plane-parallel atmosphere $r \simeq R$ and

$$4\pi \int_0^\infty H_\nu \, d\nu = \text{constant} = \sigma \, T_{\text{eff}}$$

Integrating radiative transfer equation in plane-parallel geometry

$$\mu \frac{dI_\nu}{dx} = -(\kappa_\nu + \sigma_\nu)(I_\nu - S_\nu)$$

over all angles, gives:

$$\frac{1}{2} \int_{-1}^{1} \mu \frac{dI_\nu}{dx} \, d\mu = -\frac{1}{2} \int_{-1}^{1} (\kappa_\nu + \sigma_\nu)(I_\nu - S_\nu) \, d\mu$$

$$\frac{d}{dx} \left[\frac{1}{2} \int_{-1}^{1} \mu \, I_\nu \, d\mu \right] = -(\kappa_\nu + \sigma_\nu)(J_\nu - S_\nu)$$

$$\frac{dH_\nu}{dx} = -(\kappa_\nu + \sigma_\nu)(J_\nu - S_\nu)$$

Radiative Equilibrium – II

Integrating over frequency:

$$\frac{d}{dx}\int_0^\infty H_\nu \, d\nu = -\int_0^\infty (\kappa_\nu + \sigma_\nu)(J_\nu - S_\nu)\, d\nu$$

The integral on the left-hand side is constant and therefore:

$$\int_0^\infty (\kappa_\nu + \sigma_\nu)(J_\nu - S_\nu)\, d\nu = 0$$

Substituting

$$S_\nu = \frac{\kappa_\nu}{\kappa_\nu + \sigma_\nu} B_\nu \; + \; \frac{\sigma_\nu}{\kappa_\nu + \sigma_\nu} J_\nu$$

gives

$$\int_0^\infty \left[(\kappa_\nu + \sigma_\nu) J_\nu \; - \; \kappa_\nu \, B_\nu \; - \; \sigma_\nu \, J_\nu \right] \, d\nu = 0$$

$$\int_0^\infty \kappa_\nu \left[J_\nu \; - \; B_\nu(T) \right] \, d\nu = 0$$

At every depth, the temperature T has to assume a value which satisfies the above equation.

Grey Approximation

Approximate radiative equilibrium integral using a frequency-averaged absorption coefficient $\bar{\kappa}$ placed in front of the integral:

$$\bar{\kappa} \int_0^\infty \left[J_\nu - B_\nu(T) \right] \, d\nu = 0$$

With

$$J = \int_0^\infty J_\nu \, d\nu \quad H = \int_0^\infty H_\nu \, d\nu \quad K = \int_0^\infty K_\nu \, d\nu \quad B = \int_0^\infty B_\nu \, d\nu = \frac{\sigma T^4}{\pi}$$

radiative equilibrium then implies

$$J = B$$

$$4\pi H = \sigma {T_{\mathrm{eff}}}^4.$$

In LTE and radiative equilibrium $S = B = J$ and with a mean optical depth $d\bar{\tau} = \bar{\kappa} \, dx$

$$J(\bar{\tau}) = \Lambda_{\bar{\tau}} \left[J(t) \right] = \frac{1}{2} \int_0^\infty J(t) \, E_1(|t - \bar{\tau}|) \, dt$$

with an exact solution

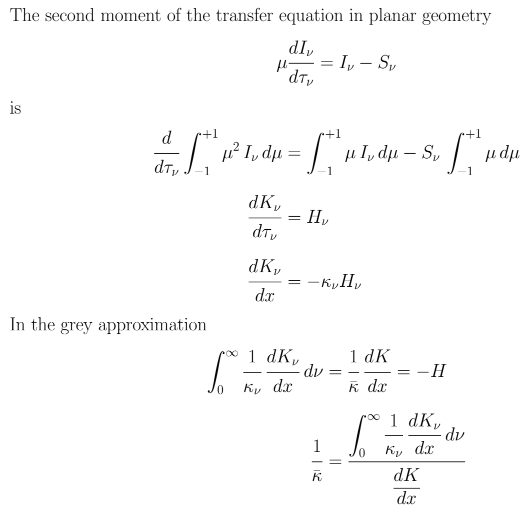
$$J(\bar{\tau}) = B(\bar{\tau}) = (\sigma/\pi) T(\bar{\tau})^4 = \text{constant} \times \left[\bar{\tau} + q(\bar{\tau}) \right]$$

and boundary condition

$$T(\bar{\tau})^4 = 0.75 \, {T_{\mathrm{eff}}}^4 \left[\bar{\tau} + q(\bar{\tau}) \right]$$

where $q(\bar{\tau}) \sim 2/3$ is Hopf's function.

