

Fundamental Stellar Parameters

Radiative Transfer

Stellar Atmospheres

Hydrostatic and Radiative Equilibrium
Grey Approximation and Mean Opacity
Convection
Classical Stellar Atmospheres
Synthetic Spectra
Solar Abundances

Equations of Stellar Structure

Nuclear Reactions in Stellar Interiors

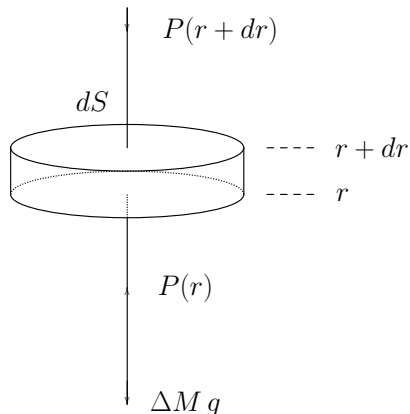
Stellar Atmospheres Introduction

Synthetic stellar spectra used for the determination of chemical element abundances were, until recently, based on one-dimensional model stellar atmospheres.

- At any optical (or geometric) depth, gravity is exactly balanced by pressure (gas, electron and radiation) – hydrostatic equilibrium
- A constant temperature is maintained at every depth; radiative energy received from below is equal to the energy radiated to layers above although frequency redistribution will have occurred – radiative equilibrium.
- Atmosphere assumed to be static and convection treated with crude mixing length theory, using an empirically determined mixing length.
- Opacities needed are dependent on abundances sought.

Model stellar atmosphere may be obtained empirically, in the case of the Sun, by direct measurement of limb-darkening; for other cases where only F_{ν} is observable, an iterative calculation is needed with some realistic estimate used as a starting approximation.

Hydrostatic Equilibrium – I



Consider a cylindrical element of mass ΔM and surface area dS between radii r and $r + dr$ in a star.

Mass of gas in the star at smaller radii = $m = m(r)$.

Hydrostatic Equilibrium – II

Radial forces acting on the element

- Gravity (inward)

$$F_g = -\frac{G m \Delta M}{r^2}$$

- Pressure Difference (outward – net force due to difference in pressure between upper and lower faces)

$$\begin{aligned} F_p &= P(r) dS - P(r + dr) dS \\ &= P(r) dS - \left[P(r) + \frac{dP}{dr} \times dr \right] dS \\ &= -\frac{dP}{dr} \times dr dS \end{aligned}$$

If ρ is gas density in element $\Delta M = \rho dr dS$.

Inward and outward forces must balance in hydrostatic equilibrium:

$$0 = -\frac{G m \rho dr dS}{r^2} - \frac{dP}{dr} dr dS$$

$$\frac{dP}{dr} = -\frac{G m}{r^2} \rho \quad (\text{First Equation of Stellar Structure})$$

$$= -g \rho \quad (\text{in stellar atmosphere})$$

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Stellar Evolution

- └ Stellar Atmospheres
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 - └ Hydrostatic Equilibrium – II

Stellar forces acting on the element

- Gravity (down): $F_g = -\frac{GM\Delta M}{r^2}$
- Pressure Difference (outward) – net force due to difference in pressure between upper and lower faces:

$$F_p = P(r)\Delta S - P(r+\Delta r)\Delta S$$

$$= P(r)\Delta S - \left[P(r) + \frac{dP}{dr}\Delta r \right] \Delta S$$

$$= -\frac{dP}{dr}\Delta r \Delta S$$

If ρ is gas density in element ΔM or $\rho dr \Delta S$,
 inward and outward forces must balance in hydrostatic equilibrium:

$$0 = -\frac{dP}{dr}\Delta r \Delta S - \frac{GM\Delta M}{r^2}$$

$$\frac{dP}{dr} = -\frac{GM}{r^2}\rho \quad \text{(Star Equation of Stellar Structure)}$$

$$= -g\rho \quad \text{(in stellar atmosphere)}$$

For hydrostatic equilibrium to prevail, the pressure gradient must exactly balance the force of gravity at every point from the stellar centre to the surface. A slab of area dS , thickness dr , density ρ and mass ΔM at distance r from the stellar centre has a total mass $m(r)$ interior to it. Calculating the force of gravitation (F_g) and setting it equal to the pressure gradient force (F_p) across the slab, leads to the Equation of Hydrostatic Equilibrium. The gravitational force per unit mass in the stellar atmosphere is simply g , the acceleration due to gravity.

Radiative Equilibrium – I

Radiative equilibrium requires the flux at any depth to be constant.

$$F(r) = \int_0^\infty \oint_{4\pi} I_\nu \cos \theta d\Omega d\nu = 4\pi \int_0^\infty H_\nu d\nu$$

$$4\pi r^2 F(r) = 4\pi r^2 \cdot 4\pi \int_0^\infty H_\nu d\nu = \text{constant} = L$$

For a plane-parallel atmosphere $r \simeq R$ and

$$4\pi \int_0^\infty H_\nu d\nu = \text{constant} = \sigma T_{\text{eff}}$$

Integrating radiative transfer equation in plane-parallel geometry

$$\mu \frac{dI_\nu}{dx} = -(\kappa_\nu + \sigma_\nu)(I_\nu - S_\nu)$$

over all angles, gives:

$$\frac{1}{2} \int_{-1}^1 \mu \frac{dI_\nu}{dx} d\mu = -\frac{1}{2} \int_{-1}^1 (\kappa_\nu + \sigma_\nu)(I_\nu - S_\nu) d\mu$$

$$\frac{d}{dx} \left[\frac{1}{2} \int_{-1}^1 \mu I_\nu d\mu \right] = -(\kappa_\nu + \sigma_\nu)(J_\nu - S_\nu)$$

$$\frac{dH_\nu}{dx} = -(\kappa_\nu + \sigma_\nu)(J_\nu - S_\nu)$$

Radiative Equilibrium – II

Integrating over frequency:

$$\frac{d}{dx} \int_0^{\infty} H_{\nu} d\nu = - \int_0^{\infty} (\kappa_{\nu} + \sigma_{\nu})(J_{\nu} - S_{\nu}) d\nu$$

The integral on the left-hand side is constant and therefore:

$$\int_0^{\infty} (\kappa_{\nu} + \sigma_{\nu})(J_{\nu} - S_{\nu}) d\nu = 0$$

Substituting

$$S_{\nu} = \frac{\kappa_{\nu}}{\kappa_{\nu} + \sigma_{\nu}} B_{\nu} + \frac{\sigma_{\nu}}{\kappa_{\nu} + \sigma_{\nu}} J_{\nu}$$

gives

$$\int_0^{\infty} [(\kappa_{\nu} + \sigma_{\nu})J_{\nu} - \kappa_{\nu} B_{\nu} - \sigma_{\nu} J_{\nu}] d\nu = 0$$
$$\int_0^{\infty} \kappa_{\nu} [J_{\nu} - B_{\nu}(T)] d\nu = 0$$

At every depth, the temperature T has to assume a value which satisfies the above equation.

