Introduction

In this lecture, consideration is given to final evolution stages and this too is dependent on mass.

- The vast majority of stars become white dwarfs in which all nuclear reactions have ceased. Interest therefore centres on cooling, structural changes during cooling and the cooling time. Equally important is what the study of white dwarfs can reveal about their progenitors on the Asymptotic Giant Branch.
- Very massive stars ($\gtrsim 8\,M_\odot$) experience mass-loss throughout their evolution and for this reason their evolution is substantially different from that of lower mass stars.
- Very massive stars ($\gtrsim 8\,M_\odot$) are neutron star and black hole progenitors following a supernova explosion.
- Circumstances leading to a neutron star, as opposed to a black hole, need to be understood.

Equation of State of a Degenerate Gas – I

So far it has been assumed that stars are formed of ideal gases although degeneracy pressure has been mentioned in connection with cores formed at the centres of lower mass stars on the RGB and AGB.

Degeneracy pressure resists gravitational collapse and in connection with white dwarfs, the final evolution stage for most stars, this idea now needs to be developed.

- At high densities, gas particles become so close that interactions between them can no longer be ignored.
- Beyond the regime where pressure ionisation becomes important, as pressure on a highly ionised gas is further increased the **Pauli Exclusion Principle** becomes relevant.
- No more than two electrons of opposite spin can occupy the same quantum cell.
- The quantum cell of an electron is defined in phase-space and specified by three spatial coordinates (q_i) and three momentum coordinates (p_i) where $(i = 1, 2, 3)$.
- The volume of allowed phase-space for an electron quantum cell is

  $$\Delta q_1\,\Delta q_2\,\Delta q_3\,\Delta p_1\,\Delta p_2\,\Delta p_3 \;= h^3$$

  which can be occupied by at most two electrons.

Equation of State of a Degenerate Gas – II

Consider the centre of a star as ρ increases:

- Electrons become confined to a smaller phase-space.
- Eventually two electrons with opposite spins occupy the same state.
- Lowest energy states will fill first and phase-space becomes full up to some critical momentum p_0.

Consider electrons in volume $V = \Delta q_1 \, \Delta q_2 \, \Delta q_3$ having momenta in the range p to $p + dp$:

- Volume of momentum-space occupied by the electrons is the volume of a spherical shell of radius p and thickness dp or $4\,\pi\,p^2\,dp$.
- The volume of phase-space V_{ph} occupied by the considered electrons is the product of the momentum and position space volumes

$$V_{\mathrm{ph}} = 4\,\pi\,p^2\,V\,dp.$$

- To get the number of quantum states in V_{ph} it is necessary to divide by the phase-space volume of each quantum state h^3, giving $4\,\pi\,p^2\,V\,dp/h^3$.

Equation of State of a Degenerate Gas – III

- Let $N_p\,dp$ be the actual number of electrons in V having momenta in the range p to $p+dp$; then Pauli's Exclusion Principle leads to

$$N_p\,dp \leqslant \frac{8\,\pi\,p^2\,V}{h^3}\,dp.$$

- Consider a completely degenerate gas in which all momentum states up to p_0 are filled while all higher momentum states are empty; the total number of electrons in this degenerate gas would then be

$$\mathcal{N} = \frac{8\,\pi\,V}{h^3}\int_0^{p_0} p^2\,dp = \frac{8\,\pi\,{p_0}^3\,V}{3\,h^3}$$

- Pressure exerted by completely degenerate gas on surrounding environment is the rate of transfer of momentum across unit area of the the interface with that environment:

$$\mathcal{P} = \frac{1}{3}\int_0^{p_0} \frac{N_p}{V}\,p\,v_p\,dp$$

where v_p is the velocity of an electron having momentum p.

Equation of State of a Degenerate Gas – IV

- Special relativistic relations between velocity and momentum are:

$$p = \frac{m_e v_p}{(1 - {v_p}^2/c^2)^{1/2}} \quad \text{or equivalently}$$

$$v_p = \frac{p/m_e}{\left[1 + p^2/(m_e\, c)^2\right]^{1/2}}$$

where m_e is the electron mass.

- Substituting for N_p and v_p in the above expression for the electron degeneracy pressure gives

$$\mathcal{P} = \frac{8\,\pi}{3\,h^3\,m_e} \int_0^{p_0} \frac{p^4}{\left[1 + p^2/(m_e\, c)^2\right]^{1/2}}\, dp.$$

- For a non-relativistic degenerate gas, $p_0 \ll m_e\, c$ and so

$$\mathcal{P} = \frac{8\,\pi}{3\,h^3\,m_e} \int_0^{p_0} p^4\, dp. = \frac{8\,\pi\,{p_0}^5}{15\,h^3\,m_e}$$

Equation of State of a Degenerate Gas – V

- For a relativistic degenerate gas, $p_0 \gg m_e\, c$ and so

$$\mathcal{P} = \frac{8\,\pi\,c}{3\,h^3}\int_0^{p_0} p^3\,dp. = \frac{8\,\pi\,c\,{p_0}^4}{12\,h^3}.$$

- Defining

$$n_e = \mathcal{N}/V = \frac{8\,\pi {p_0}^3}{3\,h^3}$$

the electron degeneracy pressures for non-relativistic and relativistic case are

$$\mathcal{P} = \frac{1}{20}\left(\frac{3}{\pi}\right)^{2/3}\frac{h^2\,{n_e}^{5/3}}{m_e} \quad \text{and} \quad \mathcal{P} = \frac{1}{8}\left(\frac{3}{\pi}\right)^{1/3} h\,c\,{n_e}^{4/3}$$

respectively.

- The aim is to obtain an equation of state for a degenerate gas; this is achieved by converting n_e to the mass-density ρ.

- For each m_H there is one electron and for each He and heavier elements there is approximately one electron for every $2\,m_H$; therefore

$$n_e = \frac{\rho\,X}{m_H} + \frac{\rho\,(1-X)}{2\,m_H} = \frac{\rho\,(1+X)}{2\,m_H}.$$

Equation of State of a Degenerate Gas – VI

On substituting for n_{e} the expressions for degeneracy pressure (equations of state) become

$$\mathcal{P} = K_1\,\rho^{5/3} \qquad \text{and} \qquad \mathcal{P} = K_2\,\rho^{4/3}$$

for the non-relativistic and relativistic case respectively. where

$$K_1 = \frac{h^2}{20\,m_{\mathrm{e}}}\left(\frac{3}{\pi}\right)^{2/3}\left(\frac{1+X}{2\,m_{\mathrm{H}}}\right)^{5/3} \qquad \text{and}$$

$$K_2 = \frac{h\,c}{8}\left(\frac{3}{\pi}\right)^{1/3}\left(\frac{1+X}{2\,m_{\mathrm{H}}}\right)^{4/3}.$$

In an electron degenerate gas the pressure depends only on ρ and chemical composition; it is independent of T.