Rosseland Mean Opacity - I



The second moment of the transfer equation in planar geometry

$$\mu \frac{dI_\nu}{d\tau_\nu} = I_\nu - S_\nu$$

is

$$\frac{d}{d\tau_\nu} \int_{-1}^{+1} \mu^2 \, I_\nu \, d\mu = \int_{-1}^{+1} \mu \, I_\nu \, d\mu - S_\nu \int_{-1}^{+1} \mu \, d\mu$$

$$\frac{dK_\nu}{d\tau_\nu} = H_\nu$$

$$\frac{dK_\nu}{dx} = -\kappa_\nu H_\nu$$

In the grey approximation

$$\int_0^\infty \frac{1}{\kappa_\nu} \frac{dK_\nu}{dx} \, d\nu = \frac{1}{\bar{\kappa}} \frac{dK}{dx} = -H$$

$$\frac{1}{\bar{\kappa}} = \frac{\displaystyle\int_0^\infty \frac{1}{\kappa_\nu} \frac{dK_\nu}{dx} \, d\nu}{\dfrac{dK}{dx}}$$

Rosseland Mean Opacity - II

As $\tau \to \infty$, $K_\nu \to (1/3)\, J_\nu$, $J_\nu \to B_\nu$ and

$$\frac{dK_\nu}{dx} \to \frac{1}{3}\frac{dB_\nu}{dx} = \frac{1}{3}\frac{dB_\nu}{dT}\frac{dT}{dx}$$

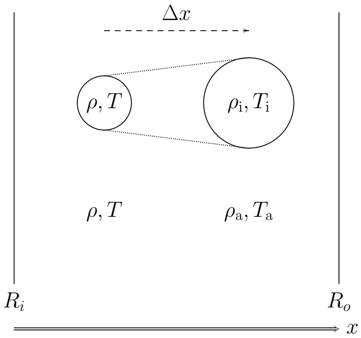
Substitution in above mean opacity expression gives definition for **Rosseland Mean Opacity**

$$\frac{1}{\bar{\kappa}_{\mathrm{Ross}}} = \frac{\displaystyle\int_0^\infty \frac{1}{\kappa_\nu}\frac{dB_\nu(T)}{dT}\,d\nu}{\displaystyle\int_0^\infty \frac{dB_\nu(T)}{dT}\,d\nu}$$

which has the properties:

- large weight for low-opacity (more transparent to radiation) regions,
- at large τ the T structure is accurately given by $T^4 = 0.75\, {T_{\mathrm{eff}}}^4\, [\tau_{\mathrm{Ross}} + q(\tau_{\mathrm{Ross}})]$,
- $\bar{\kappa}_{\mathrm{Ross}}$ used in stellar interiors and
- for stellar atmospheres using $\bar{\kappa}_{\mathrm{Ross}}$ allows an approximate temperature stratification to be obtained and used as a starting point for further iterations.

Schwarzschild Criterion for Convective Instability – I



Assume mass element ("cell") in photosphere is perturbed slowly ($v < v_\mathrm{sound}$) so it rises adiabatically (no energy exchanged with surroundings):

- Ambient density decreases ($\rho \to \rho_\mathrm{a}$).
- Cell expands and its internal density also decreases ($\rho \to \rho_\mathrm{i}$).

There are two cases:

- $\rho_\mathrm{i} > \rho_\mathrm{a}$ cell falls back → stable.
- $\rho_\mathrm{i} < \rho_\mathrm{a}$ cell rises further → unstable.

Schwarzschild Criterion for Convective Instability – II

At the end of adiabatic expansion, the density and temperature of the cell are ρ_i and T_i respectively. Identifying environmental quantities as “**rad**iative” and those in the cell as “**ad**iabatic” we make the substitutions: $\rho_\mathrm{i} \rightarrow \rho_\mathrm{ad}$, $T_\mathrm{i} \rightarrow T_\mathrm{ad}$, $\rho_\mathrm{a} \rightarrow \rho_\mathrm{rad}$ and $T_\mathrm{a} \rightarrow T_\mathrm{rad}$.

Change in cell density when rising over radial distance Δx

$$\Delta\rho_\mathrm{ad} = \left[\frac{d\rho}{dx}\right]_\mathrm{ad} \Delta x$$

that is we obtain stability if

$$|\,\Delta\rho_\mathrm{ad}\,| \;<\; |\,\Delta\rho_\mathrm{rad}\,|$$

or

$$\left[\frac{d\rho}{dx}\right]_\mathrm{ad} \;<\; \left[\frac{d\rho}{dx}\right]_\mathrm{rad}$$

Schwarzschild Criterion for Convective Instability – III

We assume that the motion is slow ($v \ll v_{\mathrm{sound}}$) so that pressure equilibrium prevails; pressure inside cell is equal to the pressure outside

$$P_{\mathrm{rad}} = P_{\mathrm{ad}}$$

From the equation of state $P \sim \rho T$ and so

$$\rho_{\mathrm{rad}}\, T_{\mathrm{rad}} = \rho_{\mathrm{ad}}\, T_{\mathrm{ad}}$$

and an equivalent criterion for stability against convection is

$$\left|\frac{dT}{dx}\right|_{\mathrm{ad}} > \left|\frac{dT}{dx}\right|_{\mathrm{rad}} .$$