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Stellar Evolution

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Integrating over frequency:

$$\frac{d}{dz} \int_0^\infty R_\nu d\nu = - \int_0^\infty (\kappa_\nu + \sigma_\nu) L_\nu - S_\nu d\nu$$

The integral on the left-hand side is constant and therefore:

$$\int_0^\infty (\kappa_\nu + \sigma_\nu) L_\nu - S_\nu d\nu = 0$$

Substituting $S_\nu = \frac{c_\nu}{\kappa_\nu + \sigma_\nu} R_\nu + \frac{c_\nu}{\kappa_\nu + \sigma_\nu} L_\nu$

gives

$$\int_0^\infty [(\kappa_\nu + \sigma_\nu) L_\nu - c_\nu R_\nu - c_\nu L_\nu] d\nu = 0$$

$$\int_0^\infty \kappa_\nu [L_\nu - R_\nu(T)] d\nu = 0$$

At every depth, the temperature T has to assume a value which satisfies the above equation.

No energy is generated in a static stellar atmosphere, the process of radiation transfer simply redistributes radiation in frequency. The monochromatic flux integrated over all frequencies is therefore invariant throughout a stellar atmosphere and is the stellar luminosity once further integrated over the area of the star. Accordingly, at each depth in a stellar atmosphere, there must be a specific temperature which ensures (given the monochromatic opacity at each frequency) that the monochromatic flux integrated over all frequencies received from below is the same as that radiated to the layer above.

Grey Approximation

Approximate radiative equilibrium integral using a frequency-averaged absorption coefficient $\bar{\kappa}$ placed in front of the integral:

$$\bar{\kappa} \int_0^\infty [J_\nu - B_\nu(T)] d\nu = 0$$

With

$$J = \int_0^\infty J_\nu d\nu \quad H = \int_0^\infty H_\nu d\nu \quad K = \int_0^\infty K_\nu d\nu \quad B = \int_0^\infty B_\nu d\nu = \frac{\sigma T^4}{\pi}$$

radiative equilibrium then implies

$$J = B$$

$$4\pi H = \sigma T_{\text{eff}}^4.$$

In LTE and radiative equilibrium $S = B = J$ and with a mean optical depth $d\bar{\tau} = \bar{\kappa} dx$

$$J(\bar{\tau}) = \Lambda_{\bar{\tau}} [J(t)] = \frac{1}{2} \int_0^\infty J(t) E_1(|t - \bar{\tau}|) dt$$

with an exact solution

$$J(\bar{\tau}) = B(\bar{\tau}) = (\sigma/\pi)T(\bar{\tau})^4 = \text{constant} \times [\bar{\tau} + q(\bar{\tau})]$$

and boundary condition

$$T(\bar{\tau})^4 = 0.75 T_{\text{eff}}^4 [\bar{\tau} + q(\bar{\tau})]$$

where $q(\bar{\tau}) \sim 2/3$ is Hopf's function.

Stellar Evolution

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└ Grey Approximation and Mean Opacity

└ Grey Approximation

Approximate solution of the radiative transfer equation using a frequency averaged absorption coefficient κ is given to first order by the form

$$J = H + \frac{1}{4\pi} \int \kappa(\nu) I(\nu) d\nu$$

With

$$J = \frac{1}{4\pi} \int I(\nu) d\nu \quad \kappa = \frac{1}{4\pi} \int \kappa(\nu) I(\nu) d\nu \quad \kappa = \frac{1}{4\pi} \int \kappa(\nu) I(\nu) d\nu$$

radiative equilibrium implies

$$J = H$$

In LTE and radiative equilibrium $\kappa = \kappa_{\text{eff}} = \kappa$ and with a mean optical depth $\bar{\tau} = 0.75$

$$J(\bar{\tau}) = \frac{1}{4\pi} \int I(\bar{\tau}, \nu) d\nu = \frac{1}{4\pi} \int \kappa(\nu) I(\bar{\tau}, \nu) d\nu$$

with an exact solution

$$J(\bar{\tau}) = H(\bar{\tau}) = \frac{1}{4\pi} \int I(\bar{\tau}, \nu) d\nu = \frac{1}{4\pi} \int \kappa(\nu) I(\bar{\tau}, \nu) d\nu$$

and boundary condition

$$J(\bar{\tau}) = \frac{1}{4\pi} \int I(\bar{\tau}, \nu) d\nu = \frac{1}{4\pi} \int \kappa(\nu) I(\bar{\tau}, \nu) d\nu$$

where $g(\bar{\tau}) = 0.75$ in the grey approximation.

In the Eddington Approximation $dK_{\nu}/d\tau_{\nu} = H_{\nu}$ which may be integrated over τ_{ν} to give $K = H(\tau + c)$ in the grey approximation where all quantities are independent of frequency and H is independent of depth. Here c is a constant. Since the Eddington Approximation also gives $J_{\nu} = 3K_{\nu}$ it follows that $J = 3H(\tau + c)$ and so $T(\bar{\tau})^4 = 0.75 T_{\text{eff}}^4 [\bar{\tau} + c]$.

Since $J = S = B$ in the grey approximation, the Schwarzschild-Milne equation for the Eddington Flux gives for $\bar{\tau} = 0$

$$H = \frac{1}{2} \int_0^{\infty} J(t) E_2(t) dt = \frac{1}{2} 3H \left[\int_0^{\infty} t E_2(t) dt + c \int_0^{\infty} E_2(t) dt \right]$$

Stellar Evolution

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└ Grey Approximation

Approximate radiative equilibrium using a frequency averaged absorption coefficient κ instead of κ_ν at each ν

$$\int_0^\infty (2 - \kappa_\nu T) d\nu = 0$$

With

$$j_\nu = \int_0^\infty \kappa_\nu d\nu \quad \kappa_\nu = \int_0^\infty \kappa_\nu d\nu \quad \kappa_\nu = \int_0^\infty \kappa_\nu d\nu \quad \frac{j_\nu}{\kappa_\nu} = \frac{j}{\kappa}$$

radiative equilibrium implies

$$j = \kappa$$

$$j = \kappa$$

In LTE and radiative equilibrium $F = \kappa = j$ and with a mean optical depth $\tau = 0.5$

$$j(\tau) = \kappa(\tau) \left[1 + \frac{1}{2} \int_0^\infty \kappa_\nu \kappa_\nu d\nu \right]$$

with an exact solution

$$j(\tau) = \kappa(\tau) \left[1 + \frac{1}{2} \int_0^\infty \kappa_\nu \kappa_\nu d\nu \right]$$

and boundary condition

$$j(\tau) = \kappa(\tau) \left[1 + \frac{1}{2} \int_0^\infty \kappa_\nu \kappa_\nu d\nu \right]$$

where $g(\tau) = \kappa(\tau)$ is the mean

The standard integral

$$\int_0^\infty t^s E_n dt = \frac{s!}{s+n}$$

gives

$$\int_0^\infty t E_n dt = \frac{1}{3} \quad \text{and} \quad \int_0^\infty E_n dt = \frac{1}{2}$$

Therefore $c = 2/3$.

