Introduction

Stellar evolution is determined by the reactions which take place within stars:

- Binding energy per nucleon determines the most stable isotopes and therefore the most probable end products of fusion and fission reactions.
- For fusion to occur, quantum mechanical tunneling through the repulsive Coulomb barrier must occur so that the strong nuclear force (which is a short-range force) can take over and hold the two nuclei together.
- Hydrogen is converted to helium by the PP-Chain and CNO-Cycle.
- In due course, helium is converted to carbon and oxygen through the 3α-reaction.
- Other processes, such as neutron capture reactions, produce heavier elements.

Binding Energy Per Nucleon – I

The general description of a nuclear reaction is

$$\mathrm{I}(\mathrm{A_i}, \mathrm{Z_i}) + \mathrm{J}(\mathrm{A_j}, \mathrm{Z_j}) \leftrightarrow \mathrm{K}(\mathrm{A_k}, \mathrm{Z_k}) + \mathrm{L}(\mathrm{A}_\ell, \mathrm{Z}_\ell)$$

where

- A_n is the baryon number, nucleon number or nuclear mass of nucleus N and
- Z_n is the nuclear charge of nucleus N.

The nucleus of any element (or isotope) N is uniquely defined by the two integers A_n and Z_n. Note also that anti-particles have the opposite charge to their corresponding particle.

In any nuclear reaction, the following must be conserved:

- the baryon number (protons, neutrons and their anti-particles),
- the lepton number (electrons, positrons, neutrinos and anti-neutrinos) and
- charge.

Binding Energy Per Nucleon – II

The total mass of a nucleus is known to be less than the total mass of the constituent nucleons and so:

- There is a decrease in mass whenever a nucleus is formed from constituent nucleons.
- The mass-deficit is released as energy according to Einstein's equation $E = mc^2$.
- The difference is known as the *binding energy* of the nucleus.

Thus if a nucleus is composed of Z protons and N neutrons, its binding energy is

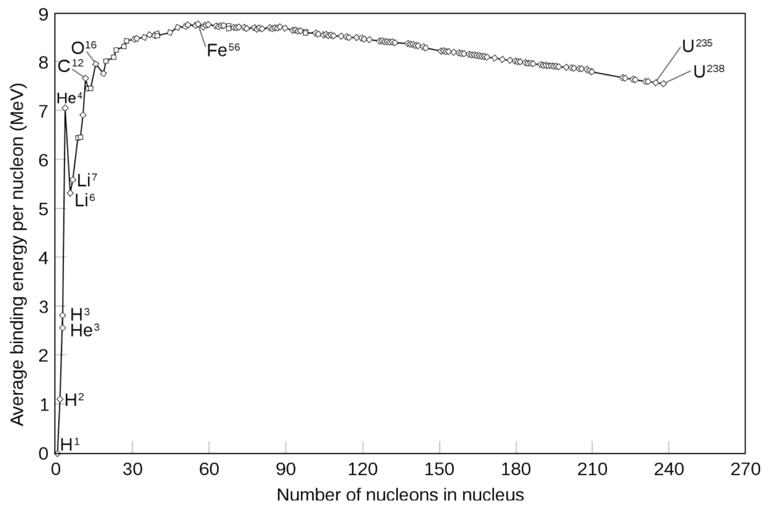
$$Q(Z,N) \equiv \left[Z\, m_{\mathrm{p}} + N\, m_{\mathrm{n}} - M(Z,N)\right]\, c^2 .$$

A more significant quantity is the total binding energy per nucleon

$$Q(Z,N)/(Z+N) = Q(Z,N)/A$$

which is considered relative to the hydrogen binding energy per nucleon which is of course zero.