In stars where electron degeneracy occurs:

- There is no sharp transition between relativistically and non-relativisitically degenerate gas.
- Similarly there is no sharp transition between an ideal gas and one that is electron degenerate.
- Partial degeneracy requires a more complex solution.

Chandrasekhar Mass – I

Recall that the polytropic equation of state

$$P = K\,\rho^{\gamma} = K\,\rho^{(n+1)/n}$$

applies to a relativistic degenerate gas if $K_2 = K$ and $n = 3$. The mass of a polytropic star was found to be

$$M = -4\,\pi\,\alpha^3\,\rho_c\,{\xi_1}^2 \left[\frac{d\theta}{d\xi}\right]_{\xi=\xi_1} \qquad \text{where} \qquad \alpha^2 = \frac{(n+1)\,K}{4\,\pi\,G\,{\rho_c}^{(n-1)/n}}$$

and other symbols have the same meanings as before.

Define

$$M_n = -{\xi_1}^2 \left[\frac{d\theta}{d\xi}\right]_{\xi=\xi_1} \qquad \text{and} \qquad R_n = \xi_1$$

gives

$$\alpha^2 = \left(\frac{R}{R_n}\right)^2 = \frac{(n+1)\,K}{4\,\pi\,G\,{\rho_c}^{(n-1)/n}}$$

$${\rho_c}^{n-1} = \left[\frac{(n+1)\,K}{4\,\pi\,G}\right]^n \left(\frac{R}{R_n}\right)^{-2n}$$

Chandrasekhar Mass – II

and

$$M = 4\,\pi\,\left(\frac{R}{R_n}\right)^3\,\rho_{\rm c}\,M_n$$

$${\rho_{\rm c}}^{n-1} = \left(\frac{1}{4\,\pi}\right)^{n-1}\,\left(\frac{M}{M_n}\right)^{n-1}\,\left(\frac{R}{R_n}\right)^{-3(n-1)}$$

Eliminating ${\rho_{\rm c}}^{n-1}$ between the above equation and the equation at the bottom of the previous slide gives

$$\left(\frac{1}{4\,\pi}\right)^{n-1}\,\left(\frac{M}{M_n}\right)^{n-1}\,\left(\frac{R}{R_n}\right)^{-3(n-1)} = \left[\frac{(n+1)\,K}{4\,\pi\,G}\right]^n\,\left(\frac{R}{R_n}\right)^{-2n}$$

$$\left(\frac{G\,M}{M_n}\right)^{n-1}\,\left(\frac{R}{R_n}\right)^{3-n} = \frac{[(n+1)\,K]^n}{4\,\pi\,G}$$

For non-relativistic degeneracy, $n = 1.5$; substituting this into the above relation shows that $M \propto R^{-3}$. A remarkable property of white dwarfs has therefore been indentified; as the mass increases, the radius decreases.

Chandrasekhar Mass – III

As mass is accreted by a white dwarf, its radius becomes smaller and eventually the point is reached where the degeneracy is relativistic; this is the maximum mass a white dwarf can have and is known as the Chandrasekhar Mass ($M_{\rm ch}$).

An electron-degenerate core which is relativistic in the sense that $v_p \rightarrow c$ is still a polytrope but $n = 3$ and so

$$\left(\frac{G\,M}{M_3}\right)^2 = \frac{[4\,K]^3}{4\,\pi\,G}$$

$$M = 4\,\pi\,M_3\left(\frac{K}{\pi\,G}\right)^{3/2}$$

For a relativistic electron degenerate gas, $K = K_2$ and so on substituting

$$M_{\rm ch} = \frac{M_3\sqrt{1.5}}{4\,\pi}\left(\frac{h\,c}{G\,{m_{\rm H}}^{4/3}}\right)^{3/2}\left(\frac{1+X}{2}\right)^2$$

From the solution of the Lane Emden Equation for $n = 3$, $M_3 = 2.01676$ and substituting for other constants gives

$$M_{\rm ch} = 5.699\left(\frac{1+X}{2}\right)^2\,M_\odot.$$

which for $X \approx 0$ (*i.e.* a He or C/O core) implies $M_{\rm ch} \simeq 1.42\,M_\odot$

White Dwarf Cooling – I

Without nuclear reactions, a white dwarf will slowly cool over time, radiating away its thermal energy.

- In normal stars the mean free path for photons is much greater than that of electrons or heavier particles; consequently energy transport is mainly by radiative diffusion.
- In a white dwarf, degenerate electrons can travel long distances before losing energy in a collision with a nucleus, since the vast majority majority of lower-energy electron states are already occupied.
- Thus, in a white dwarf energy is carried by electron conduction (similar to conduction in metals) rather than by radiation.
- Electron conduction is so efficient that the interior of a white dwarf is nearly isothermal, with the temperature dropping significantly only in the non-degenerate surface layers.
- The thin ($\sim$ 1% of the white dwarf radius) non-degenerate envelope transfers heat less efficiently and acts as an insulating "blanket" allowing energy to leak out slowly.
- A steep temperature gradient near the surface results in the outer non-degenerate envelope being convective.
- The initial temperature of a white dwarf may be estimated by recalling that it forms from the contraction of a thermally unsupported stellar core, a process which is eventually stopped by degeneracy pressure.

White Dwarf Cooling – II

- By the Virial Theorem, just before reaching the point of equilibrium, the thermal energy ($E_{\rm th}$) will equal half of the potential energy:

$$E_{\rm th} \sim \frac{1}{2}\frac{G\,M^2}{R}.$$

- For a pure He composition, the number of nuclei in the core is $M/4m_{\rm H}$ and the number of electrons is is $M/2m_{\rm H}$. The total thermal energy is therefore

$$E_{\rm th} = \frac{3}{2}\mathcal{N}\,k\,T = \frac{3}{2}\frac{M}{m_{\rm H}}\left(\frac{1}{2}+\frac{1}{4}\right)\,k\,T = \frac{9}{8}\frac{M}{m_{\rm H}}\,k\,T,$$

so that

$$k\,T \sim \frac{4}{9}\frac{G\,M\,m_{\rm H}}{R}.$$

- It was previously shown that a degenerate non-relativistic star is well represented by a $n = 1.5$ polytrope for which $R \propto M^{-1/3}$ and therefore $k\,T \propto M^{4/3}$.