Rosseland Mean Opacity - I

The second moment of the transfer equation in planar geometry

$$\mu \frac{dI_\nu}{d\tau_\nu} = I_\nu - S_\nu$$

is

$$\frac{d}{d\tau_\nu} \int_{-1}^{+1} \mu^2 I_\nu d\mu = \int_{-1}^{+1} \mu I_\nu d\mu - S_\nu \int_{-1}^{+1} \mu d\mu$$

$$\frac{dK_\nu}{d\tau_\nu} = H_\nu$$

$$\frac{dK_\nu}{dx} = -\kappa_\nu H_\nu$$

In the grey approximation

$$\int_0^\infty \frac{1}{\kappa_\nu} \frac{dK_\nu}{dx} d\nu = \frac{1}{\bar{\kappa}} \frac{dK}{dx} = -H$$

$$\frac{1}{\bar{\kappa}} = \frac{\int_0^\infty \frac{1}{\kappa_\nu} \frac{dK_\nu}{dx} d\nu}{\frac{dK}{dx}}$$

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Stellar Evolution

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└ Rosseland Mean Opacity - I

The second moment of the transfer equation in plane geometry

$$\frac{dK_\nu}{d\tau_\nu} = K_\nu - H_\nu$$

or

$$\frac{d}{d\tau_\nu} \int_{-1}^1 \mu^2 I_\nu d\mu = \int_{-1}^1 \mu^2 k_\nu I_\nu d\mu - k_\nu \int_{-1}^1 I_\nu d\mu$$

$$\frac{dK_\nu}{d\tau_\nu} = K_\nu - H_\nu$$

$$\frac{dK_\nu}{d\tau_\nu} = K_\nu - H_\nu$$

In the grey approximation

$$\int_{-1}^1 \mu^2 \frac{dI_\nu}{d\tau_\nu} d\mu = \frac{d}{d\tau_\nu} \int_{-1}^1 \mu^2 I_\nu d\mu - 2H_\nu$$

$$\frac{d}{d\tau_\nu} \int_{-1}^1 \mu^2 I_\nu d\mu = 2H_\nu$$

The result $dK_\nu/d\tau_\nu = H_\nu$ follows because the second integral on the right-hand side of the line above is clearly zero.

Rosseland Mean Opacity - II

As $\tau \rightarrow \infty$, $K_\nu \rightarrow (1/3) J_\nu$, $J_\nu \rightarrow B_\nu$ and

$$\frac{dK_\nu}{dx} \rightarrow \frac{1}{3} \frac{dB_\nu}{dx} = \frac{1}{3} \frac{dB_\nu}{dT} \frac{dT}{dx}$$

Substitution in above mean opacity expression gives definition for **Rosseland Mean Opacity**

$$\frac{1}{\bar{\kappa}_{\text{Ross}}} = \frac{\int_0^\infty \frac{1}{\kappa_\nu} \frac{dB_\nu(T)}{dT} d\nu}{\int_0^\infty \frac{dB_\nu(T)}{dT} d\nu}$$

which has the properties:

- large weight for low-opacity (more transparent to radiation) regions,
- at large τ the T structure is accurately given by $T^4 = 0.75 T_{\text{eff}}^4 [\tau_{\text{Ross}} + q(\tau_{\text{Ross}})]$,
- $\bar{\kappa}_{\text{Ross}}$ used in stellar interiors and
- for stellar atmospheres using $\bar{\kappa}_{\text{Ross}}$ allows an approximate temperature stratification to be obtained and used as a starting point for further iterations.

Stellar Evolution

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 - └ Rosseland Mean Opacity - II

As $\tau \rightarrow \infty$, $K_r \rightarrow (1/3)k_B T_e \rightarrow K_r$, and

$$\frac{dK_r}{d\tau} = \frac{1}{3} \frac{dK_r}{d\tau} = \frac{1}{3} \frac{dK_r}{d\tau}$$

Substitution in above mean opacity expression gives definition for **Rosseland Mean Opacity**

$$\frac{1}{\kappa_{\text{Ross}}} = \frac{\int_0^{\infty} \frac{\kappa_{\nu} dB_{\nu}}{dB_{\nu}} d\nu}{\int_0^{\infty} \frac{dB_{\nu}}{dT} d\nu}$$

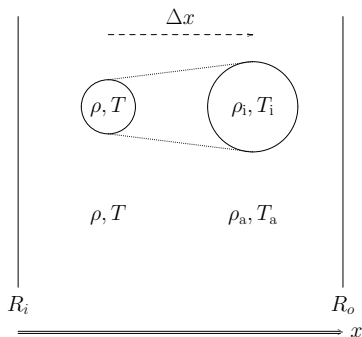
which has the properties

- large weight for low-opacity (more transparent to radiation) regions.
- at large τ the κ structure is accurately given by $\kappa^* = 0.751 E_0^2 [\mu_{\text{H}} + g(\mu_{\text{H}})]$.
- κ_{Ross} used in stellar interior and
- for stellar atmospheres using κ_{Ross} allows an approximate temperature stratification to be obtained and used as a starting point for further iteration.

Note that the denominator on the right-hand side of the expression for the Rosseland Mean Opacity is obtained using

$$\frac{d}{dx} \int_0^{\infty} K_{\nu} d\nu = \frac{1}{3} \frac{dT}{dx} \int_0^{\infty} \frac{dB_{\nu}}{dT} d\nu.$$

Schwarzschild Criterion for Convective Instability – I



Assume mass element (“cell”) in photosphere is perturbed slowly ($v < v_{\text{sound}}$) so it rises adiabatically (no energy exchanged with surroundings):

- Ambient density decreases ($\rho \rightarrow \rho_a$).
- Cell expands and its internal density also decreases ($\rho \rightarrow \rho_i$).

There are two cases:

- $\rho_i > \rho_a$ cell falls back \rightarrow stable.
- $\rho_i < \rho_a$ cell rises further \rightarrow unstable.

Schwarzschild Criterion for Convective Instability – II

At the end of adiabatic expansion, the density and temperature of the cell are ρ_i and T_i respectively. Identifying environmental quantities as “**radiative**” and those in the cell as “**adiabatic**” we make the substitutions: $\rho_i \rightarrow \rho_{\text{ad}}$, $T_i \rightarrow T_{\text{ad}}$, $\rho_a \rightarrow \rho_{\text{rad}}$ and $T_a \rightarrow T_{\text{rad}}$.

