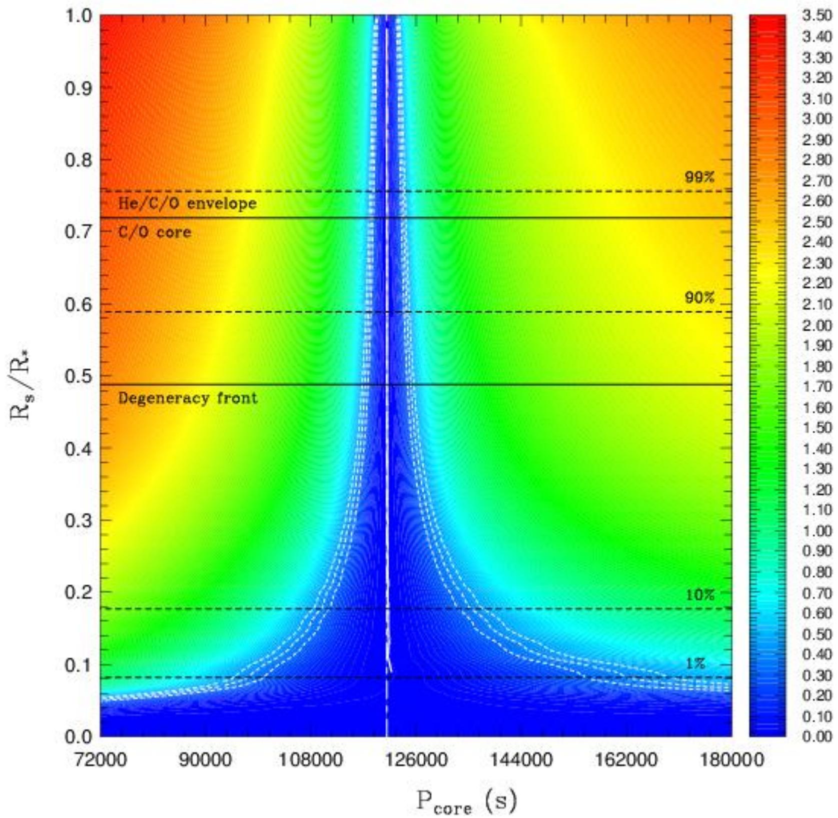
Asteroseismology as Probe of Internal Rotation in Pulsating White Dwarf and Hot Subdwarf Stars

G. Fontaine[1], S. Charpinet[2], P. Brassard[1], V. Van Grootel[3], and N. Giammichele[1]

[1]Département de Physique, Université de Montréal; [2]IRAP/OMP, Université de Toulouse
[3]Institut d'Astrophysique et de Géophysique, Université de Liège



Internal rotation profile of the very hot (T_{eff} ~ 128,000 K) GW Vir white dwarf PG 1159-035 inferred from seismic analysis. This is depicted by the very nearly vertical solid white curve. The region where the profile is reliably measured covers some 90% of the radius, corresponding to about 99% of the mass of the star. This is unique in all of asteroseismology. PG 1159-035 is found to *globally* rotate rigidly and extremely slowly (by white dwarf standards), with a period of 33.67 ± 0.24 h. Given the nearly complete coverage in mass, this allows for a realistic estimate of the total angular momentum of 1.25×10^{39} kg m^2 s^{-1}, which is more than 150 times smaller than the angular momentum of the Sun. <u>This is an extremely small value, which implies that, for all practical purposes, the star has lost essentially all of its initial angular momentum in previous (RGB, AGB, post-AGB) evolutionary phases</u>.

Pulsations and rotation in stars

1) Within the framework of the linear theory of stellar pulsations, a **mode** is defined in terms of 3 "quantum numbers", k, l, and m, the first one giving the number of nodes in the radial direction of the eigenfunction, and the others (l and m) being the indices of the spherical harmonic function which specifies the angular geometry of the mode.

2) Non-rotating (spherical) stars have eigenfrequencies that are $(2l+1)$-fold **degenerate** in m.