Binding Energy Per Nucleon – III

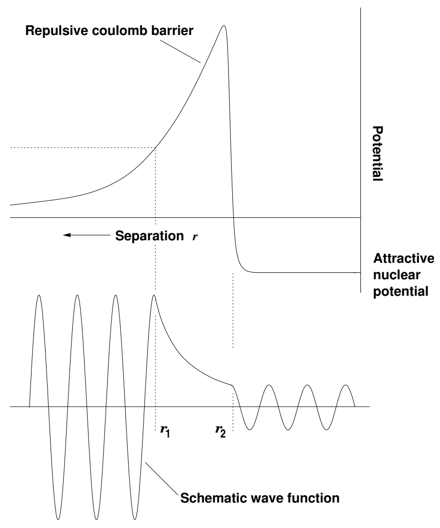


Binding Energy Per Nucleon – IV

The variation of binding energy per nucleon with baryon number A:

- General trend is an increase of Q with atomic mass up to A = 56 (Fe), followed by a slow monotonic decline.
- There is a steep rise from H to ^{2}H, ^{3}He to ^{4}He.
- Fusion of H to He should release a larger amount of energy per unit mass than say fusion of He to C.
- Energy may be gained by the fusion of lighter elements to form heavier elements, up to Fe.
- Energy may also be gained from the fission of heavy nuclei to form lighter nuclei, down to Fe.

Coulomb Barrier Penetration – I



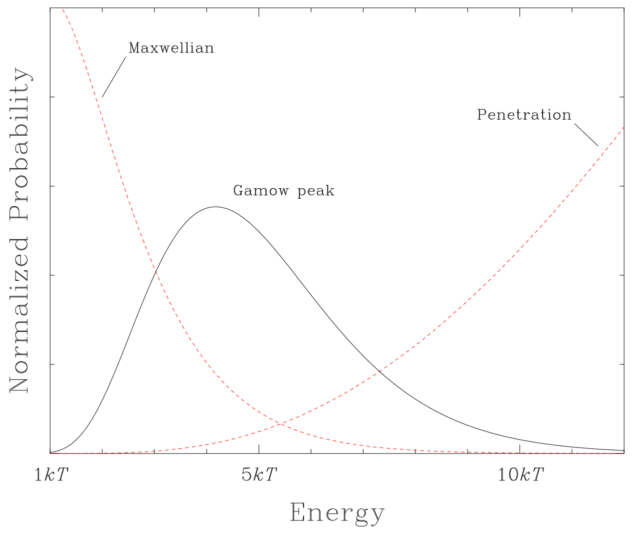
Coulomb Barrier Penetration – II

Conditions under which fusion can occur are:

- Nuclei interact through the four forces of physics, though only the electromagnetic and the strong nuclear forces are important here.
- Two positively charged nuclei must overcome a Coulomb barrier (a long range force $\propto r^2$) to achieve a separation where the strong nuclear attractive force dominates ($\sim 10^{-15}$m, which is the typical size of a nucleus).
- Height of the Coulomb barrier is estimated by $z_1\, z_2\, e^2/(4\,\pi\,\epsilon_0\, r)$ where z_1 and z_2 are the numbers of protons in the two nuclei, e is the electronic charge (1.6×10^{-19} C) and ϵ_0 is the permittivity of free space $= 8.85 \times 10^{-12}\,\mathrm{C}^2\,\mathrm{N}^{-1}\,\mathrm{m}^{-2}$.
- There is a finite probability for a particle to penetrate the Coulomb barrier as if a "tunnel" existed; this quantum mechanical effect was discovered by Gamow (1928) in connection with radioactivity.
- Penetration probability as computed by Gamow is $\exp(-\pi z_1 z_2 e^2/(\epsilon_0\, h\nu))$ which increases as the particle velocity v (and hence energy $h\nu$) increase.
- For an ideal gas, v will have a Maxwellian distribution and so the fusion probability is given by

$$P_{\mathrm{fusion}} \;\propto\; \exp(-\frac{\pi\, z_1\, z_2\, e^2}{\epsilon_0\, h\,\nu})\;\exp(-\frac{m\,v^2}{2\,k\,T}).$$

Gamow Peak – I

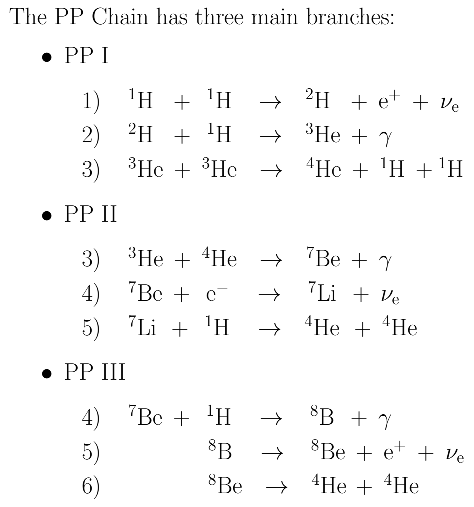

Maxwellian
Penetration
Gamow peak
Normalized Probability
1kT
5kT
10kT
Energy

Gamow Peak – II

The Gamow peak is the product of the Maxwellian distribution and tunnelling probability (P_{fusion}):