Change in cell density when rising over radial distance Δx

$$\Delta\rho_{\text{ad}} = \left[\frac{d\rho}{dx} \right]_{\text{ad}} \Delta x$$

that is we obtain stability if

$$|\Delta\rho_{\text{ad}}| < |\Delta\rho_{\text{rad}}|$$

or

$$\left[\frac{d\rho}{dx} \right]_{\text{ad}} < \left[\frac{d\rho}{dx} \right]_{\text{rad}}$$

Schwarzschild Criterion for Convective Instability – III

We assume that the motion is slow ($v \ll v_{\text{sound}}$) so that pressure equilibrium prevails; pressure inside cell is equal to the pressure outside

$$P_{\text{rad}} = P_{\text{ad}}$$

From the equation of state $P \sim \rho T$ and so

$$\rho_{\text{rad}} T_{\text{rad}} = \rho_{\text{ad}} T_{\text{ad}}$$

and an equivalent criterion for stability against convection is

$$\left| \frac{dT}{dx} \right|_{\text{ad}} > \left| \frac{dT}{dx} \right|_{\text{rad}} .$$

The equation of hydrostatic equilibrium and the perfect gas equation are

$$dP_{\text{gas}} = -g \rho(x) dx \quad P_{\text{gas}} = \frac{k}{m_{\text{H}} \bar{\mu}} \rho T$$

give

$$\left| \frac{dT}{dx} \right| = \left| \frac{dT}{dP} \frac{dP}{dx} \right| = \left| \frac{dT}{dP} g \rho \right| = \left| \frac{dT}{dP} \right| g \frac{m_{\text{H}} \bar{\mu}}{kT} P = g \frac{m_{\text{H}} \bar{\mu}}{k} \left| \frac{d \ln T}{d \ln P} \right|$$

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Stellar Evolution

- └ Stellar Atmospheres
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We assume that the medium is slow ($\tau \ll \tau_{\text{conv}}$) so that pressure equilibrium prevails; pressure inside cell is equal to the pressure outside.

$$P_{\text{in}} = P_{\text{out}}$$

From the equation of state $P = \rho T$ and so

$$\rho_{\text{in}} T_{\text{in}} = \rho_{\text{out}} T_{\text{out}}$$

and an equivalent criterion for stability against convection is

$$\left(\frac{dP}{dz} \right)_{\text{in}} > \left(\frac{dP}{dz} \right)_{\text{out}}$$

The equation of hydrostatic equilibrium and the perfect gas equation are

$$dP_{\text{in}} = -g \rho(z) dz \quad P_{\text{in}} = \frac{\rho}{\mu} RT$$

give

$$\left(\frac{dT}{dz} \right)_{\text{in}} > \left(\frac{dT}{dz} \right)_{\text{out}} = \left(\frac{dT}{dz} \right)_{\text{out}} + \left(\frac{dT}{dz} \right)_{\text{rad}} \quad P = \frac{\rho}{\mu} \frac{RT}{M} \left(\frac{dT}{dz} \right)_{\text{in}}$$

Since $P_{\text{gas}} \propto \rho T$ and pressures are equal in the rising gas bubble and its surrounds, the criterion for stability against convection expressed terms of the density gradient may also (and more conveniently) be expressed in terms of the corresponding temperature gradient.

Symbols introduced for the first time: m_{H} — mass of the hydrogen atom (1.67×10^{-27}) kg and $\bar{\mu}$ — mean molecular weight.

As pressures inside and outside a rising gas bubble are equal, pressure may replace depth as the independent variable and it is customary to state the Schwarzschild Criterion in terms of the adiabatic and radiative gradients of the natural logarithm of temperature with respect to the natural logarithm of pressure.

Schwarzschild Criterion for Convective Instability – IV

Therefore the criterion for stability against convection in a stellar atmosphere or envelope, known as the **Schwarzschild Criterion** is:

$$\left| \frac{d \ln T}{d \ln P} \right|_{\text{ad}} > \left| \frac{d \ln T}{d \ln P} \right|_{\text{rad}}$$

or instability occurs when the temperature gradient in a stellar atmosphere or envelope is larger than the adiabatic gradient, leading to convection. In an adiabatic process

$$P \sim \rho^\gamma \quad \text{where} \quad \gamma = C_p/C_v$$

and from the perfect gas equation of state

$$P_{\text{gas}} = \frac{k}{m_{\text{H}} \bar{\mu}} \rho T,$$

$$P^{\gamma-1} \sim T^\gamma$$

and so

$$\left| \frac{d \ln T}{d \ln P} \right|_{\text{ad}} = \frac{\gamma - 1}{\gamma} = 1 - \frac{1}{\gamma}$$

Schwarzschild Criterion for Convective Instability – V

In a plasma which is completely ionised or neutral $\gamma = 5/3$ and

$$\left| \frac{d \ln T}{d \ln P} \right|_{\text{ad}} = 0.4$$

With changing degree of ionisation:

- Plasma undergoes phase transition.
- Specific heats C_p and C_v are no longer constant.
- Adiabatic exponent γ changes.

In general

$$\left| \frac{d \ln T}{d \ln P} \right|_{\text{ad}} = \frac{2 + X_{\text{ion}}(1 - X_{\text{ion}}) \left(\frac{5}{2} + \frac{E_{\text{ion}}}{kT} \right)}{5 + X_{\text{ion}}(1 - X_{\text{ion}}) \left(\frac{5}{2} + \frac{E_{\text{ion}}}{kT} \right)^2}$$

where

$$X_{\text{ion}} = \frac{n_e}{n_p + n_H}$$

is the degree of ionisation and E_{ion} is ionisation potential. For $X_{\text{ion}} = 0$ or 1

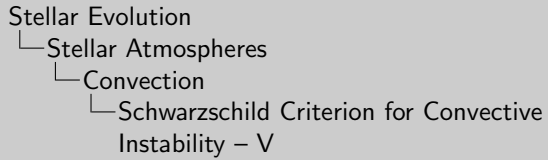
$$\left| \frac{d \ln T}{d \ln P} \right|_{\text{ad}} = 0.40$$

For $X_{\text{ion}} = 0.5$, a minimum

$$\left| \frac{d \ln T}{d \ln P} \right|_{\text{ad}} = 0.07$$

is reached.

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In a plasma which is completely ionized or neutral $\gamma = 5/3$ and $\frac{dP}{dr} = \frac{g}{\gamma - 1}$

With changing degree of ionization

- Thomson scattering opacity increases
- Radiative flux F_{rad} and κ_{rad} are no longer constant
- Diffusion equation is adequate

In general

$$\frac{dP}{dr} = \frac{g}{\gamma - 1} = \frac{g_{rad}}{\gamma - 1} + \frac{g_{scat}}{\gamma - 1} = \frac{g_{rad}}{\gamma - 1} + \frac{3}{4} \frac{F_{rad}}{c} \frac{1}{\gamma - 1}$$

where

$$g_{rad} = \frac{4\pi c}{3} \frac{K_{rad}}{K_{scat}}$$

is the degree of ionization and K_{rad} is constant provided the X_{ion} is not too small

For $X_{ion} = 0.5$ a minimum

$$\frac{dP}{dr} = \frac{g_{rad}}{\gamma - 1} = \frac{g_{rad}}{2}$$

is reached

In an adiabatic process, gas pressure and density are related by an exponent γ which is the ratio of specific heat at constant pressure (C_p) to the specific heat at constant volume (C_v). The ionisation state of a gas and relative abundances of the elements then determine γ and whether the radiative temperature gradient is higher than the adiabatic temperature gradient which allows convection.