3) A **slowly rotating** star has en eigenmode spectrum that is NOT degenerate in m as a result of the destruction of the spherical symmetry. To first order, with slow rotation considered as a perturbation, one can show that

$$\sigma_{klm} \simeq \sigma_{kl} - m\Delta\sigma_{kl} \quad , \qquad (1)$$

where,

σ_{klm}: angular frequency of mode (k, l, m)
σ_{kl}: angular frequency of the degenerate mode (k, l) in the absence of rotation,

and where the frequency spacing is given by,

$$\Delta\sigma_{k\ell} = \int_0^R \Omega_{\rm rot}(r) K_{k\ell}(r) dr \quad . \qquad (2)$$

This leads to a set of **equally-spaced frequencies** with a splitting between adjacent frequency components given by $\Delta\sigma_{kl}$.

In the above expression, $\Omega_{\rm rot}(r)$ is the (assumed) spherically symmetric rotation law, and $K_{k\ell}$ is the so-called **first-order rotation kernel** which plays the role of a weight function. It is given by,

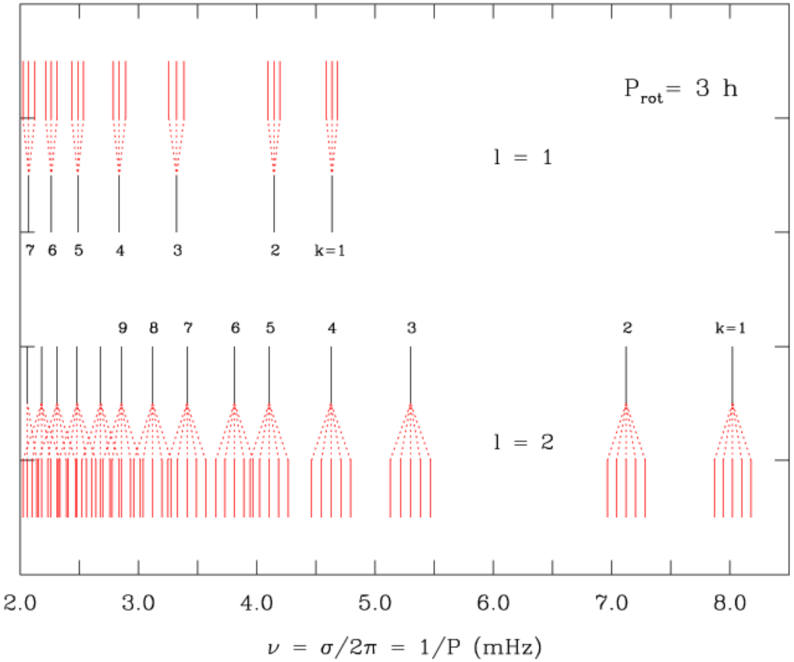
$$K_{k\ell}(r) = \frac{\xi_r^2 + [\ell(\ell+1) - 1]\xi_h^2 - 2\xi_r\xi_h}{\int_0^R \{\xi_r^2 + \ell(\ell+1)\xi_h^2\}\rho r^2 dr} \rho r^2 \quad , \quad (3)$$