The equation of hydrostatic equilibrium and the perfect gas equation are

$$dP_{\mathrm{gas}} = -g\,\rho(x)\,dx \qquad P_{\mathrm{gas}} = \frac{k}{m_{\mathrm{H}}\bar{\mu}}\,\rho T$$

give

$$\left|\frac{dT}{dx}\right| = \left|\frac{dT}{dP}\,\frac{dP}{dx}\right| = \left|\frac{dT}{dP}\,g\,\rho\right| = \left|\frac{dT}{dP}\right|\,g\,\frac{m_{\mathrm{H}}\bar{\mu}}{kT}\,P = g\,\frac{m_{\mathrm{H}}\bar{\mu}}{k}\,\left|\frac{d\,\ln\,T}{d\,\ln\,P}\right|$$

Schwarzschild Criterion for Convective Instability – IV

Therefore the criterion for stability against convention in a stellar atmosphere or enevelope, known as the **Schwarzschild Criterion** is:

$$\left|\frac{d\ \ln\ T}{d\ \ln\ P}\right|_{\mathrm{ad}} > \left|\frac{d\ \ln\ T}{d\ \ln\ P}\right|_{\mathrm{rad}}$$

or instability occurs when the temperature gradient in a stellar atmosphere or envelope is larger than the adiabatic gradient, leading to convection. In an adiabatic process

$$P \sim \rho^{\gamma} \quad \text{where} \quad \gamma = C_{\mathrm{p}}/C_{\mathrm{v}}$$

and from the perfect gas equation of state

$$P_{\mathrm{gas}} = \frac{k}{m_{\mathrm{H}}\bar{\mu}}\rho T,$$

$$P^{\gamma-1} \sim T^{\gamma}$$

and so

$$\left|\frac{d\ \ln\ T}{d\ \ln\ P}\right|_{\mathrm{ad}} = \frac{\gamma-1}{\gamma} = 1 - \frac{1}{\gamma}$$

Schwarzschild Criterion for Convective Instability – V

In a plasma which is completely ionised or neutral $\gamma = 5/3$ and

$$\left|\frac{d\ \ln\ T}{d\ \ln\ P}\right|_{\rm ad} = 0.4$$

With changing degree of ionisation:

- Plasma undergoes phase transition.
- Specific heats C_p and C_v are no longer constant.
- Adiabatic exponent γ changes.

In general

$$\left|\frac{d\ \ln\ T}{d\ \ln\ P}\right|_{\rm ad} = \frac{2 + X_{\rm ion}(1 - X_{\rm ion})\left(\frac{5}{2} + \frac{E_{\rm ion}}{kT}\right)}{5 + X_{\rm ion}(1 - X_{\rm ion})\left(\frac{5}{2} + \frac{E_{\rm ion}}{kT}\right)^2}$$

where

$$X_{\rm ion} = \frac{n_{\rm e}}{n_{\rm p} + n_{\rm H}}$$

is the degree of ionisation and $E_{\rm ion}$ is ionisation potential. For $X_{\rm ion} = 0$ or 1

$$\left|\frac{d\ \ln\ T}{d\ \ln\ P}\right|_{\rm ad} = 0.40$$

For $X_{\rm ion} = 0.5$, a minimum

$$\left|\frac{d\ \ln\ T}{d\ \ln\ P}\right|_{\rm ad} = 0.07$$

is reached.

Conditions for Convection

For a grey atmosphere or a stellar interior

$$T^4 = 0.75\, {T_{\rm eff}}^4 \left[\tau_{\rm Ross} + \frac{2}{3}\right]$$

$$\left|\frac{dT}{dx}\right|_{\rm rad} = \frac{\bar{\kappa}_{\rm Ross}}{4T^3}\,\frac{3}{4}\,{T_{\rm eff}}^4$$

$$\left|\frac{d\ln T}{d\ln P}\right|_{\rm rad} = \frac{k}{g\,\bar{\mu}\,m_H}\,\frac{\bar{\kappa}_{\rm Ross}}{4T^3}\,\frac{3}{4}\,{T_{\rm eff}}^4 = \frac{3}{16}\left(\frac{T_{\rm eff}}{T}\right)^4 \bar{\kappa}_{\rm Ross}\,H \simeq \frac{3}{16}\,\frac{P}{\sigma g \rho T^4}\,\bar{\kappa}_{\rm Ross}\,F$$

where $H = kT/(g\,\bar{\mu}\,m_H)$ is the pressure scale height.