White Dwarf Cooling – III

- Specifically, the solution of the Lane Emden Equation for a $n = 1.5$ polytrope yields $\xi_1 = 3.6538$ and $|d\theta/d\xi|_{\xi=\xi_1} = -0.20325$ which with the previously derived result

$$\left(\frac{G\,M}{M_n}\right)^{n-1}\left(\frac{R}{R_n}\right)^{3-n} = \frac{[(n+1)\,K]^n}{4\,\pi\,G}$$

gives

$$R = 2.679 \times 10^{17} M^{-1/3}$$

with $K = K_1$, $R_{1.5} = \xi_1$ and $M_{1.5} = \xi_1{}^2\,|d\theta/d\xi|_{\xi=\xi_1}$.

- Substituting for R in the above expression for kT gives

$$k\,T \sim \frac{4}{9} G\,m_{\mathrm{H}}\,(2.679 \times 10^{17})^{-1}\,M^{4/3} = 1.848 \times 10^{-55}\,M^{4/3}$$

- For a white dwarf having $M = 0.5\,M_{\odot}$

$$k\,T \sim 1.848 \times 10^{-15}\,\text{Joules}$$

$$T \sim 1.34 \times 10^{8}\,\text{K}.$$

- Clearly a just-formed degenerate core is a very hot object with thermal emission that peaks at X-ray wavelengths; this radiation ionises the layers of gas that were blown off during the AGB phase, giving rise to planetary nebulae.

White Dwarf Cooling – IV

Energy radiated away from the surface of a white dwarfs is the thermal energy stored in the still classical gas of nuclei within the star's volume.

- Degeneracy of the electron gas limits almost completely the ability of electrons to lose their kinetic energies.
- An upper limit to the cooling rate is obtained by neglecting the envelope and assuming a uniform temperature throughout.
- The rate at which $E_{\rm th}$ decreases is then determined by L from

$$L = 4\,\pi\,R^2\,\sigma\,T^4 \sim -\frac{dE_{\rm th}}{dt} = -\frac{3}{8}\,\frac{M\,k}{m_{\rm H}}\frac{dT}{dt}$$

  where only the thermal contribution of nuclei has been included.
- On integrating and assuming a fixed radius, the time required for a white dwarf to cool from $T = T_1$ to $T = T_2$ is

$$\tau_{12} = \frac{3}{8}\,\frac{M\,k}{m_{\rm H}}\,\frac{1}{4\,\pi\,R^2\,\sigma}\,\frac{1}{3}\,\left[\frac{1}{{T_2}^3} - \frac{1}{{T_1}^3}\right]$$

- Substituting numerical values gives

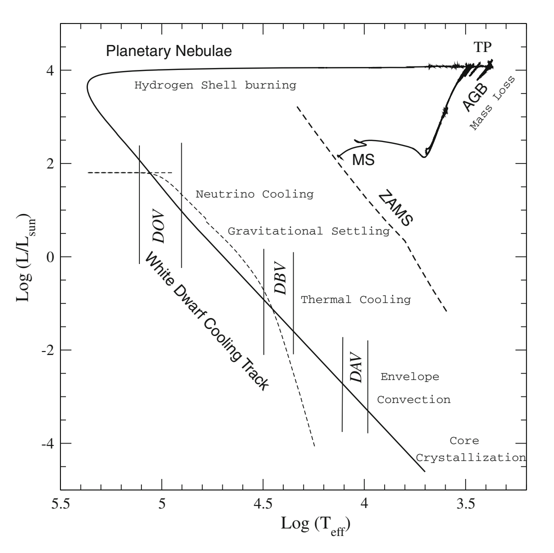
$$\tau_{12} \sim 6.41E + 24\,\left(\frac{M}{M_\odot}\right)^{5/3}\,\left[\frac{1}{{T_2}^3} - \frac{1}{{T_1}^3}\right] \quad \text{seconds.}$$

White Dwarf Cooling – V

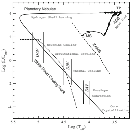
Thus, even with an unrealistically efficient cooling it would take a 1 $M_\odot$ white dwarf $\sim 1Gyr$ to cool to $\sim 10^3\,\mathrm{K}$.

- In reality, the insulation provided by the non-degenerate envelope results in a T_{eff} that is significantly lower than the interior temperature and this lowers the cooing rate.
- Furthermore, as a white dwarf cools, it crystallises in a gradual process that starts at the centre and moves outwards.
- The regular crystal structure is maintained by the mutual electrostatic repulsion of the nuclei; it minimises their energy as they vibrate about their average position in the lattice.
- As the nuclei undergo this phase change, the latent heat that is released is added to the thermal balance, further slowing down the decline in temperature.

White Dwarf Cooling – VI



White Dwarf Cooling – VI



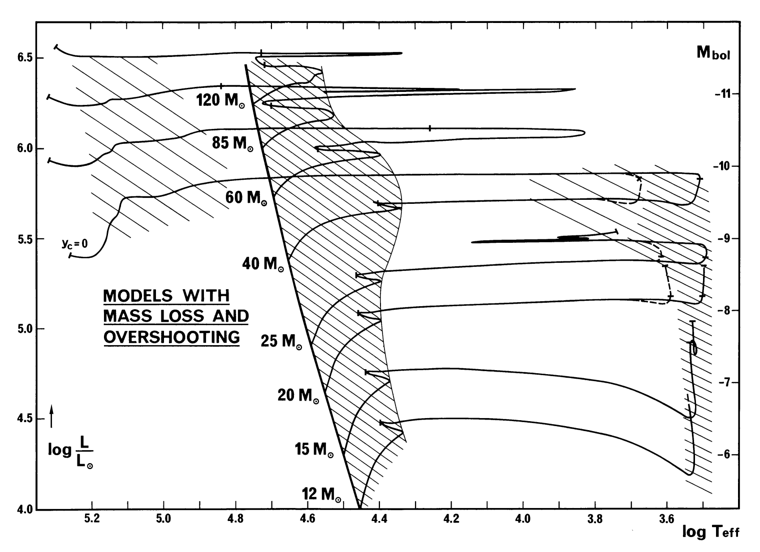
A Hertzsprung-Russell Diagram showing the evolution of a 5 $M_{\odot}$ star from the Zero Age Main Sequence through to core crystalisation at the end of the white dwarf cooling track.