- Area under the Gamow peak determines the reaction rate.
- The higher the electric charges of the interacting nuclei, the greater the repulsive force; hence the higher T has to be before reactions occur.
- Highly charged nuclei are obviously more massive and so reactions between light nuclei occur at a lower T than reactions between heavy nuclei.

PP Chain – I



The PP Chain has three main branches:

- PP I

$$\begin{array}{lllll}
1) & {}^{1}\mathrm{H} + {}^{1}\mathrm{H} & \rightarrow & {}^{2}\mathrm{H} + \mathrm{e}^{+} + \nu_{\mathrm{e}} \\
2) & {}^{2}\mathrm{H} + {}^{1}\mathrm{H} & \rightarrow & {}^{3}\mathrm{He} + \gamma \\
3) & {}^{3}\mathrm{He} + {}^{3}\mathrm{He} & \rightarrow & {}^{4}\mathrm{He} + {}^{1}\mathrm{H} + {}^{1}\mathrm{H}
\end{array}$$

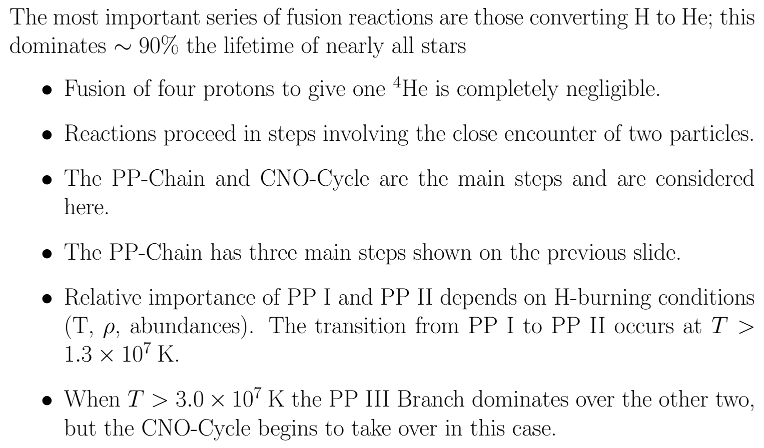
- PP II

$$\begin{array}{lllll}
3) & {}^{3}\mathrm{He} + {}^{4}\mathrm{He} & \rightarrow & {}^{7}\mathrm{Be} + \gamma \\
4) & {}^{7}\mathrm{Be} + \mathrm{e}^{-} & \rightarrow & {}^{7}\mathrm{Li} + \nu_{\mathrm{e}} \\
5) & {}^{7}\mathrm{Li} + {}^{1}\mathrm{H} & \rightarrow & {}^{4}\mathrm{He} + {}^{4}\mathrm{He}
\end{array}$$

- PP III

$$\begin{array}{lllll}
4) & {}^{7}\mathrm{Be} + {}^{1}\mathrm{H} & \rightarrow & {}^{8}\mathrm{B} + \gamma \\
5) & {}^{8}\mathrm{B} & \rightarrow & {}^{8}\mathrm{Be} + \mathrm{e}^{+} + \nu_{\mathrm{e}} \\
6) & {}^{8}\mathrm{Be} & \rightarrow & {}^{4}\mathrm{He} + {}^{4}\mathrm{He}
\end{array}$$

PP Chain – II



The most important series of fusion reactions are those converting H to He; this dominates $\sim 90\%$ the lifetime of nearly all stars

- Fusion of four protons to give one ^{4}He is completely negligible.
- Reactions proceed in steps involving the close encounter of two particles.
- The PP-Chain and CNO-Cycle are the main steps and are considered here.
- The PP-Chain has three main steps shown on the previous slide.
- Relative importance of PP I and PP II depends on H-burning conditions (T, ρ, abundances). The transition from PP I to PP II occurs at $T > 1.3 \times 10^{7}$ K.
- When $T > 3.0 \times 10^{7}$ K the PP III Branch dominates over the other two, but the CNO-Cycle begins to take over in this case.