Conditions for convection:

- Adiabatic Gradient $[(d\ln T/d\ln P)_{\rm ad}]$ small (γ-effect);
- Radiative Gradient $[(d\ln T/d\ln P)_{\rm rad}]$ large (κ-effect) when $\bar{\kappa}_{\rm Ross}$ large (F or H large are less important);
- strong absorption from the $n = 2$ level in hydrogen at $T \simeq 9000$K gives large $\bar{\kappa}_{\rm Ross}$ and hence hydrogen convection zone in Sun;
- molecular phase transitions occur in stars cooler than the Sun and molecular opacities are increasingly important, convection therefore becomes more and more important and
- scale-height also becomes important in red giants and supergiants.

Mixing Length Theory – I

A simple approach to a complicated phenomenon:

- Suppose atmosphere becomes unstable at $r = r_0$ and mass element (cell) rises for a characteristic distance ℓ (**mixing length**) to $r_0 + \ell$.
- Cell excess energy is released into the ambient medium.
- Cell cools, sinks back down, absorbs energy and rises again.
- The mixing length (ℓ) is assumed to be equal for all cells.
- The velocity (v) of all cells is equal.

Note that the last two assumptions are made for ad hoc simplicity; there is no real justification for them beyond the fact that Mixing Length Theory provides a remarkably good representation of convection in stellar atmospheres and envelopes.

Mixing Length Theory – II

Given the pressure scale height

$$H = \frac{kT}{(g\,\bar{\mu}\,m_H)}$$

we adopt an adjustable parameter $\alpha = \ell/H$ as a parameterisation of the mixing length ($\alpha \sim 0.5 \to 2.0\,\mathrm{km/s}$) and calculate the consequent energy flux.

For a cell moving up with speed v

$$\text{Flux of energy transferred} = \text{mass flow} \times \text{heat transferred per gram}$$

$$F_{\text{conv}} = \rho\, v \times dQ = \rho\, v\, C_p\, \Delta T$$

We need to estimate v from the work done by the buoyancy force

$$|f_{\text{b}}| = g\,|\Delta\rho|$$

by relating the cell – surroundings density difference $(\Delta\rho)$ to the known temperature difference (ΔT). In pressure equilibrium, the equation of state

$$P = \frac{\rho\, k\, T}{\bar{\mu}\, m_{\text{H}}}$$

gives

$$\frac{dP}{P} = \frac{d\rho}{\rho} + \frac{dT}{T} - \frac{d\bar{\mu}}{\bar{\mu}} = 0$$

$$d\rho = -\rho\left(\frac{dT}{T} - \frac{d\bar{\mu}}{\bar{\mu}}\right) = -\rho\,\frac{dT}{T}\left(1 - \frac{d\ln\bar{\mu}}{d\ln T}\right)$$

Mixing Length Theory – III

Work done by buoyancy force

$$w = \int_0^{\ell} |f_\mathrm{b}| \, d(\Delta r) = \int_0^{\ell} g \, |\Delta\rho| \, d(\Delta r)$$

where

$$|\Delta\rho| = \frac{\rho}{T}\,\Delta T \left|1 - \frac{d\ln\bar{\mu}}{d\ln T}\right| \qquad \text{and} \qquad \Delta T = \left[\left|\frac{dT}{dr}\right|_\mathrm{rad} - \left|\frac{dT}{dr}\right|_\mathrm{ad}\right] \Delta r.$$

Assume integrand above is constant over integration interval (crude approximation):

$$w = g\,\frac{\rho}{T}\left|1 - \frac{d\ln\bar{\mu}}{d\ln T}\right| \left[\left|\frac{dT}{dr}\right|_\mathrm{rad} - \left|\frac{dT}{dr}\right|_\mathrm{ad}\right] \times \frac{1}{2}\,\ell^2.$$

Cell kinetic energy must be work done by buoyancy force:

$$v = \left[\frac{g}{T}\left|1 - \frac{d\ln\bar{\mu}}{d\ln T}\right|\right]^{1/2} \left[\left|\frac{dT}{dr}\right|_\mathrm{rad} - \left|\frac{dT}{dr}\right|_\mathrm{ad}\right]^{1/2} \ell$$

and so the convective flux is given by

$$F_\mathrm{conv} = \rho\, v\, C_\mathrm{p}\, \Delta T = \rho\, C_\mathrm{p} \left[\frac{g}{T}\left|1 - \frac{d\ln\bar{\mu}}{d\ln T}\right|\right]^{1/2} \left[\left|\frac{dT}{dr}\right|_\mathrm{rad} - \left|\frac{dT}{dr}\right|_\mathrm{ad}\right]^{3/2} \ell^2.$$

From the equations of state and hydrostatic equilibrium

$$\left|\frac{dT}{dr}\right| = \frac{g\, m_\mathrm{H}\, \bar{\mu}}{k}\left|\frac{d\ln T}{d\ln P}\right| = \frac{T}{H}\left|\frac{d\ln T}{d\ln P}\right|,$$

Mixing Length Theory – IV

a final expression for the convective flux is

$$F_{\rm conv} = \rho\, C_{\rm p}\, \alpha^2\, T \left[g\, H \left| 1 - \frac{d \ln \bar{\mu}}{d \ln T} \right| \right]^{1/2} \left[\left| \frac{d\ \ln\ T}{d\ \ln\ P} \right|_{\rm rad} - \left| \frac{d\ \ln\ T}{d\ \ln\ P} \right|_{\rm ad} \right]^{3/2} .$$

We require that the total energy flux is given by

$$F = F_{\rm rad} + F_{\rm conv} = \sigma\, {T_{\rm eff}}^4$$

but as temperature stratification was initially calculated assuming

$$F = F_{\rm rad} = \sigma\, {T_{\rm eff}}^4$$

a correction $\Delta T(\tau)$ must be iteratively applied to correct the T stratification if the convection instability criterion is satisfied.