Massive Star Evolution – I

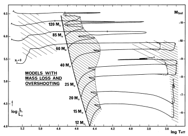
The evolution of massive ($M \gtrsim 8\,M_\odot$) is different from lower mass stars in several important ways:

- As T_c reaches 5×10^8 K, non-degenerate C ignition can occur in the core provided a core mass of $M_{\text{CO-CORE}} > 1.06\,M_\odot$ can develop; this in turn requires and initial stellar mass of $\gtrsim 8\,M_\odot$.
- The fate of stars having $8M_\odot \gtrsim M \gtrsim 11\,M_\odot$ is uncertain; they develop O-Ne cores after central C-burning and their structure is then similar to AGB stars with degenerate CO cores; such stars have been named "Super-AGB Stars" in recent years.
- Stars with higher initial masses ($M \gtrsim 11\,M_\odot$) also ignite and burn fuels heavier than C until a Fe core is formed; this collapses and results in a supernova explosion.
- Mass-loss by stellar winds becomes important during all evolution phases, including the Main Sequence.
- For stars having initial masses ($M \gtrsim 30\,M_\odot$), mass-loss rates $\dot{M}$ are so large that $\tau_{\text{ml}} = M/\dot{M} > \tau_{\text{nuc}}$ and as a consequence mass-loss determines the evolution.

Massive Star Evolution – II



Massive Star Evolution – II



Evolution of massive stars ($12 - 120\,M_\odot$) calculated with mass loss and a moderate amount of convective overshooting ($0.25\,H_\mathrm{p}$). The shaded region corresponds to long-lived evolution phases on the main sequence, and during core He burning as a red supergiant ($\mathrm{at}\ \log T_\mathrm{eff} < 4.0$) or as a Wolf-Rayet star ($\mathrm{at}\ \log T_\mathrm{eff} > 4.8$). Stars with initial mass $M > 40\,M_\odot$ are assumed to lose their entire envelope due to luminous blue variable episodes and never become red supergiants.

Massive Star Evolution – III

OB-Type Main Sequence and Blue Supergiant (BSG) stars have fast radiation driven stellar winds.

- P_{rad} at frequencies corresponding to spectrum absorption lines, where the interaction between photons and matter is strong, causes an outward acceleration.
- If photons transfer their entire momentum to the wind then
$$\dot{M}\, v_\infty < \frac{L}{c}$$
gives an upper limit to $\dot{M}$, where v_∞ is the terminal wind velocity.
- Comparison between $\dot{M}$ from the above formula and more accurate estimates suggests photon momentum is efficiently transferred to the stellar wind.
- The efficiency is attributed to Doppler broadening of spectral lines caused by wind acceleration so that outflowing atoms and ions can absorb photons of a higher frequency.
- Radiation-driven mass-loss is also dependent on metallicity because it is mostly the lines of heavier elements that contribute to the line driving.

Massive Star Evolution – IV

BSG stars evolve into red supergiants (RSG) during H-shell burning unless mass-loss removes such a large fraction of the envelope that this is no longer possible.

- RSG stars have a slow but copious stellar wind.
- Stellar winds in RSG stars are probably driven in the same way as AGB star "superwinds"; that is a combination of stellar pulsations and radiation pressure on dust particles that form in the cool outer atmosphere.
- Theoretical predictions are lacking but observations suggest $\dot{M}$ as high as $10^{-4}\,M_\odot\,\mathrm{yr}^{-1}$.
- Stars with $M \lesssim 40\,M_\odot$ spend a large fraction of their core He-burning period as RSG stars.
- During the RSG phase mass-loss can remove a large part of, or even the entire, envelope of $M \lesssim 40\,M_\odot$ stars exposing the He-core as a Wolf-Rayet (WR) star.

Massive Star Evolution – V

Observations reveal an upper limit to L that depends on $T_{\rm eff}$

- In particular, there are no RSG stars having $\log\,(L/L_\odot) > 5.8$, corresponding the RSG luminosity of a $40\,M_\odot$.
- Apparently stars with $M \gtrsim 40\,M_\odot$ do not become RSG stars; this upper limit in the HRD is known as the **Humphreys-Davidson Limit**.
- At $T_{\rm eff} > 10000\,{\rm K}$, the maximum L increases gradually to $\log\,(L/L_\odot) > 6.8$ at $T_{\rm eff} = 40000\,{\rm K}$ (O stars).
- The Humphreys-Davidson limit is interpreted as a generalised Eddington limit. When L exceeds the classical Eddington limit

$$L_{\rm Edd} = \frac{4\,\pi\,c\,G\,M}{\kappa}$$

where κ is the electron-scattering opacity, the outward force due to $P_{\rm rad}$ on free electrons exceeds the inward gravitational force on nuclei.

Massive Star Evolution – VI

- Electrostatic coupling between electrons and ions means that outer layers are accelerated outwards and the star becomes unstable.
- Actual opacity in the atmosphere is larger than the electron scattering κ and decreases with T.
- Therefore the L at which the $P_{\rm rad}$ limit is reached is lower than $L_{\rm Edd}$ and the decrease of the Humphreys-Davidson limit with $T_{\rm eff}$ is explained qualitatively.
- Luminous stars near the Humphreys-Davidson limit are observed to be very unstable, undergoing large excursions in the HRD with $\dot{M} \gtrsim 10^{-3}\,M_\odot\,{\rm yr}^{-1}$ during outbursts.
- Such objects are known as luminous blue variables (LBVs), examples of which are η Carina and P Cygni.
- Stars losing mass due to LBV outbursts are destined to become Wolf-Rayet Stars.
- Strong LBV mass-loss prevents them from ever becoming RSG stars.

Massive Star Evolution – VII

Wolf-Rayet (WR) stars are hot, very luminous and have strong emission lines in their spectra.

- Emission indicates strong, optically thick stellar winds with $\dot{M} \sim 10^{-5} - 10^{-4}\, M_\odot\,\mathrm{yr}^{-1}$.
- WR stars surrounded by circumstellar nebulae of ejected material.
- Winds are probably driven by radiation pressure as for O stars, but multiple photon scattering in the optically thick outflow can increase $\dot{M}$ to well above the single scattering limit.
- WR spectra reveal increased CNO abundances, indicating that they are exposed H- or He-burning cores of massive stars.