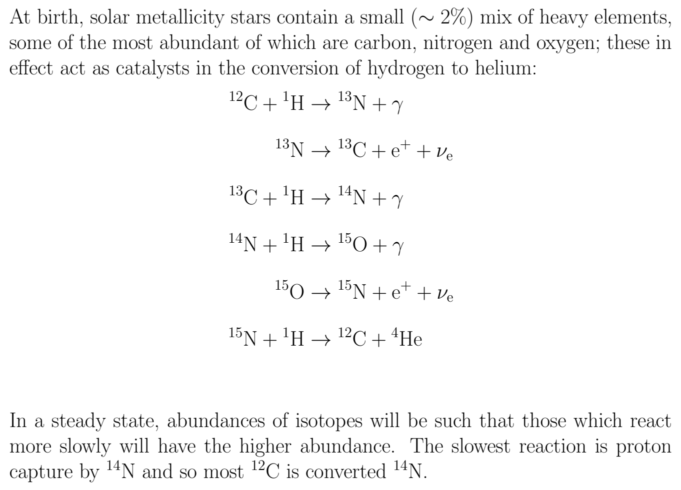
PP Chain – III

Energy released in the formation of an α-particle by the fusion of four protons is the mass difference of four protons and an α-particle:

$$Q_{\mathrm{p-p}} = \left[4\,M(^{1}\mathrm{H}) - M(^{4}\mathrm{He})\right]\,c^2 = 26.7\,\mathrm{MeV}$$

- The fusion of four protons to give an α-particle involves the conversion of two protons into neutrons.
- Spin conservation requires that two neutrinos also be emitted which carry energy away from the reaction site.
- Neutrino emission directly confirms the occurrence of nuclear reactions in the solar interior. No other direct test of nuclear reactions is possible.
- Mean neutrino energy flux is $\sim 0.26\,\mathrm{MeV}$ for $^{2}\mathrm{H}$ creation (PP I and PP II) and $\sim 7.2\,\mathrm{MeV}$ for $\beta-$decay (PP III).
- But as PP III makes only a small contribution, the energy released for each α-particle created is $\sim 26\,\mathrm{MeV}$.

CNO Cycle



At birth, solar metallicity stars contain a small ($\sim 2\%$) mix of heavy elements, some of the most abundant of which are carbon, nitrogen and oxygen; these in effect act as catalysts in the conversion of hydrogen to helium:

$$
\begin{aligned}
{}^{12}\mathrm{C} + {}^{1}\mathrm{H} &\rightarrow {}^{13}\mathrm{N} + \gamma \\
{}^{13}\mathrm{N} &\rightarrow {}^{13}\mathrm{C} + \mathrm{e}^{+} + \nu_{\mathrm{e}} \\
{}^{13}\mathrm{C} + {}^{1}\mathrm{H} &\rightarrow {}^{14}\mathrm{N} + \gamma \\
{}^{14}\mathrm{N} + {}^{1}\mathrm{H} &\rightarrow {}^{15}\mathrm{O} + \gamma \\
{}^{15}\mathrm{O} &\rightarrow {}^{15}\mathrm{N} + \mathrm{e}^{+} + \nu_{\mathrm{e}} \\
{}^{15}\mathrm{N} + {}^{1}\mathrm{H} &\rightarrow {}^{12}\mathrm{C} + {}^{4}\mathrm{He}
\end{aligned}
$$

In a steady state, abundances of isotopes will be such that those which react more slowly will have the higher abundance. The slowest reaction is proton capture by ${}^{14}\mathrm{N}$ and so most ${}^{12}\mathrm{C}$ is converted ${}^{14}\mathrm{N}$.