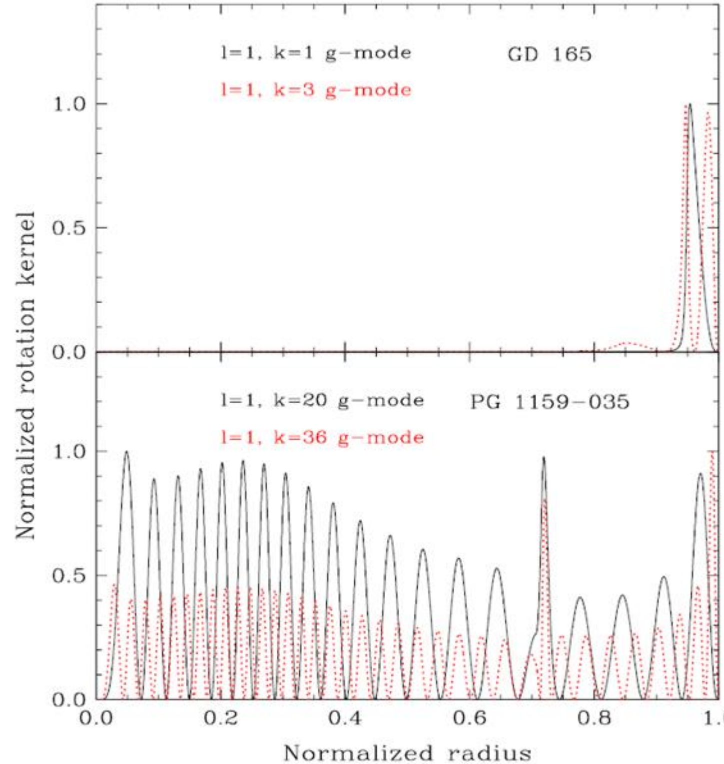
where ξ_r and ξ_h are, respectively, the real parts of the radial and horizontal components of the displacement vector,

$$\vec{\xi} = \left[\xi_r(r), \xi_h(r)\frac{\partial}{\partial\theta}, \xi_h(r)\frac{1}{\sin\theta}\frac{\partial}{\partial\phi}\right] Y_m^l(\theta,\phi)e^{i\sigma t} \quad . \qquad (4)$$

These eigenfunctions for mode (k,ℓ) refer to the unperturbed configuration, i.e., to a purely spherical (non-rotating) stellar model.

Hence, to first order, slow spherically symmetric ($\Omega_{\rm rot}(r)$) rotation produces a set of $2\ell+1$ equally-spaced frequency components out of a degenerate mode with indices (k,ℓ). The spacing between two adjacent components of this multiplet (modes that differ by $|\Delta m| = 1$) can be computed from the unperturbed eigenfunctions as can be appreciated from equations (2) and (3). This fine structure within a multiplet is usually referred to as **rotational splitting**.





Example of rotational splitting in a representative ZZ Ceti star model. The low-order *g*-mode frequency spectrum for both dipole and quadrupole modes, with and without rotation turned on, are illustrated. It is assumed that the star rotates as a solid body and with a (relatively) short period of 3 h. Degenerate dipole modes split into triplets, while quadrupole modes split into quintuplets. The spacings between adjacent components within a multiplet are the same in frequency space, as shown here.

Normalized rotation kernel (weight function) as a function of depth for representative gravity modes in our seismic model of the ZZ Cet pulsator GD 165 (upper panel) and in our seismic model of the GW Vir pulsator PG 1159-035 (lower panel). In GD 165, the weight function has appreciable amplitude in the outer 20% of the radius of the star, meaning that gravity modes are useful probes of the rotation profile only in these outer layers. In contrast, the weight function samples the full stellar model for PG 1159-035, meaning that gravity modes are sensitive to the entire rotation profile, from the center to the surface in that case.