Conditions for Convection

For a grey atmosphere or a stellar interior

$$T^4 = 0.75 T_{\text{eff}}^4 \left[\tau_{\text{Ross}} + \frac{2}{3} \right]$$

$$\left| \frac{dT}{dx} \right|_{\text{rad}} = \frac{\bar{\kappa}_{\text{Ross}}}{4T^3} \frac{3}{4} T_{\text{eff}}^4$$

$$\left| \frac{d \ln T}{d \ln P} \right|_{\text{rad}} = \frac{k}{g \bar{\mu} m_H} \frac{\bar{\kappa}_{\text{Ross}}}{4T^3} \frac{3}{4} T_{\text{eff}}^4 = \frac{3}{16} \left(\frac{T_{\text{eff}}}{T} \right)^4 \bar{\kappa}_{\text{Ross}} H \simeq \frac{3}{16} \frac{P}{\sigma g \rho T^4} \bar{\kappa}_{\text{Ross}} F$$

where $H = kT/(g \bar{\mu} m_H)$ is the pressure scale height.

Conditions for convection:

- Adiabatic Gradient $[(d \ln T/d \ln P)_{\text{ad}}]$ small (γ -effect);
- Radiative Gradient $[(d \ln T/d \ln P)_{\text{rad}}]$ large (κ -effect) when $\bar{\kappa}_{\text{Ross}}$ large (F or H large are less important);
- strong absorption from the $n = 2$ level in hydrogen at $T \simeq 9000\text{K}$ gives large $\bar{\kappa}_{\text{Ross}}$ and hence hydrogen convection zone in Sun;
- molecular phase transitions occur in stars cooler than the Sun and molecular opacities are increasingly important, convection therefore becomes more and more important and
- scale-height also becomes important in red giants and supergiants.

Stellar Evolution

└ Stellar Atmospheres

└└ Convection

└└└ Conditions for Convection

For a grey atmosphere in a stellar interior

$$P^* = 0.75 P_* \left[\frac{\mu_{\text{atm}}}{\mu} \right]$$

$$\frac{dP^*}{dx} = \frac{\rho_{\text{atm}}}{\mu_{\text{atm}}} g$$

$$\left| \frac{dP^*}{dx} \right| = \frac{\rho_{\text{atm}}}{\mu_{\text{atm}}} g \approx \frac{\rho_{\text{atm}}}{\mu_{\text{atm}}} \frac{GM}{R^2} \approx \frac{2}{3} \left(\frac{P^*}{\mu_{\text{atm}}} \right)^2 \frac{1}{H_p} \text{ since } H_p \approx \frac{2}{3} \frac{P^*}{\rho_{\text{atm}} g}$$

where $H_p = 2P^*/(\rho_{\text{atm}} g)$ is the pressure scale height.

Conditions for convection:

- Adiabatic Gradient $|d \ln T / d \ln P^*|_{\text{ad}} > \text{const.}$ (sufficient)
- Radiative Gradient $|d \ln T / d \ln P^*|_{\text{rad}} < \text{const.}$ (necessary) when $H_{\text{ph}} \ll H_p$ (if $H_p \ll H$ layer size is important)
- strong absorption from the $n = 2$ level in hydrogen at $T \approx 10000$ gives large H_{ph} and hence the upper convection zone is thin.
- molecular phase transitions occur at sizes smaller than the flux and radiative gradient are increasingly important, convection therefore becomes more and more important and
- scale height also becomes important to set depth and temperature.

The second line follows using

$$d\tau_{\text{ROSS}} = \kappa_{\text{ROSS}} dx.$$

Mixing Length Theory – I

A simple approach to a complicated phenomenon:

- Suppose atmosphere becomes unstable at $r = r_0$ and mass element (cell) rises for a characteristic distance ℓ (**mixing length**) to $r_0 + \ell$.
- Cell excess energy is released into the ambient medium.
- Cell cools, sinks back down, absorbs energy and rises again.
- The mixing length (ℓ) is assumed to be equal for all cells.
- The velocity (v) of all cells is equal.

Note that the last two assumptions are made for ad hoc simplicity; there is no real justification for them beyond the fact that Mixing Length Theory provides a remarkably good representation of convection in stellar atmospheres and envelopes.

Mixing Length Theory – II

Given the pressure scale height

$$H = \frac{kT}{(g \bar{\mu} m_H)}$$

we adopt an adjustable parameter $\alpha = \ell/H$ as a parameterisation of the mixing length ($\alpha \sim 0.5 \rightarrow 2.0$ km/s) and calculate the consequent energy flux.

For a cell moving up with speed v

Flux of energy transferred = mass flow \times heat transferred per gram

$$F_{\text{conv}} = \rho v \times dQ = \rho v C_p \Delta T$$

We need to estimate v from the work done by the buoyancy force

$$|f_b| = g |\Delta\rho|$$

by relating the **cell – surroundings** density difference ($\Delta\rho$) to the known temperature difference (ΔT). In pressure equilibrium, the equation of state

$$P = \frac{\rho k T}{\bar{\mu} m_H}$$

gives

$$\frac{dP}{P} = \frac{d\rho}{\rho} + \frac{dT}{T} - \frac{d\bar{\mu}}{\bar{\mu}} = 0$$

$$d\rho = -\rho \left(\frac{dT}{T} - \frac{d\bar{\mu}}{\bar{\mu}} \right) = -\rho \frac{dT}{T} \left(1 - \frac{d \ln \bar{\mu}}{d \ln T} \right)$$

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- └ Stellar Atmospheres
 - └ Convection
 - └ Mixing Length Theory - II

Close the pressure scale height

$$\delta l = \frac{\delta T}{\rho g}$$

we adopt an adjusted parameter $\alpha = 1/2$ as a contribution to the mixing length
 $(\alpha = 0.1$ to 0.3 often) and calculate the convective energy flux.

The α will modify the scale height h

$$h_{\text{mix}} = \rho g \alpha \delta l = \rho g \alpha \frac{\delta T}{\rho g}$$

We need to estimate δT from the work done by the buoyancy force

$$\delta W = \rho g \delta l$$

In addition, the α **introduces** energy difference δQ to the buoyancy temperature difference δT . In general equilibrium, the equation of state

$$p = \frac{\rho k T}{\mu m_H}$$

gives

$$\frac{\delta p}{p} = \frac{\delta \rho}{\rho} + \frac{\delta T}{T} = 0$$

$$\delta T = -T \left(\frac{\delta \rho}{\rho} \right) = -\frac{\delta p}{\rho g} \left(1 - \frac{\alpha}{2} \right)$$

The heat transferred per gram as the bubble of gas rises a height ℓ is $dQ = C_p \Delta T$ where C_p is the specific heat at constant pressure.