Note also that $F_{\rm conv} \sim T^{1.5}$ whereas $F_{\rm rad} \sim {T_{\rm eff}}^4$ and convection is therefore not effective in the atmospheres of hot stars.

Classical Stellar Atmosphere Models – I

Parameters defining a model stellar atmosphere are $T_{\rm eff}$, $\log g$ and abundances of chemical elements. Procedure for iteratively computing a stellar atmosphere is:

- Define about 100 depth-points (layers) in the atmosphere having monotonically increasing geometric depth. Each layer is assumed to have uniform properties in the horizontal direction.

- Apply the following steps to all depth-points.

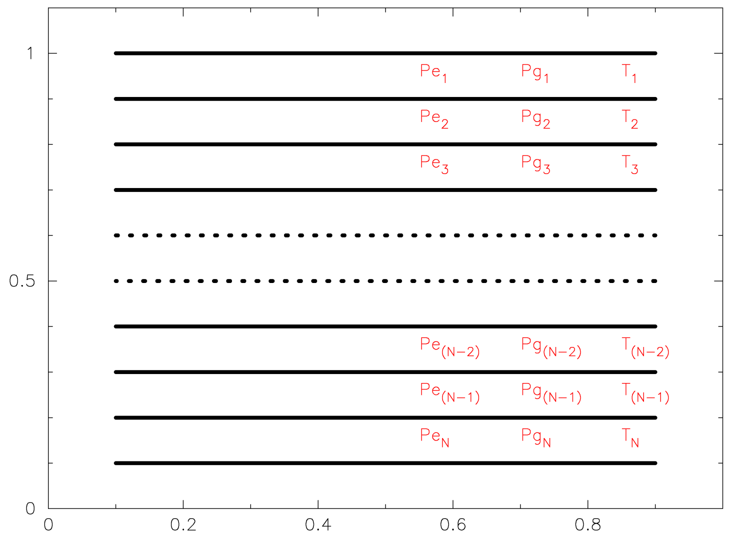
  1) Establish initial values for $T(x)$, $\kappa_\nu(x)$, $B_\nu\,[T(x)]$, $P(x)$ and $\rho(x)$ using the Grey Approximation already discussed.
  2) Solve the equation of radiative transfer to determine $J_\nu(x)$ and $H_\nu(x)$ at each depth-point.
  3) Are the conditions of radiative equilibrium and flux conservation

  $$\int_0^\infty \kappa_\nu(x)\,[J_\nu(x) - B_\nu(x)]\;d\nu = 0 \qquad \text{and} \qquad 4\,\pi \int_0^\infty H_\nu(x)\,d\nu = \sigma\,{T_{\rm eff}}^4$$

  satisfied at each depth? If so, we have a converged model stellar atmosphere and calculations may cease. Otherwise corrections need to be made to current estimates of $T(x)$, $\kappa_\nu(x)$, $B_\nu\,[T(x)]$, $P(x)$ and $\rho(x)$ before returning to Step 2.

Once a converged model atmosphere is available, we know T, $P_{\rm gas}$ and $P_{\rm e}$ at every depth-point and all other quantities of interest may be calculated.

Classical Stellar Atmosphere Models – II



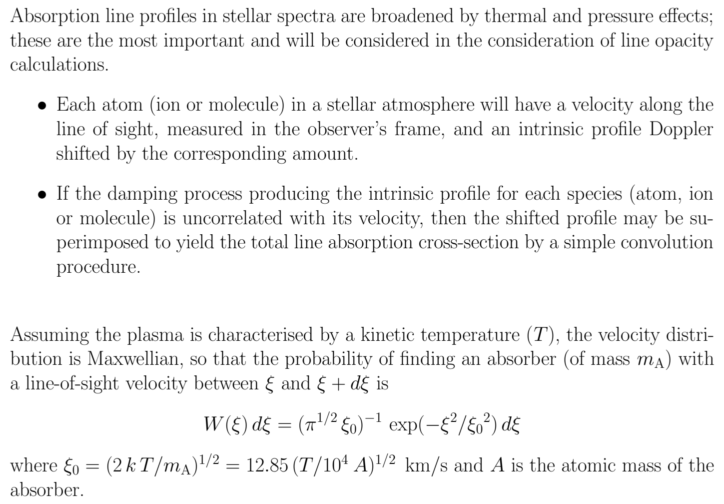
Calculation of Synthetic Spectra – I

Bound-bound transitions in atoms, ions or molecules can give rise to absorption lines seen in stellar spectra; steps involved in using this line in an abundance determination for the chemical element responsible (or partly responsible) for the line are:

- Compute a model stellar atmosphere as already described or, in case of the Sun, deduce a model atmosphere empirically from limb darkening measurements.
- For each depth-point in the stellar atmosphere it is then necessary to
    1) Establish ionisation fractions and fractions associated into molecules for the chemical element of interest.
    2) Determine the population of the lower level involved in the transition responsible for the spectral line of interest.
    3) Calculate line opacities at every frequency point used to define the synthetic spectrum.
    4) Repeat steps 1–3 for all other lines to be included in the synthetic spectrum.
    5) Deduce a total line opacity, to which all spectral lines contribute, for every frequency point used to define the synthetic spectrum.
- Solve the radiative transfer equation at all frequencies of interest, with line opacity added to the continuous opacity, to obtain the emergent synthetic spectrum.

An abundance has to be assumed in a line opacity calculation and several iterations of the calculation described above would be needed before the observed spectrum is matched.

Calculation of Synthetic Spectra – II



Absorption line profiles in stellar spectra are broadened by thermal and pressure effects; these are the most important and will be considered in the consideration of line opacity calculations.

- Each atom (ion or molecule) in a stellar atmosphere will have a velocity along the line of sight, measured in the observer's frame, and an intrinsic profile Doppler shifted by the corresponding amount.
- If the damping process producing the intrinsic profile for each species (atom, ion or molecule) is uncorrelated with its velocity, then the shifted profile may be superimposed to yield the total line absorption cross-section by a simple convolution procedure.

Assuming the plasma is characterised by a kinetic temperature (T), the velocity distribution is Maxwellian, so that the probability of finding an absorber (of mass m_{A}) with a line-of-sight velocity between ξ and $\xi + d\xi$ is

$$W(\xi)\, d\xi = (\pi^{1/2}\, \xi_0)^{-1}\ \exp(-\xi^2/{\xi_0}^2)\, d\xi$$

where $\xi_0 = (2\, k\, T/m_{\mathrm{A}})^{1/2} = 12.85\, (T/10^4\ A)^{1/2}$ km/s and A is the atomic mass of the absorber.

Calculation of Synthetic Spectra – III

If an atom (ion or molecule) having a velocity component ξ in the observer's direction is observed at frequency ν, it is absorbing at frequency $\nu\,[1-(\xi/c)]$ in is own frame and the line absorption coefficient for that atom is $\ell(\nu-\xi\nu/c)$. The total absorption coefficient at frequency ν for all species of that type as seen by the observer is therefore given by the convolution integral

$$\ell(\nu) = \int_{-\infty}^{+\infty} \ell(\nu-\xi\nu/c)\, W(\xi)\, d\xi$$

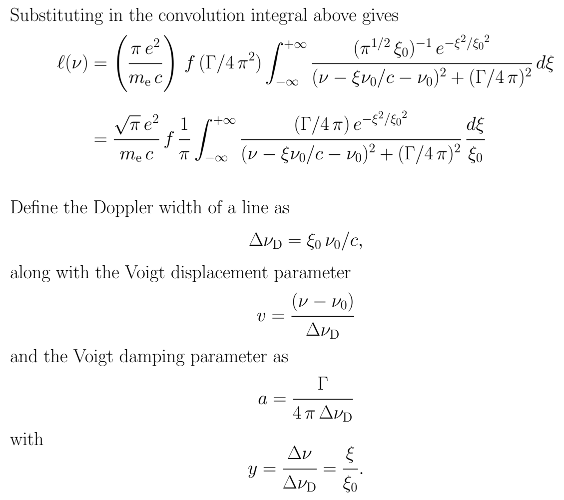
which can be applied to any absorption profile to allow for the effects of Doppler broadening.

In general, pressure broadening is important in stellar atmospheres; this is the damping phenomenon and its frequency dependence is given by the Lorentz profile

$$\ell(\nu) = \left(\frac{\pi\, e^2}{m_e\, c}\right)\, f \frac{(\Gamma/4\,\pi^2)}{(\nu-\nu_0)^2 + (\Gamma/4\,\pi)^2}$$

where f is the transition oscillator strength, ν_0 the line-centre frequency and Γ the damping constant.