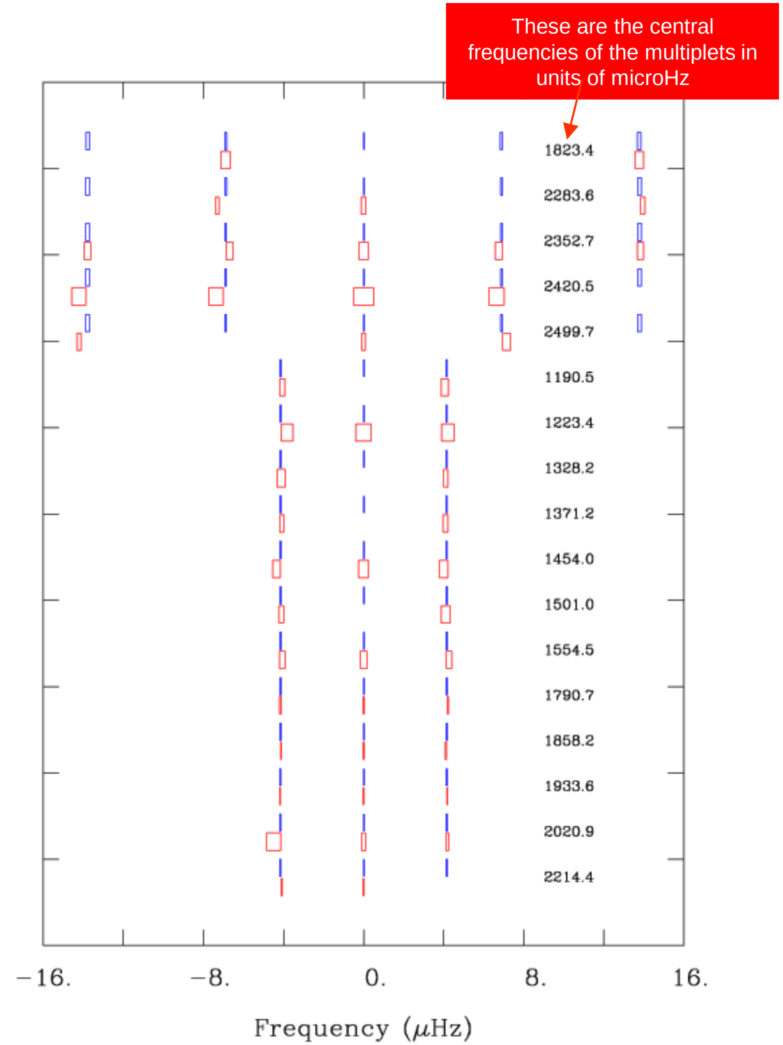
Pulsations and rotation in stars (cont'd)

4) Given a credible seismic model for a slowly rotating pulsating star (this implies that each frequency multiplet has been properly identified in terms of its basic indices k and ℓ, and that the corresponding unperturbed eigenfunctions ξ_r and ξ_h are available), and given a rotation law $\Omega_{\rm rot}(r)$, it is possible to define a goodness-of-fit merit function inspired from a χ^2 approach such that,

$$S^2 = \sum_{i=1}^{N_{\rm obs}} \left(\frac{\Delta\sigma_{\rm obs}^{(i)} - \Delta\sigma_{\rm th}^{(i)}}{\delta_{\rm obs}^{(i)}} \right)^2 , \qquad (5)$$

where $\Delta\sigma_{\rm obs}^{(i)}$ is the i^{th} observed frequency spacing between two rotationally-split frequency components within a multiplet (modes with the same values of k and ℓ, but different values of m), $\delta_{\rm obs}^{(i)}$ is its associated uncertainty, and $\Delta\sigma_{\rm th}^{(i)}$ is the computed spacing for the same two modes according to the previous recipe.

The merit function S^2 is minimized in parameter space to obtain the best possible fit. In the particular case of solid body rotation, parameter space reduces to a single dimension, and S^2 is optimized as a function of the uniform rotation period $P_{\rm rot}$ $(=2\pi/\Omega_{\rm rot})$.



PG 1159-035 is the best observed of all of the pulsating white dwarfs, having been the target of several multisite campaigns from 1989 through 2002. The Fourier transform of its light curve exhibits at least 17 distinct frequency groups showing fine structure caused by rotational splitting. Twelve of those structures correspond to dipole (*l*=1) modes, and five others correspond to quadrupole (*l*=2) modes. The observed rotational multiplets are shown in red in the RHS figure, while our solution, assuming slow uniform rotation, is given by the blue boxes.

-- Our method consists in analyzing the available rotational data under the assumptions that 1) the star rotates slowly, and 2) it rotates as a solid body.

-- The inferred uniform rotation period, *P*(solid), can be used after the fact to verify if the star indeed rotates slowly, in which case *P*(solid) should be much larger than the periods of the pulsation modes used in the experiment.