WR stars are classified into several sub-types on the basis of surface abundances:

- **WNL Stars** – have photospheric H (with $X_{\mathrm{H}} < 0.4$) with increased He and N abundances, consistent with CNO-Cycle equilibrium values.
- **WNE Stars** – have He and N abundances similar to those in WNL stars except H is absent ($X_{\mathrm{H}} = 0$).
- **WC Stars** – have no H, little or no N, and increased He, C and O abundances (consistent with partial He-burning).
- **WO Stars** – are similar to WC stars with strongly increased O abundances (as expected for nearly complete He-burning).

WNL $\rightarrow$ WNE $\rightarrow$ WC $\rightarrow$ WO appear to form an evolutionary sequence in which deeper and deeper layers become exposed, as the envelope is "peeled-off" by mass-loss.

Massive Star Evolution – VIII

Massive star evolution may be summarised:

$M \lesssim 15\,M_\odot$	MS(OB) $\rightarrow$ RSG $\rightarrow$ (BSG in blue loop?) $\rightarrow$ RSG $\rightarrow$ SN II Mass-loss is relatively unimportant, only a few $M_\odot$ are lost during the entire evolution.
$15\,M_\odot \lesssim M \lesssim 25\,M_\odot$	MS(O) $\rightarrow$ BSG $\rightarrow$ RSG $\rightarrow$ SN II Mass-loss is strong during the RSG phase, but not strong enough to remove the entire H-rich envelope.
$25\,M_\odot \lesssim M \lesssim 40\,M_\odot$	MS(O) $\rightarrow$ BSG $\rightarrow$ RSG $\rightarrow$ WNL $\rightarrow$ WNE $\rightarrow$ WC $\rightarrow$ SN Ib The H-rich envelope is removed during the RSG stage, resulting in the formation of a WR star.
$M \gtrsim 40\,M_\odot$	MS(O) $\rightarrow$ BSG $\rightarrow$ LBV $\rightarrow$ WNL $\rightarrow$ WNE $\rightarrow$ WC $\rightarrow$ SN Ib/c A LBV phase blows off the envelope before the RSG can be reached.

Mass limits listed above are for $Z = 0.02$ and are metallicity dependent.

Massive Star Evolution – IX

Evolution of photospheric properties already described correspond to H- and He-burning phases of massive stars.

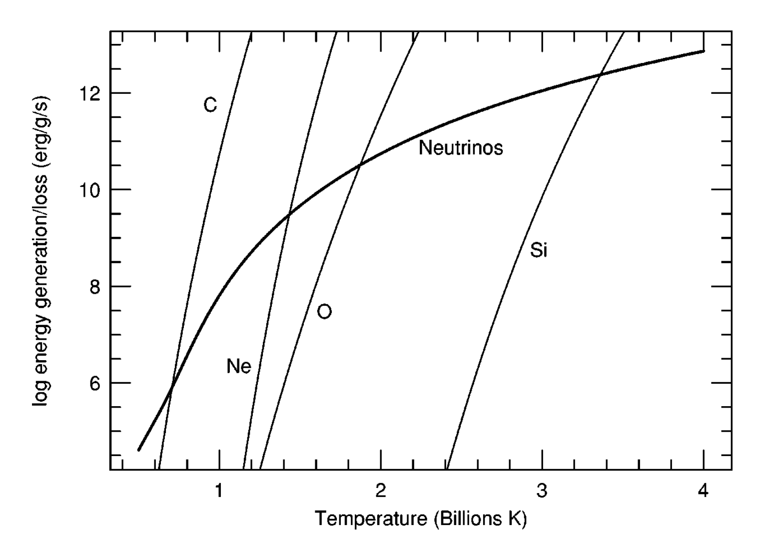
- Once a C/O core has formed which is massive enough ($> 1.06\,M_\odot$) to undergo C-ignition, subsequent evolution of the core is a series of alternating nuclear burning and core contraction cycles in quick succession.
- Due to strong neutrino losses, the core evolution is sped-up enormously. Less than $\sim 10^3$ years pass between the onset of C-burning and the formation of a Fe core.
- The stellar envelope does not have time to respond to changes in the core whose mass is fixed following C-ignition; the evolution of the envelope is now practically disconnected from that of the core.
- As a result, the position of a massive star in the HRD remains almost unchanged during C-burning and beyond.
- Exclusive attention to core evolution is required from this point onwards.

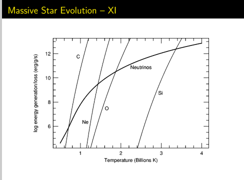
Massive Star Evolution – X

At high T and ρ, several weak interaction processes result in spontaneous neutrino production:

- **Pair annihilation** occurs at $T > 10^9$ K. Energetic photons can undergo pair creation ($\gamma + \gamma \leftrightarrow e^+ + e^-$), but once in every 10^{19} cases the pair annihilates into neutrinos ($e^+ + e^- \rightarrow \nu + \bar{\nu}$).
- There is a small probability of a $\nu\bar{\nu}$ pair production when a γ-ray photon is scattered off an electron ($\gamma + e^- \rightarrow e^- + \nu + \bar{\nu}$), known as the **Photo-Neutrino Process**.
- A plasma oscillation quantum (or plasmon) may decay in electron-degenerate gas at high densities ($\rho \gtrsim 10^6\,\mathrm{g/cm^3}$) into a $\nu\bar{\nu}$ pair, known as the **Plasma-Neutrino Process**.
- Bremsstrahlung neutrinos (at low T and very high ρ) are inelastic ("free-free") scattering of an electron in the Coulomb field of a nucleus, producing a $\nu\bar{\nu}$ pair instead of the usual γ photon.