PP Chain and CNO Cycle Temperature Dependence – I

The PP-Chain and CNO-Cycle have very different energy generation rate temperature dependencies. Because energy generation in the CNO-Cycle is limited by the slowest reaction, energy generation rates are of the form:

$$\epsilon^{\mathrm{PP}} = (\epsilon^{\mathrm{PP}})_0\, \rho {X_{\mathrm{H}}}^2\, T^4 \quad \text{and}$$

$$\epsilon^{\mathrm{CNO}} = (\epsilon^{\mathrm{CNO}})_0\, \rho X_{\mathrm{H}}\, Z\, f_{\mathrm{N}}\, T^{18}.$$

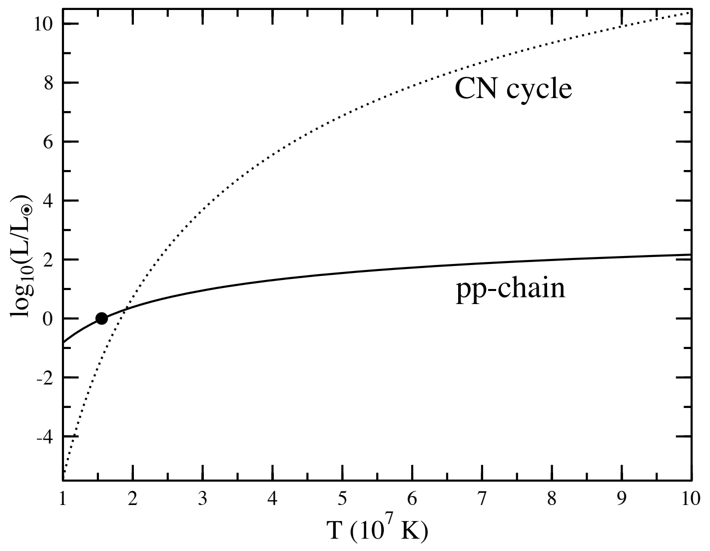
Here $(\epsilon^{\mathrm{PP}})_0$ and $(\epsilon^{\mathrm{CNO}})_0$ are constants of proportionality, X_{H} is the hydrogen mass-fraction, Z is the metal mass-fraction and f_{N} is the nitrogen fraction in the metal mass-fraction.

Equating the two gives the temperature at which the PP-Chain and CNO-Cycle generate energy at the same rate:

$$T \simeq \left[\frac{(\epsilon^{\mathrm{PP}})_0\, X_{\mathrm{H}}}{(\epsilon^{\mathrm{CNO}})_0\, Z\, f_{\mathrm{N}}} \right]^{1/14} \mathrm{K} \qquad \text{and approximately}$$

$$T \sim 1.7 \times 10^7\, \mathrm{K}.$$

PP Chain and CNO Cycle Temperature Dependence – II



Helium Burning – Triple-α Reaction

The simplest reaction in a helium gas should be the fusion of two helium nuclei, but there is no stable configuration with $A = 8$:

- The half life of ^{8}Be is 2.6×10^{-16} seconds
- A third helium nucleus can be added to ^{8}Be before decay, forming ^{12}C by the "triple-alpha" reaction

$$^{4}\mathrm{He} + {}^{4}\mathrm{He} \rightarrow {}^{8}\mathrm{Be}$$

$$^{8}\mathrm{Be} + {}^{4}\mathrm{He} \rightarrow {}^{12}\mathrm{C} + \gamma.$$

- The existence of an energy level in the ^{12}C nucleus with an energy close to the energy of the combining ^{8}Be and ^{4}He nuclei was proposed by Hoyle and subsequently found by experiment; this allows the "triple-alpha" reaction to proceed to the expected rate.
- Energy released by the two-stage "triple-alpha" is

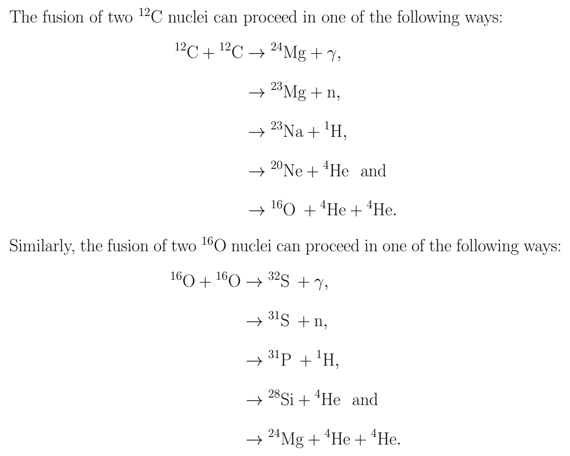
$$Q_{3\alpha} = \left[3\,M(^{4}\mathrm{He}) - M(^{12}\mathrm{C})\right]\,c^2 = 7.275\,\mathrm{MeV}$$

  or about 10% the energy generated per unit mass when converting hydrogen to helium.