Calculation of Synthetic Spectra – IV



Substituting in the convolution integral above gives

$$\ell(\nu) = \left(\frac{\pi\, e^2}{m_{\rm e}\, c}\right)\, f\,(\Gamma/4\,\pi^2) \int_{-\infty}^{+\infty} \frac{(\pi^{1/2}\,\xi_0)^{-1}\, e^{-\xi^2/\xi_0{}^2}}{(\nu - \xi\nu_0/c - \nu_0)^2 + (\Gamma/4\,\pi)^2}\, d\xi$$

$$= \frac{\sqrt{\pi}\, e^2}{m_{\rm e}\, c}\, f\, \frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{(\Gamma/4\,\pi)\, e^{-\xi^2/\xi_0{}^2}}{(\nu - \xi\nu_0/c - \nu_0)^2 + (\Gamma/4\,\pi)^2}\, \frac{d\xi}{\xi_0}$$

Define the Doppler width of a line as

$$\Delta\nu_{\rm D} = \xi_0\, \nu_0/c,$$

along with the Voigt displacement parameter

$$v = \frac{(\nu - \nu_0)}{\Delta\nu_{\rm D}}$$

and the Voigt damping parameter as

$$a = \frac{\Gamma}{4\,\pi\, \Delta\nu_{\rm D}}$$

with

$$y = \frac{\Delta\nu}{\Delta\nu_{\rm D}} = \frac{\xi}{\xi_0}.$$

Calculation of Synthetic Spectra – V

Then the line absorption coefficient due to the selected line and atmospheric depth point is

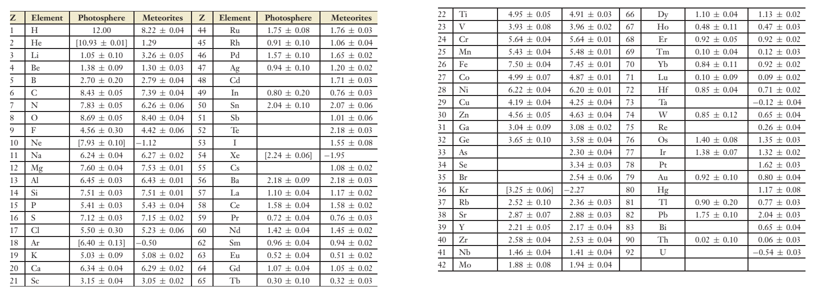
$$\ell(\nu) = \frac{\sqrt{\pi}\, e^2\, f}{m_{\mathrm{e}}\, c\, \Delta\nu_{\mathrm{D}}}\ \frac{a}{\pi} \int_{-\infty}^{+\infty} \frac{e^{-y^2}}{(v-y)^2 + a^2}\, dy$$

The integral

$$H(a,v) = \frac{a}{\pi} \int_{-\infty}^{\infty} \frac{e^{-y^2}}{(v-y)^2 + a^2}\, dy$$

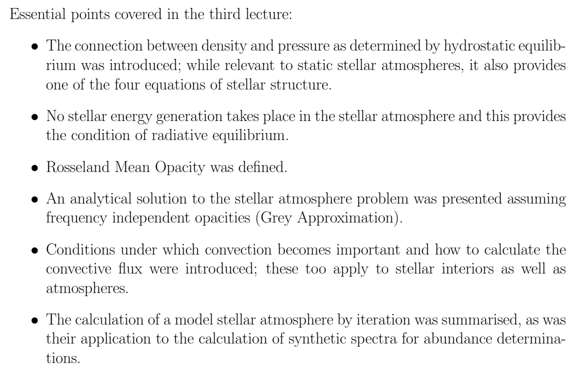
is known as the Voigt Function; it needs to be evaluated rapidly for synthetic stellar spectrum calculations.