Massive Star Evolution – XI





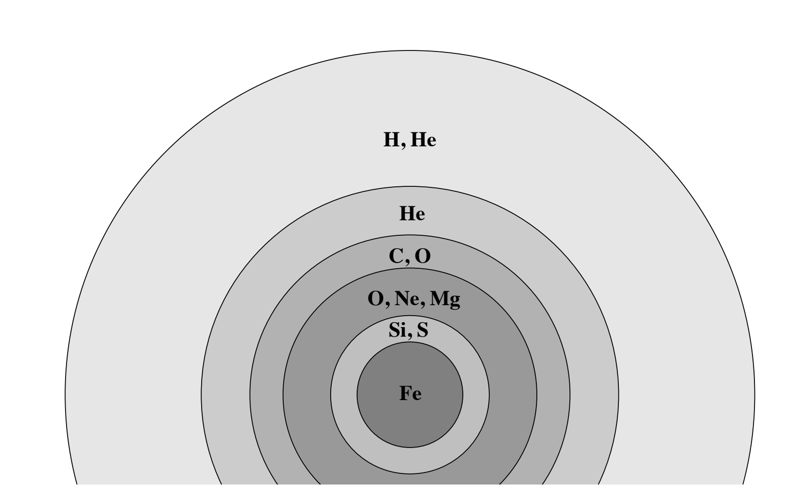
Energy generation rate and neutrino loss rate during the advanced evolution of a massive star. The intersection of the nuclear burning lines with the neutrino loss line defines the burning temperature of the corresponding fuel. The lifetime of each burning stage is approximately equal to the energy generated by nuclear burning ($\sim 4.0, 1.1, 5.0 \text{and } 1.9 \times 10^{17}\,\mathrm{erg/gram}$ for C-, Ne-, O- and Si- burning respectively) divided by the energy generated per gram per second at balanced power as defined by the intersecions of ϵ_{nuc} and ϵ_{ν}. Thus the lifetime ranges from several 10^3 years for C-burning to about a day for Si-burning!

Massive Star Evolution – XII

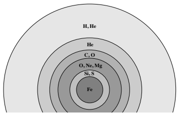
Once $T_c \gtrsim 5 \times 10^8$ K, neutrino losses are the most important energy leak from the stellar centre, removing it much more rapidly than photon diffusion or convection can transport it to the surface.

- Therefore $L_\nu \gg L$ so that during nuclear burning $L_{\rm nuc} = \dot{E}_{\rm nuc} \approx L_\nu$, leading to a much shorter timescale $\tau_{\rm nuc} = E_{\rm nuc}/L_\nu \ll E_{nuc}/L$.
- Similarly, the rate of core contraction (on the thermal timescale) between burning cycles also speeds up; $\dot{E}_{\rm gr} \approx L_\nu$ so that $\tau_{\rm th} = E_{\rm gr}/L_\nu \ll E_{nuc}/L$.
- Therefore the rate of evolution speeds up and accelerates as the core contracts and heats up.
- Note that nuclear burning in the presence of neutrino losses is stable. A small perturbation ($\delta T > 0$) would increase the local heat content ($\epsilon_{\rm nuc} > \epsilon_\nu$), leading to expansion and cooling of the core until thermal equilibrium is re-established.
- Advanced burning stages leading to the Fe core have already been discussed.

Pre-Supernova Structure – I



Pre-Supernova Structure – I



Schematic overview of the "onion-skin" structure of a massive star at the end of its evolution.

Pre-Supernova Structure – II

The following general picture of a massive star, immediately before a supernova explosion, therefore emerges.

- After exhaustion of a fuel (*e.g.* carbon) in the centre, the core contracts and burning continues in a shell around it.
- Neutrino losses speed-up the contraction and heating of the core, until the next fuel (*e.g.* neon) is ignited in the centre.
- At each subsequent burning stage, the outer burning shells have advanced outward, while neutrino cooling is more efficient, resulting in a smaller burning core (in mass) than the previous stage.
- Eventually an "onion-skin" structure of type shown in the previous slide remains, layers of heavier nuclei at increasing depth, separated by burning shells.

Core Collapse in Massive Stars – I

At the end of Si-burning, when $T_c \gtrsim 4 \times 10^9\,\mathrm{K}$, the composition of the central core is determined by nuclear statistical equilibrium; the mixture of nuclei has reached the minimum possible binding energy at that temperature.

- Minimum binding energy is achieved by having a core of ^{56}Fe nuclei.
- No further energy can be extracted by nuclear fusion; the core has become inert.
- Because of neutrino cooling during the late evolution stages, the core has a considerable degree of electron degeneracy.
- However, the high temperature and density ($\gtrsim 10^9\,\mathrm{g/cm^3}$) mean that electrons are always relativistic.
- As a consequence, core contraction cannot be stopped and must continue on the very rapid neutrino-mediated thermal timescale.
- Furthermore, since relativistic electron gas dominates the pressure, the adiabatic exponent $\gamma_{\mathrm{ad}} \approx 4/3$; the Fe core is therefore very close to a state of dynamical instability.

Core Collapse in Massive Stars – II

Two processes occur at high ρ and T that contribute to accelerating the already rapid contraction into a dynamical collapse of the core:

- Electron Captures:
  - At very high ρ, free electrons can be captured and bound into otherwise β-unstable heavy nuclei; this process (known as inverse β-decay) occurs when the most energetic electrons have energies high enough to overcome the difference in nuclear binding energy.
  - As a result, the composition becomes increasingly neutron-rich, a process known as **neutronisation**.
  - Furthermore, the electron pressure decreases which can destroy the precarious state of hydrostatic equilibrium and trigger the collapse of the core.
  - For a composition of predominantly ^{56}Fe one would expect $M_{\rm Ch} = 5.7\,/{\mu_e}^2\,M_\odot \approx 1.2\,M_\odot$; electron capture increases μ_e and thus decreases $M_{\rm Ch}$, facilitating collapse by bringing $M_{\rm Ch}$ below the current core mass.
  - Stars with initial masses $M \lesssim 11\,M_\odot$ develop O-Ne-Mg cores; if these can grow through shell burning to $1.37\,M_\odot$, electrons are captured by ^{24}Mg and ^{20}Ne which then brings about core collapse as above.

Core Collapse in Massive Stars – III

- Photodisintegration
  - If $T_{\mathrm{c}} \rightarrow 10^{10}\,\mathrm{K}$, photon energy is high enough to break up heavy nuclei; in particular, $^{56}\mathrm{Fe}$ is disintegrated as

$$^{56}\mathrm{Fe} + \gamma \quad \leftrightarrow \quad 13\,^{4}\mathrm{He} + 4\,n.$$

  - The above reaction is in statistical equilibrium and abundances of nuclei involved are determined by a Saha-type equation, the balance shifting towards the right-hand side as T increases.
  - The process is thus similar to the ionisation of hydrogen, and results in a lowering of γ_{ad} to below the critical value of $4/3$ leading to a dynamically unstable core; this process dominates in relatively massive iron cores.
  - $^{56}\mathrm{Fe}$ photodisintegration requires about $2\,\mathrm{MeV}$ per nucleon; this is absorbed from the radiation field and thus ultimately from the internal energy of the gas.
  - As a result, the pressure decreases quite drastically, triggering an almost free-fall collapse of the core.