- The temperature dependence is astounding:

$$\epsilon^{3\alpha} \propto \rho^2\,T^{40}$$

Carbon and Oxygen Burning – I



The fusion of two ^{12}C nuclei can proceed in one of the following ways:

$$\begin{aligned} {}^{12}\mathrm{C} + {}^{12}\mathrm{C} &\to {}^{24}\mathrm{Mg} + \gamma, \\ &\to {}^{23}\mathrm{Mg} + \mathrm{n}, \\ &\to {}^{23}\mathrm{Na} + {}^{1}\mathrm{H}, \\ &\to {}^{20}\mathrm{Ne} + {}^{4}\mathrm{He} \quad \text{and} \\ &\to {}^{16}\mathrm{O} \ + {}^{4}\mathrm{He} + {}^{4}\mathrm{He}. \end{aligned}$$

Similarly, the fusion of two ^{16}O nuclei can proceed in one of the following ways:

$$\begin{aligned} {}^{16}\mathrm{O} + {}^{16}\mathrm{O} &\to {}^{32}\mathrm{S} \ + \gamma, \\ &\to {}^{31}\mathrm{S} \ + \mathrm{n}, \\ &\to {}^{31}\mathrm{P} \ + {}^{1}\mathrm{H}, \\ &\to {}^{28}\mathrm{Si} + {}^{4}\mathrm{He} \quad \text{and} \\ &\to {}^{24}\mathrm{Mg} + {}^{4}\mathrm{He} + {}^{4}\mathrm{He}. \end{aligned}$$

Carbon and Oxygen Burning – II

- Carbon burning requires $T > 5 \times 10^8\,\mathrm{K}$ and oxygen burning $T > 10^9\,\mathrm{K}$.
- Interactions between $^{12}\mathrm{C}$ and $^{16}\mathrm{O}$ are negligible as at the temperatures required to penetrate the Coulomb barrier, $^{12}\mathrm{C}$ nuclei are quickly destroyed by interacting with themselves.
- Branching ratios for the $^{12}\mathrm{C}$ and $^{16}\mathrm{O}$ reactions are temperature dependent probabilities.
- $^{12}\mathrm{C} + {}^{12}\mathrm{C} \quad \rightarrow \quad \sim 13\,\mathrm{MeV} \;\; (\sim 5.2 \times 10^{13}\,\mathrm{J\,kg^{-1}})$.
- $^{16}\mathrm{O} + {}^{16}\mathrm{O} \quad \rightarrow \quad \sim 16\,\mathrm{MeV} \;\; (\sim 4.8 \times 10^{13}\,\mathrm{J\,kg^{-1}})$.
- Protons, neutrons and α-particles produced are immediately captured by heavy nuclei, creating a variety of isotopes by secondary reactions.

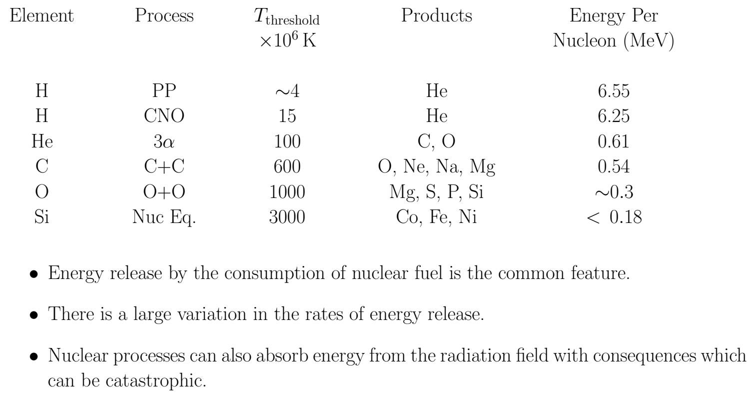
Nuclear Statistical Equilibrium

- Two ^{28}Si nuclei could fuse to give ^{56}Fe which is at the end of the fusion chain.
- But the T required to penetrate the Coulomb Barrier is now higher than that at which ^{28}Si photodisintegrates ($T \sim 3 \times 10^9\,$K).
- Light particles emitted by ^{28}Si photodisintegration are absorbed by other ^{28}Si nuclei.
- Another example of photodisintegration is the reaction