Mixing Length Theory – III

Work done by buoyancy force

$$w = \int_0^\ell |f_b| d(\Delta r) = \int_0^\ell g |\Delta \rho| d(\Delta r)$$

where

$$|\Delta \rho| = \frac{\rho}{T} \Delta T \left| 1 - \frac{d \ln \bar{\mu}}{d \ln T} \right| \quad \text{and} \quad \Delta T = \left[\left. \frac{dT}{dr} \right|_{\text{rad}} - \left. \frac{dT}{dr} \right|_{\text{ad}} \right] \Delta r.$$

Assume integrand above is constant over integration interval (crude approximation):

$$w = g \frac{\rho}{T} \left| 1 - \frac{d \ln \bar{\mu}}{d \ln T} \right| \left[\left. \frac{dT}{dr} \right|_{\text{rad}} - \left. \frac{dT}{dr} \right|_{\text{ad}} \right] \times \frac{1}{2} \ell^2.$$

Cell kinetic energy must be work done by buoyancy force:

$$v = \left[\frac{g}{T} \left| 1 - \frac{d \ln \bar{\mu}}{d \ln T} \right| \right]^{1/2} \left[\left. \frac{dT}{dr} \right|_{\text{rad}} - \left. \frac{dT}{dr} \right|_{\text{ad}} \right]^{1/2} \ell$$

and so the convective flux is given by

$$F_{\text{conv}} = \rho v C_p \Delta T = \rho C_p \left[\frac{g}{T} \left| 1 - \frac{d \ln \bar{\mu}}{d \ln T} \right| \right]^{1/2} \left[\left. \frac{dT}{dr} \right|_{\text{rad}} - \left. \frac{dT}{dr} \right|_{\text{ad}} \right]^{3/2} \ell^2.$$

From the equations of state and hydrostatic equilibrium

$$\left. \frac{dT}{dr} \right| = \frac{g m_H \bar{\mu}}{k} \left. \frac{d \ln T}{d \ln P} \right| = \frac{T}{H} \left. \frac{d \ln T}{d \ln P} \right|,$$

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 - └ Convection
 - └ Mixing Length Theory – III

Work done by buoyancy force

$$= - \int_{\Delta r}^{\Delta r + \Delta r} \rho g \Delta r d(\Delta r)$$

where

$$\Delta r = \frac{1}{2} \Delta r \quad \frac{\partial \Delta r}{\partial t} = \Delta r \quad \text{and} \quad \Delta r = \left[\frac{\rho}{\rho_0} \frac{\Delta r}{\rho_0} \right] =$$

Assume constant value of convective velocity (constant velocity approximation)

$$= \frac{1}{2} \left[\frac{\rho}{\rho_0} \frac{\Delta r}{\rho_0} \right] = \frac{1}{2} \left[\frac{\rho}{\rho_0} \frac{\Delta r}{\rho_0} \right] = \frac{1}{2} \left[\frac{\rho}{\rho_0} \frac{\Delta r}{\rho_0} \right] =$$

Use Boussinesq approximation for the buoyancy force

$$= \frac{1}{2} \left[\frac{\rho}{\rho_0} \frac{\Delta r}{\rho_0} \right] = \frac{1}{2} \left[\frac{\rho}{\rho_0} \frac{\Delta r}{\rho_0} \right] = \frac{1}{2} \left[\frac{\rho}{\rho_0} \frac{\Delta r}{\rho_0} \right] =$$

and use the continuity equation to give

$$\rho_{\text{conv}} = \rho_0 \left(\frac{\Delta r}{\rho_0} \right) = \rho_0 \left(\frac{\Delta r}{\rho_0} \right) = \rho_0 \left(\frac{\Delta r}{\rho_0} \right) =$$

From the equations of state and hydrostatic equilibrium

$$\left[\frac{\rho}{\rho_0} \frac{\Delta r}{\rho_0} \right] = \left[\frac{\rho}{\rho_0} \frac{\Delta r}{\rho_0} \right] = \left[\frac{\rho}{\rho_0} \frac{\Delta r}{\rho_0} \right] =$$

In making the crude assumption that the integrand is constant in the work done by the buoyancy force, the integrand remaining is $\Delta r d(\Delta r)$ which when evaluated gives $(1/2) \ell^2$. In fact as the equation is written down, the work done by the buoyancy force is the work done per unit volume. Removing the density (ρ) then leaves the square of the velocity required. The final result needed (bottom line on the slide) comes from the equations of hydrostatic equilibrium and perfect gas as derived on a previous slide labelled “Schwarzschild Criterion for Convective Instability — III”.

Mixing Length Theory – IV

a final expression for the convective flux is

$$F_{\text{conv}} = \rho C_p \alpha^2 T \left[g H \left| 1 - \frac{d \ln \bar{\mu}}{d \ln T} \right| \right]^{1/2} \left[\left| \frac{d \ln T}{d \ln P} \right|_{\text{rad}} - \left| \frac{d \ln T}{d \ln P} \right|_{\text{ad}} \right]^{3/2}.$$

We require that the total energy flux is given by

$$F = F_{\text{rad}} + F_{\text{conv}} = \sigma T_{\text{eff}}^4$$

but as temperature stratification was initially calculated assuming

$$F = F_{\text{rad}} = \sigma T_{\text{eff}}^4$$

a correction $\Delta T(\tau)$ must be iteratively applied to correct the T stratification if the convection instability criterion is satisfied.

Note also that $F_{\text{conv}} \sim T^{1.5}$ whereas $F_{\text{rad}} \sim T_{\text{eff}}^4$ and convection is therefore not effective in the atmospheres of hot stars.

Classical Stellar Atmosphere Models – I

Parameters defining a model stellar atmosphere are T_{eff} , $\log g$ and abundances of chemical elements. Procedure for iteratively computing a stellar atmosphere is:

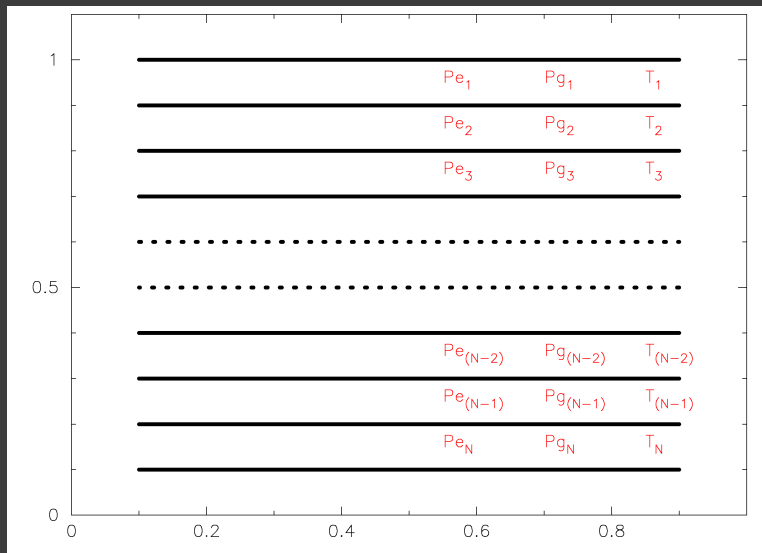
- Define about 100 depth-points (layers) in the atmosphere having monotonically increasing geometric depth. Each layer is assumed to have uniform properties in the horizontal direction.
- Apply the following steps to all depth-points.
 - 1) Establish initial values for $T(x)$, $\kappa_\nu(x)$, $B_\nu[T(x)]$, $P(x)$ and $\rho(x)$ using the Grey Approximation already discussed.
 - 2) Solve the equation of radiative transfer to determine $J_\nu(x)$ and $H_\nu(x)$ at each depth-point.
 - 3) Are the conditions of radiative equilibrium and flux conservation

$$\int_0^\infty \kappa_\nu(x) [J_\nu(x) - B_\nu(x)] d\nu = 0 \quad \text{and} \quad 4\pi \int_0^\infty H_\nu(x) d\nu = \sigma T_{\text{eff}}^4$$

satisfied at each depth? If so, we have a converged model stellar atmosphere and calculations may cease. Otherwise corrections need to be made to current estimates of $T(x)$, $\kappa_\nu(x)$, $B_\nu[T(x)]$, $P(x)$ and $\rho(x)$ before returning to Step 2.