Core Collapse in Massive Stars – IV

Collapse is extremely rapid, taking $\sim 10\,\mathrm{m\,sec}$, because of the short dynamical timescale at the prevailing $\rho \sim 10^{10}\,\mathrm{g/cm^3}$ at which collapse is initiated.

- During collapse, T and P keep rising, but never enough to reverse the collapse until nuclear densities are reached.
- Further photodisintegrations can occur due to increasing photon energies.
- Electron captures on to protons

  $$^{1}\mathrm{H} + \mathrm{e}^{-} \quad \rightarrow \quad \mathrm{n} + \nu$$

  inside heavy nuclei continues the process of neutronisation, creating more and more neutron-rich nuclei.
- Neutron-rich nuclei gradually merge, creating what is essentially a gigantic stellar-mass nucleus, as $\rho \rightarrow \sim 10^{14}\,\mathrm{g/cm^3}$.
- Composition inside the core becomes predominantly neutrons, which become degenerate and thereby modify the equation of state to suddenly become "stiff" (*i.e.* the neutron gas becomes almost incompressible).
- Core collapse thereby terminates at $R_{\mathrm{c}} \approx 20\,\mathrm{km}$.

Core Collapse and Supernova Explosion Energetics – I

Gravitational energy released during core collapse is

$$E_{\rm grav} \approx -\frac{G\,M_{\rm c}^2}{R_{\rm c,i}} + \frac{G\,M_{\rm c}^2}{R_{\rm c,f}} \approx \frac{G\,M_{\rm c}^2}{R_{\rm c,f}} \approx 3 \times 10^{53}\,{\rm erg}$$

assuming homologous collapse of a core of $M_{\rm c} \approx 1.4\,M_\odot$ from initial radius $R_{\rm c,i} \approx R_{\rm WD}$ to final radius $R_{\rm c,f} \approx 20\,{\rm km} \ll R_{\rm c,i}$. By comparison, energy required to expel the envelope which has no time to respond to the core collapse, is

$$E_{\rm env} = \int_{M_{\rm c}}^{M} \frac{G\,m}{r}\,dm \ll \frac{G\,M^2}{R_{\rm c,i}} \approx 3 \times 10^{52}\,{\rm erg}$$

for $M = 10\,M_\odot$. $E_{\rm env}$ comes down to $\sim 10^{50}$ ergs when a realistic mass distribution in the envelope is taken into account.

Core Collapse and Supernova Explosion Energetics – II

The question is therefore how such a small fraction of the collapse energy can be transformed into kinetic energy of the envelope.

- When the inner part of the core is compressed to ~ 1.5 times the nuclear density, it bounces back (**core bounce**).
- As the velocity of the inner core material is reversed, it encounters matter from the still free-falling outer part of the core.
- If the collision were perfectly elastic, the outer core would bounce back to its initial radius even if the inner core were stationary.
- The outward motion of the inner core during core bounce gives the possibility of a "super-elastic" outer core bounce which might conceivably explode the star.
- Infalling outer core material is supersonic and its encounter with the inner core bounce creates a shock wave that steepens as it travels outward into regions of lower density.
- Kinetic energy stored in the shock wave was once thought to give rise to a so-called **prompt explosion** which blows off the envelope.

Core Collapse and Supernova Explosion Energetics – III

It is currently thought that such a prompt explosion does not occur for two reasons

1. As the shock wave travels through the infalling matter which mostly consists of iron-group nuclei, it heats them up and effectively disintegrates them into protons and neutrons.

    - The binding energy of a ^{56}Fe nucleus is about 9 MeV/nucleon.
    - Disintegration of a $1.4\,M_\odot$ core (1.7×10^{57} nucleons) requires $\sim 2 \times 10^{52}$ ergs.
    - All shock energy can then be absorbed in the core before it reaches the envelope.

2. Electron captures by free protons created behind the shock produce energetic neutrinos.

    - Neutrinos carry away the larger fraction of the energy released in the collapse, especially as the shock moves into relatively low density ($< 10^{12}$ g/cm^3) regions from where they can readily escape.
    - As a result, the shock wave "fizzles out" before it reaches the stellar envelope and no prompt explosion occurs.

Core Collapse and Supernova Explosion Energetics – IV

The role played by neutrinos during core collapse requires closer examination:

- Neutrinos produced before core collapse had energies of the order of the thermal energy of the electrons.
- During core collapse, neutrino production by neutronization dominates and so these neutrinos have energies of the typical order of the relativistic electron Fermi energy.
- A previous result gave the electron degeneracy pressure ($\mathcal{P}$), for the relativistic case, in terms of the Fermi momentum (p_0) as

$$\mathcal{P} = \frac{8\,\pi\,c\,{p_0}^4}{12\,h^3} = \frac{1}{8}\left(\frac{3}{\pi}\right)^{1/3} h\,c\,{n_\mathrm{e}}^{4/3} \quad \text{where} \quad n_\mathrm{e} = \frac{8\,\pi\,{p_0}^3}{3\,h^3}.$$

- The Fermi momentum is then

$$p_0 = \left(\frac{3}{8\,\pi}\right)^{1/3} h\,{n_\mathrm{e}}^{1/3}$$

which is related to the Fermi Energy of degenerate relativistic electrons through $E_\mathrm{F} = c\,p_0$

Core Collapse and Supernova Explosion Energetics – V

The energy of neutrinos produced during core collapse is therefore approximately:

$$\frac{E_\nu}{m_e\,c^2} \approx \frac{E_F}{m_e\,c^2} = \frac{p_0}{m_e\,c} = \frac{h}{m_e\,c}\left(\frac{3}{8\,\pi}\,\frac{\rho}{\mu_e\,m_H}\right)^{1/3} \approx 10^{-2}\left(\frac{\rho}{\mu_e}\right)^{1/3}$$

where μ_e is the mean molecular weight per free electron and $\rho = \mu_e\,m_H\,n_e$, with ρ expressed in g/cm^3.