  $$^{16}\mathrm{O} + {}^{4}\mathrm{He} \leftrightarrow {}^{20}\mathrm{Ne} + \gamma$$

  which proceeds to the left when $T > 1.5 \times 10^9\,$K and to the right when $T < 1.5 \times 10^9\,$K.
- Although reactions tend to a state of equilibrium, a leakage occurs towards the stable iron group (Fe, Co and Ni) nuclei which do not photodisintegrate until $T < 7 \times 10^9\,$K.

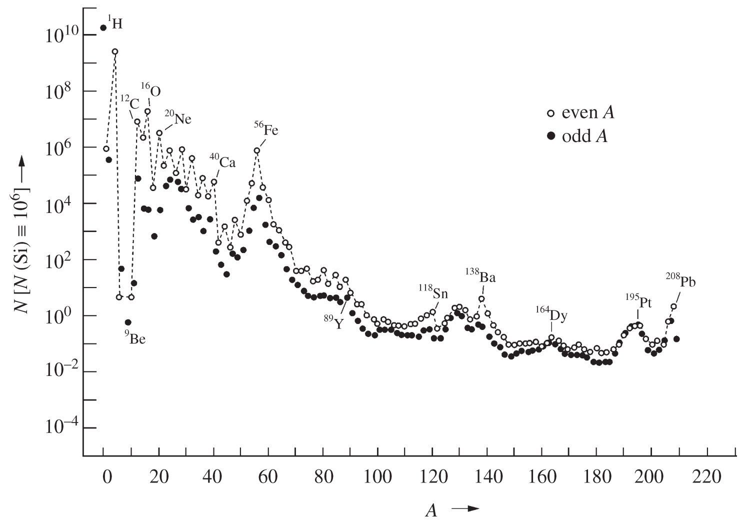
Major Nuclear Burning Processes



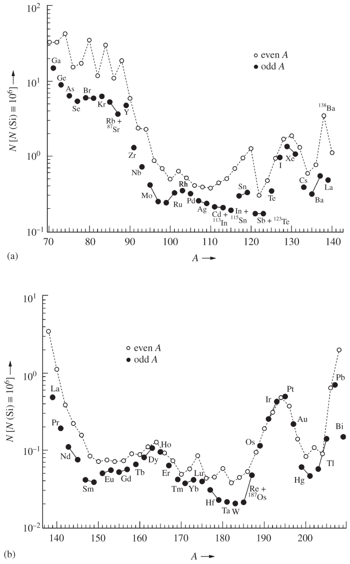
Element	Process	$T_{\text{threshold}}$ $\times 10^6$ K	Products	Energy Per Nucleon (MeV)
H	PP	$\sim$4	He	6.55
H	CNO	15	He	6.25
He	3α	100	C, O	0.61
C	C+C	600	O, Ne, Na, Mg	0.54
O	O+O	1000	Mg, S, P, Si	$\sim$0.3
Si	Nuc Eq.	3000	Co, Fe, Ni	< 0.18

- Energy release by the consumption of nuclear fuel is the common feature.
- There is a large variation in the rates of energy release.
- Nuclear processes can also absorb energy from the radiation field with consequences which can be catastrophic.

Solar System Abundances - I



Solar System Abundances - II



S-Process and R-Process – I

Elements having $Z > 28$ (nickel) are created by neutron capture:

- Neutrons are produced during C, O and Si burning.
- Neutron capture is not limited by the Coulomb barrier and it can proceed at relatively low temperatures.
- Limited by the availability of free neutrons.
- If enough neutrons are available, a chain of reactions becomes possible:

$$\mathrm{I(A, Z) + n} \rightarrow \mathrm{I}_1(A+1, Z)$$

$$\mathrm{I_1(A+1, Z) + n} \rightarrow \mathrm{I}_2(A+2, Z)$$

$$\mathrm{I_2(A+2, Z) + n} \rightarrow \mathrm{I}_3(A+3, Z) \quad \text{etc..}$$

- If a radioactive isotope is formed, it will undergo β decay, creating a new element

$$\mathrm{I_N(A+N, Z) \rightarrow J(A+N, Z+1) + e^- + \bar{\nu}_e}$$

S-Process and R-Process – II

If the new element is stable, it will resume neutron capture; otherwise, it may undergo a further series of β decays.