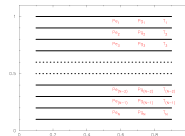
Once a converged model atmosphere is available, we know T , P_{gas} and P_e at every depth-point and all other quantities of interest may be calculated.

Classical Stellar Atmosphere Models – II



Stellar Evolution

- └ Stellar Atmospheres
 - └ Classical Stellar Atmospheres
 - └ Classical Stellar Atmosphere Models – II



It is customary to specify a model stellar atmosphere as series of plane-parallel layers. For each layer, assuming Local Thermodynamic Equilibrium (LTE), a temperature (T_i), gas pressure (P_g); and electron pressure (P_e); needs to have been determined; once these are known, all other quantities of interest such as ionisation fractions and opacities can be calculated for any specified element abundances. In LTE, temperature is monotonically increasing with increasing depth.

Calculation of Synthetic Spectra – I

Bound-bound transitions in atoms, ions or molecules can give rise to absorption lines seen in stellar spectra; steps involved in using this line in an abundance determination for the chemical element responsible (or partly responsible) for the line are:

- Compute a model stellar atmosphere as already described or, in case of the Sun, deduce a model atmosphere empirically from limb darkening measurements.
- For each depth-point in the stellar atmosphere it is then necessary to
 - 1) Establish ionisation fractions and fractions associated into molecules for the chemical element of interest.
 - 2) Determine the population of the lower level involved in the transition responsible for the spectral line of interest.
 - 3) Calculate line opacities at every frequency point used to define the synthetic spectrum.
 - 4) Repeat steps 1–3 for all other lines to be included in the synthetic spectrum.
 - 5) Deduce a total line opacity, to which all spectral lines contribute, for every frequency point used to define the synthetic spectrum.
- Solve the radiative transfer equation at all frequencies of interest, with line opacity added to the continuous opacity, to obtain the emergent synthetic spectrum.

An abundance has to be assumed in a line opacity calculation and several iterations of the calculation described above would be needed before the observed spectrum is matched.

Calculation of Synthetic Spectra – II

Absorption line profiles in stellar spectra are broadened by thermal and pressure effects; these are the most important and will be considered in the consideration of line opacity calculations.

- Each atom (ion or molecule) in a stellar atmosphere will have a velocity along the line of sight, measured in the observer's frame, and an intrinsic profile Doppler shifted by the corresponding amount.
- If the damping process producing the intrinsic profile for each species (atom, ion or molecule) is uncorrelated with its velocity, then the shifted profile may be superimposed to yield the total line absorption cross-section by a simple convolution procedure.

Assuming the plasma is characterised by a kinetic temperature (T), the velocity distribution is Maxwellian, so that the probability of finding an absorber (of mass m_A) with a line-of-sight velocity between ξ and $\xi + d\xi$ is

$$W(\xi) d\xi = (\pi^{1/2} \xi_0)^{-1} \exp(-\xi^2/\xi_0^2) d\xi$$

where $\xi_0 = (2kT/m_A)^{1/2} = 12.85 (T/10^4 A)^{1/2}$ km/s and A is the atomic mass of the absorber.

Calculation of Synthetic Spectra – III

If an atom (ion or molecule) having a velocity component ξ in the observer's direction is observed at frequency ν , it is absorbing at frequency $\nu [1 - (\xi/c)]$ in its own frame and the line absorption coefficient for that atom is $\ell(\nu - \xi\nu/c)$. The total absorption coefficient at frequency ν for all species of that type as seen by the observer is therefore given by the convolution integral

$$\ell(\nu) = \int_{-\infty}^{+\infty} \ell(\nu - \xi\nu/c) W(\xi) d\xi$$

which can be applied to any absorption profile to allow for the effects of Doppler broadening.

In general, pressure broadening is important in stellar atmospheres; this is the damping phenomenon and its frequency dependence is given by the Lorentz profile

$$\ell(\nu) = \left(\frac{\pi e^2}{m_e c} \right) f \frac{(\Gamma/4\pi^2)}{(\nu - \nu_0)^2 + (\Gamma/4\pi)^2}$$

where f is the transition oscillator strength, ν_0 the line-centre frequency and Γ the damping constant.

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Stellar Evolution

- └ Stellar Atmospheres
 - └ Synthetic Spectra
 - └ Calculation of Synthetic Spectra – III

If an atom (or molecule) having a velocity component z in the observer's direction is observed at frequency ω , it is absorbing at frequency $\omega' = \omega(1+z/c)$ in its own frame and the line absorption coefficient for that atom is $k(\omega - z/c\nu)$. The total absorption coefficient at frequency ω for all species of that type as seen by the observer is therefore given by the summation integral:

$$k(\omega) = \int_{-\infty}^{\infty} k(\omega - z/c\nu) \mathcal{N}(z) dz$$

which can be applied to any absorption profile to allow for the effects of Doppler broadening.

In general, pressure broadening is important in stellar atmospheres; this is the damping phenomenon and its frequency dependence is given by the Lorentz profile:

$$k(\omega) = \left(\frac{f_{ul}}{4\pi\nu}\right) \frac{H^2(\omega, \nu^2)}{(\omega - \nu)^2 + (H/2)^2}$$

where f is the transition oscillator strength, ν is the line-center frequency and H the damping constant.

A transition oscillator strength (f) is related to the transition probability; that is given an incident photon of the right energy, there is a probability that an electron in the lower level will jump to the upper level and thereby absorb the incident photon. Oscillator strengths may be calculated exactly for one-electron atoms (H I, He II, ...) but for more complicated atoms they need to be measured in a laboratory or calculated approximately using state of the art quantum mechanical calculations.

Calculation of Synthetic Spectra – IV

Substituting in the convolution integral above gives

$$\begin{aligned}\ell(\nu) &= \left(\frac{\pi e^2}{m_e c} \right) f (\Gamma/4\pi^2) \int_{-\infty}^{+\infty} \frac{(\pi^{1/2} \xi_0)^{-1} e^{-\xi^2/\xi_0^2}}{(\nu - \xi\nu_0/c - \nu_0)^2 + (\Gamma/4\pi)^2} d\xi \\ &= \frac{\sqrt{\pi} e^2}{m_e c} f \frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{(\Gamma/4\pi) e^{-\xi^2/\xi_0^2}}{(\nu - \xi\nu_0/c - \nu_0)^2 + (\Gamma/4\pi)^2} \frac{d\xi}{\xi_0}\end{aligned}$$

Define the Doppler width of a line as

$$\Delta\nu_D = \xi_0 \nu_0/c,$$

along with the Voigt displacement parameter

$$v = \frac{(\nu - \nu_0)}{\Delta\nu_D}$$

and the Voigt damping parameter as

$$a = \frac{\Gamma}{4\pi \Delta\nu_D}$$

with

$$y = \frac{\Delta\nu}{\Delta\nu_D} = \frac{\xi}{\xi_0}.$$

Calculation of Synthetic Spectra – V

Then the line absorption coefficient due to the selected line and atmospheric depth point is

$$\ell(\nu) = \frac{\sqrt{\pi} e^2 f}{m_e c \Delta\nu_D} \frac{a}{\pi} \int_{-\infty}^{+\infty} \frac{e^{-y^2}}{(v - y)^2 + a^2} dy$$

The integral

$$H(a, v) = \frac{a}{\pi} \int_{-\infty}^{\infty} \frac{e^{-y^2}}{(v - y)^2 + a^2} dy$$

is known as the Voigt Function; it needs to be evaluated rapidly for synthetic stellar spectrum calculations.