In the presence of heavy nuclei, neutrinos interact mainly through so-called coherent scattering with these nuclei, with a typical cross-section of the order

$$\sigma_\nu \approx 10^{-45}\,A^2\left(\frac{E_\nu}{m_e\,c^2}\right)^2\ \mathrm{cm}^2,$$

where A is the atomic mass number of the scattering nucleus. Substituting for the Fermi energy so as to express σ_ν in terms of ρ gives

$$\sigma_\nu \approx 10^{-49}\,A^2\left(\frac{\rho}{\mu_e}\right)^{2/3}\ \mathrm{cm}^2.$$

If $n = \rho/(A\,m_H)$ is the number density of nuclei, the mean free path of the neutrinos in the collapsing core can then be estimated as

$$\ell_\nu \approx \frac{1}{n\,\sigma_\nu} \approx 2\times 10^{25}\,\frac{1}{\mu_e\,A}\left(\frac{\rho}{\mu_e}\right)^{-5/3}\ \mathrm{cm}.$$

Core Collapse and Supernova Explosion Energetics – VI

Taking $\mu_e \approx 2$ and $A \approx 100$, $\ell_\nu \approx 10^7$ cm (the typical dimension of the collapsing core) when $\rho/\mu_e \approx 4 \times 10^9$ g/cm^3.

- Neutrinos can no longer escape freely at the high densities prevailing in the collapsing core.
- The core becomes opaque for neutrinos, which can only diffuse out of the core via many scattering events.
- Towards the end of the collapse phase, when $\rho > 3 \times 10^{11}$ g/cm^3, the diffusion velocity becomes smaller than the infall velocity of the gas, so that neutrinos are trapped in the core.
- Analogus to the photosphere of a star, one can define a "neutrinosphere" in the outer layers of the core where the density is low enough for the neutrinos to escape.
- Below the "neutrinoshpere", there is a neutrino trapping surface under which neutrinos are trapped.

Core Collapse and Supernova Explosion Energetics – VII

The real situation is more complicated because σ_ν depends on the neutrino energy, so that the neutrino transport problem needs to be solved in an energy-dependent way.

- The congestion of neutrinos in the core causes them to become degenerate (since neutrinos are fermions) with a high Fermi energy.
- Electron capture becomes less probable, because the new neutrinos have to occupy higher energy states.
- Therefore neutronisation effectively stops when $\rho \approx 3 \times 10^{12}\,\mathrm{g/cm^3}$.
- Only after some neutrinos have diffused out of the core can further neutronisation take place.
- The process of neutronisation therefore takes several seconds, while the collapse takes only a few milliseconds.

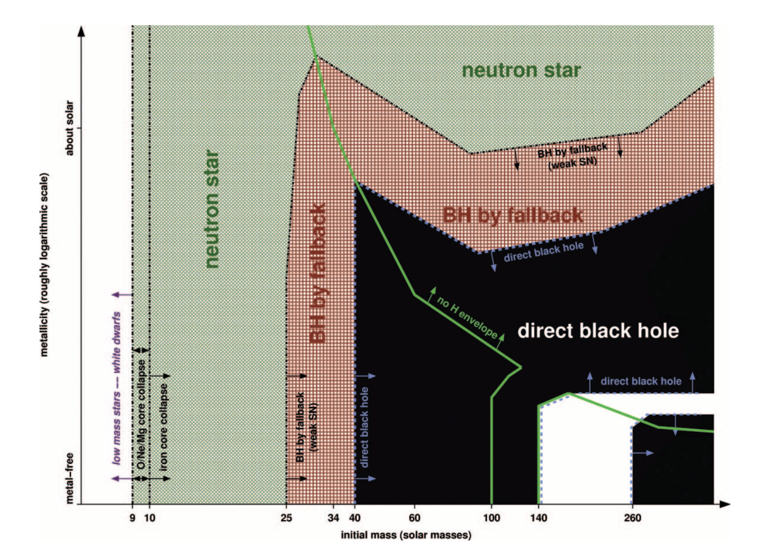
Core Collapse and Supernova Explosion Energetics – VIII

The deposition of neutrino energy in the core provides an energy source that may revive the shock wave previously discussed and cause an explosion.

- Neutrinos diffusing out of the dense core heat the region through which the former shock wave has passed and cause it to become convectively unstable.
- Convection thus provides a way to convert some of the thermal energy from neutrino deposition into kinetic energy.
- Multi-dimensional hydrodynamical calculations show that the outward force thus created can overcome the ram pressure of the outer layers that are still falling onto the core and launch a succesful explosion, but only for $M \lesssim 11\, M_\odot$.
- Alternative ways of reviving the shock and driving a successful supernova explosion are still being explored.

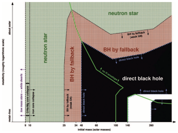
Core Collapse and Supernova Explosion Energetics – IX

Heger A *et al.* , 2003 Astrophys. J. 591, 288



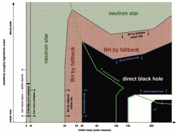
Core Collapse and Supernova Explosion Energetics – IX

Heger A *et al.*, 2003 Astrophys. J. 591, 288



Remnants of massive single stars as a function of initial metallicity (y-axis; qualitatively) and initial mass (x-axis). The thick green line separates the regimes where the stars keep their hydrogen envelope (left and lower right) from those where the hydrogen envelope is lost (upper right and small strip at the bottom between 100 and 140 $M_\odot$). The dashed blue line indicates the border of the regime of direct black hole (BH) formation (black). This domain is interrupted by a strip of pair-instability supernovae that leave no remnant (white). Outside the direct BH regime, at lower mass and higher metallicity, follows the regime of BH formation by fallback (red cross-hatching and bordered by a black dot-dashed line).

Core Collapse and Supernova Explosion Energetics – IX
Heger A *et al.* , 2003 Astrophys. J. 591, 288



Outside of this, green cross-hatching indicates the formation of neutron stars. The lowest mass neutron stars may be made by O/Ne/Mg core collapse instead of iron core collapse (vertical dot-dashed lines at the left). At even lower mass, the cores do not collapse and only white dwarfs are formed (white strip at the extreme left.

Summary

Subjects discussed in the Tenth Lecture include:

- White dwarfs are composed of electron degenerate gas; consequences of this are that their masses must be below the Chandrasekhar Mass of $\sim 1.4\,M_\odot$ and that their radii decrease with increasing mass.
- A simple method is developed for estimating white dwarf cooling although in reality structural changes within the white dwarfs are important and these have been neglected.
- Massive star $M \gtrsim 8\,M_\odot$ evolution is discussed as these are the progenitors of neutron stars and black holes.
- Core collapse supernovae are considered as the intermediate step by which massive stars end their lives as neutron stars or black holes.

Acknowledgement

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Homologous Stellar Models and Polytropes

Main Sequence Stars

Post-Main Sequence Hydrogen-Shell Burning

Post-Main Sequence Helium-Core Burning

White Dwarfs, Massive and Neutron Stars