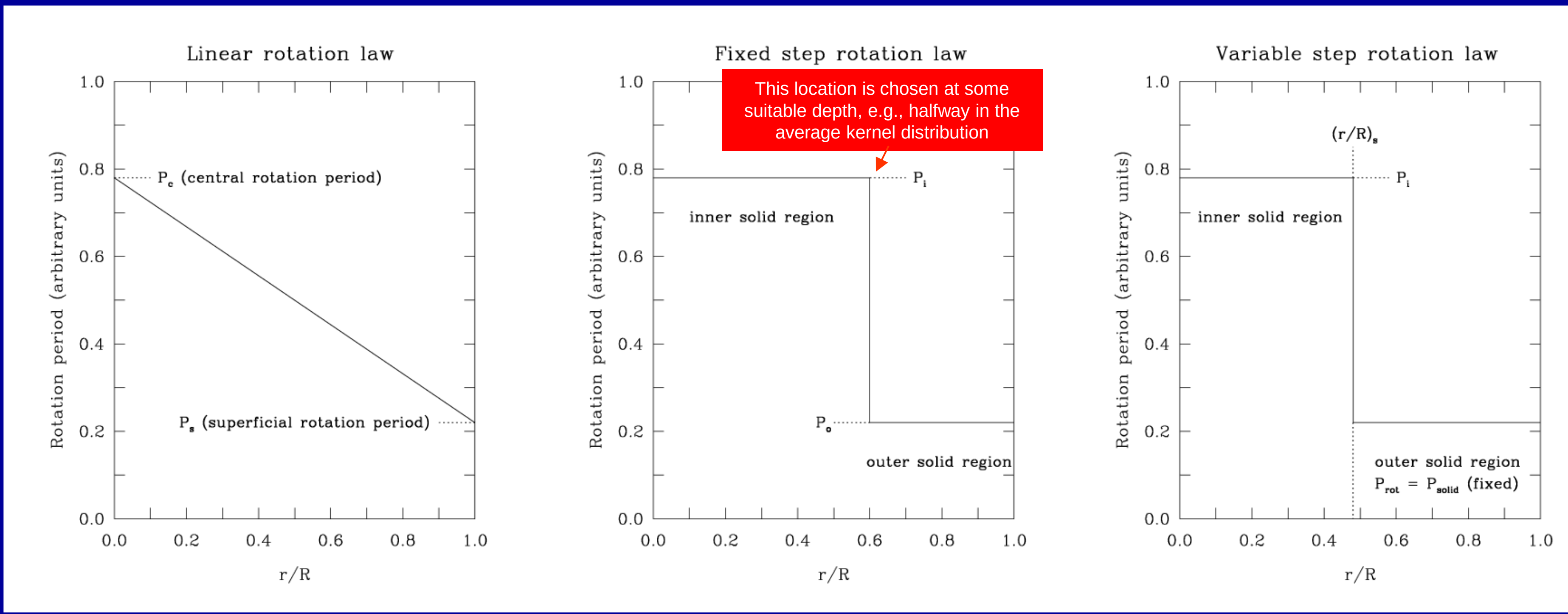
-- But how to test for rigid rotation?

Three possible ways to test for solid body rotation

$$S^2 = \sum_{i=1}^{N_{\rm obs}} \left(\frac{\Delta\sigma_{\rm obs}^{(i)} - \Delta\sigma_{\rm th}^{(i)}}{\delta_{\rm obs}^{(i)}} \right)^2$$