Solar Abundances



Z	Element	Photosphere	Meteorites	Z	Element	Photosphere	Meteorites
1	H	12.00	8.22 ± 0.04	44	Ru	1.75 ± 0.08	1.76 ± 0.03
2	He	[10.93 ± 0.01]	1.29	45	Rh	0.91 ± 0.10	1.06 ± 0.04
3	Li	1.05 ± 0.10	3.26 ± 0.05	46	Pd	1.57 ± 0.10	1.65 ± 0.02
4	Be	1.38 ± 0.09	1.30 ± 0.03	47	Ag	0.94 ± 0.10	1.20 ± 0.02
5	B	2.70 ± 0.20	2.79 ± 0.04	48	Cd		1.71 ± 0.03
6	C	8.43 ± 0.05	7.39 ± 0.04	49	In	0.80 ± 0.20	0.76 ± 0.03
7	N	7.83 ± 0.05	6.26 ± 0.06	50	Sn	2.04 ± 0.10	2.07 ± 0.06
8	O	8.69 ± 0.05	8.40 ± 0.04	51	Sb		1.01 ± 0.06
9	F	4.56 ± 0.30	4.42 ± 0.06	52	Te		2.18 ± 0.03
10	Ne	[7.93 ± 0.10]	−1.12	53	I		1.55 ± 0.08
11	Na	6.24 ± 0.04	6.27 ± 0.02	54	Xe	[2.24 ± 0.06]	−1.95
12	Mg	7.60 ± 0.04	7.53 ± 0.01	55	Cs		1.08 ± 0.02
13	Al	6.45 ± 0.03	6.43 ± 0.01	56	Ba	2.18 ± 0.09	2.18 ± 0.03
14	Si	7.51 ± 0.03	7.51 ± 0.01	57	La	1.10 ± 0.04	1.17 ± 0.02
15	P	5.41 ± 0.03	5.43 ± 0.04	58	Ce	1.58 ± 0.04	1.58 ± 0.02
16	S	7.12 ± 0.03	7.15 ± 0.02	59	Pr	0.72 ± 0.04	0.76 ± 0.03
17	Cl	5.50 ± 0.30	5.23 ± 0.06	60	Nd	1.42 ± 0.04	1.45 ± 0.02
18	Ar	[6.40 ± 0.13]	−0.50	62	Sm	0.96 ± 0.04	0.94 ± 0.02
19	K	5.03 ± 0.09	5.08 ± 0.02	63	Eu	0.52 ± 0.04	0.51 ± 0.02
20	Ca	6.34 ± 0.04	6.29 ± 0.02	64	Gd	1.07 ± 0.04	1.05 ± 0.02
21	Sc	3.15 ± 0.04	3.05 ± 0.02	65	Tb	0.30 ± 0.10	0.32 ± 0.03
22	Ti	4.95 ± 0.05	4.91 ± 0.03	66	Dy	1.10 ± 0.04	1.13 ± 0.02
23	V	3.93 ± 0.08	3.96 ± 0.02	67	Ho	0.48 ± 0.11	0.47 ± 0.03
24	Cr	5.64 ± 0.04	5.64 ± 0.01	68	Er	0.92 ± 0.05	0.92 ± 0.02
25	Mn	5.43 ± 0.04	5.48 ± 0.01	69	Tm	0.10 ± 0.04	0.12 ± 0.03
26	Fe	7.50 ± 0.04	7.45 ± 0.01	70	Yb	0.84 ± 0.11	0.92 ± 0.02
27	Co	4.99 ± 0.07	4.87 ± 0.01	71	Lu	0.10 ± 0.09	0.09 ± 0.02
28	Ni	6.22 ± 0.04	6.20 ± 0.01	72	Hf	0.85 ± 0.04	0.71 ± 0.02
29	Cu	4.19 ± 0.04	4.25 ± 0.04	73	Ta		−0.12 ± 0.04
30	Zn	4.56 ± 0.05	4.63 ± 0.04	74	W	0.85 ± 0.12	0.65 ± 0.04
31	Ga	3.04 ± 0.09	3.08 ± 0.02	75	Re		0.26 ± 0.04
32	Ge	3.65 ± 0.10	3.58 ± 0.04	76	Os	1.40 ± 0.08	1.35 ± 0.03
33	As		2.30 ± 0.04	77	Ir	1.38 ± 0.07	1.32 ± 0.02
34	Se		3.34 ± 0.03	78	Pt		1.62 ± 0.03
35	Br		2.54 ± 0.06	79	Au	0.92 ± 0.10	0.80 ± 0.04
36	Kr	[3.25 ± 0.06]	−2.27	80	Hg		1.17 ± 0.08
37	Rb	2.52 ± 0.10	2.36 ± 0.03	81	Tl	0.90 ± 0.20	0.77 ± 0.03
38	Sr	2.87 ± 0.07	2.88 ± 0.03	82	Pb	1.75 ± 0.10	2.04 ± 0.03
39	Y	2.21 ± 0.05	2.17 ± 0.04	83	Bi		0.65 ± 0.04
40	Zr	2.58 ± 0.04	2.53 ± 0.04	90	Th	0.02 ± 0.10	0.06 ± 0.03
41	Nb	1.46 ± 0.04	1.41 ± 0.04	92	U		−0.54 ± 0.03
42	Mo	1.88 ± 0.08	1.94 ± 0.04				

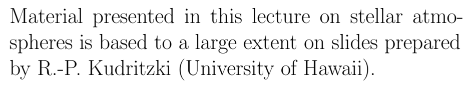
Lecture 3: Summary



Essential points covered in the third lecture:

- The connection between density and pressure as determined by hydrostatic equilibrium was introduced; while relevant to static stellar atmospheres, it also provides one of the four equations of stellar structure.
- No stellar energy generation takes place in the stellar atmosphere and this provides the condition of radiative equilibrium.
- Rosseland Mean Opacity was defined.
- An analytical solution to the stellar atmosphere problem was presented assuming frequency independent opacities (Grey Approximation).
- Conditions under which convection becomes important and how to calculate the convective flux were introduced; these too apply to stellar interiors as well as atmospheres.
- The calculation of a model stellar atmosphere by iteration was summarised, as was their application to the calculation of synthetic spectra for abundance determinations.

Acknowledgement



Material presented in this lecture on stellar atmospheres is based to a large extent on slides prepared by R.-P. Kudritzki (University of Hawaii).