Solar Abundances

Z	Element	Photosphere	Meteorites	Z	Element	Photosphere	Meteorites
1	H	12.00	8.22 ± 0.04	44	Ru	1.75 ± 0.08	1.76 ± 0.03
2	He	[10.93 ± 0.01]	1.29	45	Rh	0.91 ± 0.10	1.06 ± 0.04
3	Li	1.05 ± 0.10	3.26 ± 0.05	46	Pd	1.57 ± 0.10	1.65 ± 0.02
4	Be	1.38 ± 0.09	1.30 ± 0.03	47	Ag	0.94 ± 0.10	1.20 ± 0.02
5	B	2.70 ± 0.20	2.79 ± 0.04	48	Cd		1.71 ± 0.03
6	C	8.43 ± 0.05	7.39 ± 0.04	49	In	0.80 ± 0.20	0.76 ± 0.03
7	N	7.83 ± 0.05	6.26 ± 0.06	50	Sn	2.04 ± 0.10	2.07 ± 0.06
8	O	8.69 ± 0.05	8.40 ± 0.04	51	Sb		1.01 ± 0.06
9	F	4.56 ± 0.30	4.42 ± 0.06	52	Te		2.18 ± 0.03
10	Ne	[7.93 ± 0.10]	-1.12	53	I		1.55 ± 0.08
11	Na	6.24 ± 0.04	6.27 ± 0.02	54	Xe	[2.24 ± 0.06]	-1.95
12	Mg	7.60 ± 0.04	7.53 ± 0.01	55	Cs		1.08 ± 0.02
13	Al	6.45 ± 0.03	6.43 ± 0.01	56	Ba	2.18 ± 0.09	2.18 ± 0.03
14	Si	7.51 ± 0.03	7.51 ± 0.01	57	La	1.10 ± 0.04	1.17 ± 0.02
15	P	5.41 ± 0.03	5.43 ± 0.04	58	Ce	1.58 ± 0.04	1.58 ± 0.02
16	S	7.12 ± 0.03	7.15 ± 0.02	59	Pr	0.72 ± 0.04	0.76 ± 0.03
17	Cl	5.50 ± 0.30	5.23 ± 0.06	60	Nd	1.42 ± 0.04	1.45 ± 0.02
18	Ar	[6.40 ± 0.13]	-0.50	62	Sm	0.96 ± 0.04	0.94 ± 0.02
19	K	5.03 ± 0.09	5.08 ± 0.02	63	Eu	0.52 ± 0.04	0.51 ± 0.02
20	Ca	6.34 ± 0.04	6.29 ± 0.02	64	Gd	1.07 ± 0.04	1.05 ± 0.02
21	Sc	3.15 ± 0.04	3.05 ± 0.02	65	Tb	0.30 ± 0.10	0.32 ± 0.03

22	Ti	4.95 ± 0.05	4.91 ± 0.03	66	Dy	1.10 ± 0.04	1.13 ± 0.02
23	V	3.93 ± 0.08	3.96 ± 0.02	67	Ho	0.48 ± 0.11	0.47 ± 0.03
24	Cr	5.64 ± 0.04	5.64 ± 0.01	68	Er	0.92 ± 0.05	0.92 ± 0.02
25	Mn	5.43 ± 0.04	5.48 ± 0.01	69	Tm	0.10 ± 0.04	0.12 ± 0.03
26	Fe	7.50 ± 0.04	7.45 ± 0.01	70	Yb	0.84 ± 0.11	0.92 ± 0.02
27	Co	4.99 ± 0.07	4.87 ± 0.01	71	Lu	0.10 ± 0.09	0.09 ± 0.02
28	Ni	6.22 ± 0.04	6.20 ± 0.01	72	Hf	0.85 ± 0.04	0.71 ± 0.02
29	Cu	4.19 ± 0.04	4.25 ± 0.04	73	Ta		-0.12 ± 0.04
30	Zn	4.56 ± 0.05	4.63 ± 0.04	74	W	0.85 ± 0.12	0.65 ± 0.04
31	Ga	3.04 ± 0.09	3.08 ± 0.02	75	Re		0.26 ± 0.04
32	Ge	3.65 ± 0.10	3.58 ± 0.04	76	Os	1.40 ± 0.08	1.35 ± 0.03
33	As		2.30 ± 0.04	77	Ir	1.38 ± 0.07	1.32 ± 0.02
34	Se		3.34 ± 0.03	78	Pt		1.62 ± 0.03
35	Br		2.54 ± 0.06	79	Au	0.92 ± 0.10	0.80 ± 0.04
36	Kr	[3.25 ± 0.06]	-2.27	80	Hg		1.17 ± 0.08
37	Rb	2.52 ± 0.10	2.36 ± 0.03	81	Tl	0.90 ± 0.20	0.77 ± 0.03
38	Sr	2.87 ± 0.07	2.88 ± 0.03	82	Pb	1.75 ± 0.10	2.04 ± 0.03
39	Y	2.21 ± 0.05	2.17 ± 0.04	83	Bi		0.65 ± 0.04
40	Zr	2.58 ± 0.04	2.53 ± 0.04	90	Th	0.02 ± 0.10	0.06 ± 0.03
41	Nb	1.46 ± 0.04	1.41 ± 0.04	92	U		-0.54 ± 0.03
42	Mo	1.88 ± 0.08	1.94 ± 0.04				

Lecture 3: Summary

Essential points covered in the third lecture:

- The connection between density and pressure as determined by hydrostatic equilibrium was introduced; while relevant to static stellar atmospheres, it also provides one of the four equations of stellar structure.
- No stellar energy generation takes place in the stellar atmosphere and this provides the condition of radiative equilibrium.
- Rosseland Mean Opacity was defined.
- An analytical solution to the stellar atmosphere problem was presented assuming frequency independent opacities (Grey Approximation).
- Conditions under which convection becomes important and how to calculate the convective flux were introduced; these too apply to stellar interiors as well as atmospheres.
- The calculation of a model stellar atmosphere by iteration was summarised, as was their application to the calculation of synthetic spectra for abundance determinations.

Acknowledgement

Material presented in this lecture on stellar atmospheres is based to a large extent on slides prepared by R.-P. Kudritzki (University of Hawaii).