$$\mathrm{J(A+N,Z+1)} \rightarrow K(A+N,Z+2) + \mathrm{e}^- + \bar{\nu}_\mathrm{e},$$

$$\mathrm{K(A+N,Z+2)} \rightarrow L(A+N,Z+3) + \mathrm{e}^- + \bar{\nu}_\mathrm{e} \quad \text{etc.}$$

In the process, two types of reaction and two types of nuclei are involved:

- neutron captures and β decays and
- stable and unstable nuclei.

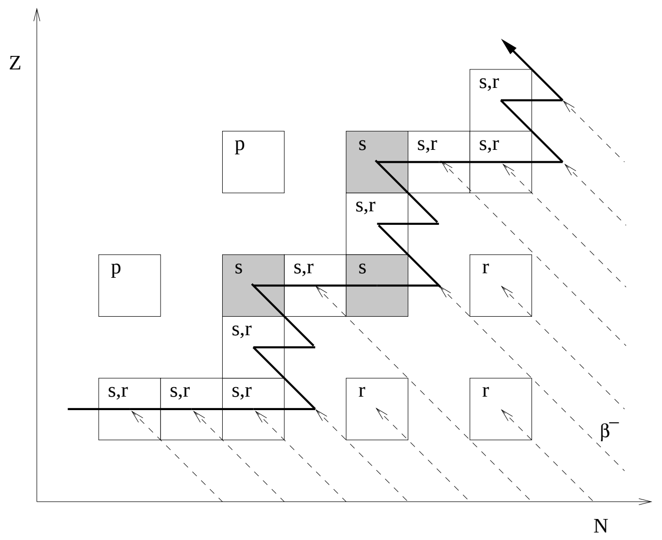
Stable nuclei may only undergo neutron captures whereas unstable nuclei may also undergo β decays; in this latter case, the outcome depends on the timescales for the two processes.

Neutron capture reactions may proceed more slowly or more rapidly than competing β decays. The different chains of reactions and products are called the **s-process** and **r-process**.

S-Process and R-Process – III

- Neutron densities of $10^5\,\mathrm{cm}^{-3}$ and $10^{23}\,\mathrm{cm}^{-3}$ are respectively required for the s-process and r-process.
- ^{12}C and ^{16}O burning in Asymptotic Giant Branch (AGB) star envelopes produce the neutron density required for the s-process.
- Nuclei formed by the s-process are always close to the line of stable nuclei in the (A, Z) diagram.
- A r-process neutron density may only be achieved in a supernova.
- Neutron capture during the r-process is so rapid that neutron-rich nuclei are formed faster than the β decay can remove them.
- Once a complete nuclear shell is formed in either process, adding further neutrons becomes only possible once β decay has converted enough neutrons into protons for a stable nucleus to be formed.
- Fission involving α decay limits the nuclear mass produced.
- About 35 nuclei are proton rich and cannot be produced by the s-process or r-process. A proton capture reaction (p-process) is involved.

S-Process and R-Process – IV

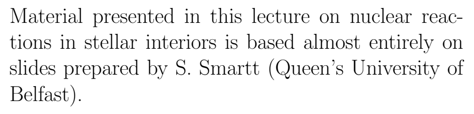


Lecture 5: Summary

Essential points covered in fifth lecture:

- Stellar evolution is determined by reactions that occur in stellar interiors.
- Binding energy per nucleon determines the most stable isotopes and therefore the most probable end products of fusion and fission reactions.
- For fusion to occur, quantum mechanical tunneling through the repulsive Coulomb barrier must occur so that the strong nuclear force (which is a short-range force) can take over and hold the two nuclei together.
- Hydrogen is converted to helium by the PP-Chain and CNO-Cycle.
- In due course, helium is converted to carbon and oxygen through the 3α-reaction.
- Other processes, such as neutron capture reactions, produce heavier elements.

Acknowledgement



Material presented in this lecture on nuclear reactions in stellar interiors is based almost entirely on slides prepared by S. Smartt (Queen's University of Belfast).