$$\Delta\sigma_{k\ell} = \int_0^R \Omega_{\rm rot}(r) K_{k\ell}(r) dr$$

$$\Omega_{\rm rot} \; (= 2\pi / P_{\rm rot})$$

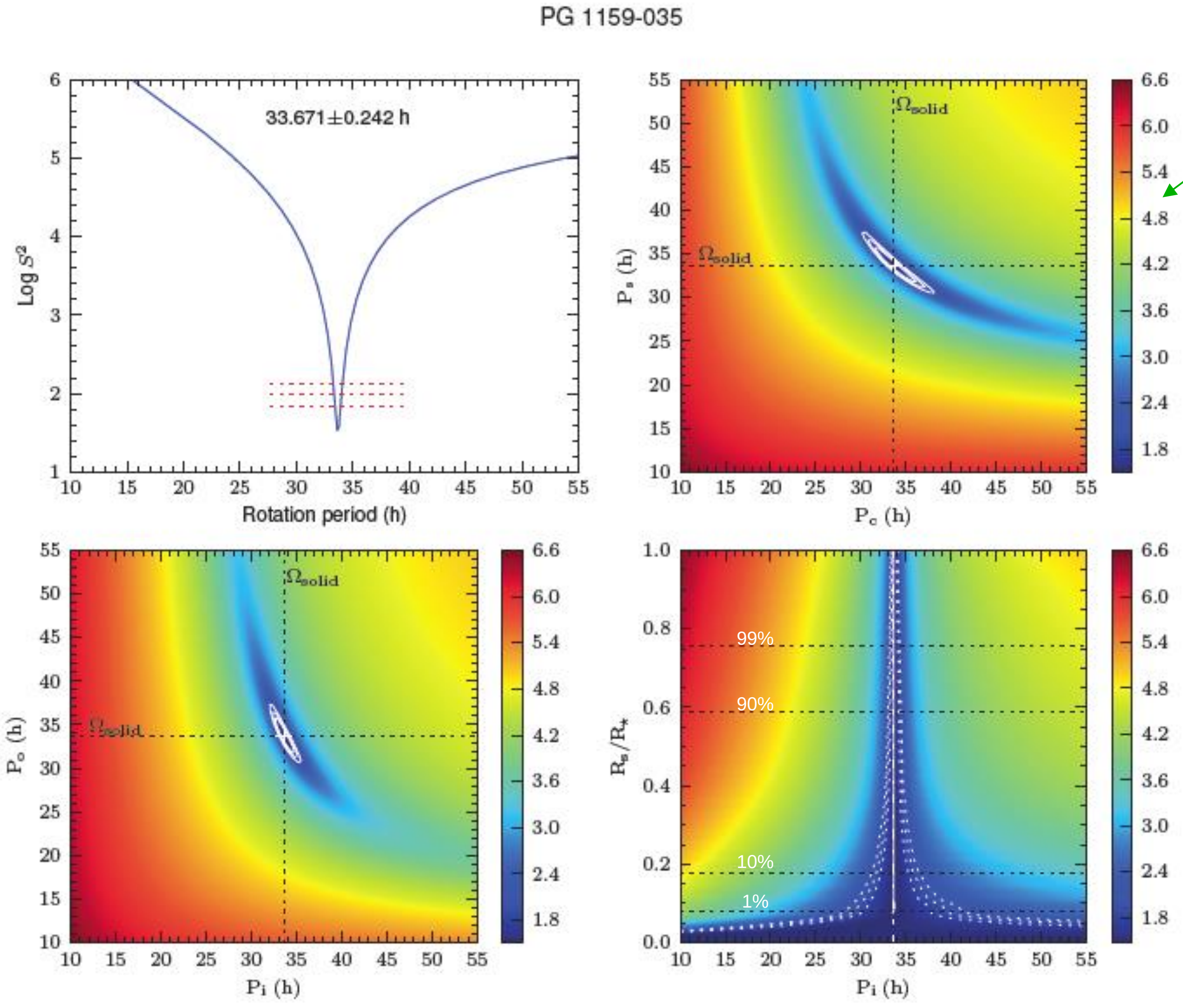


$S^2(P_c,P_s)$ should show a minimum when $P_c = P_s = P(\text{solid})$

$S^2(P_i,P_o)$ should show a minimum when $P_i = P_o = P(\text{solid})$

$S^2(P_i,r/R_s)$ should show a minimum when $P_i = P(\text{solid})$ irrespective of depth r/R_s

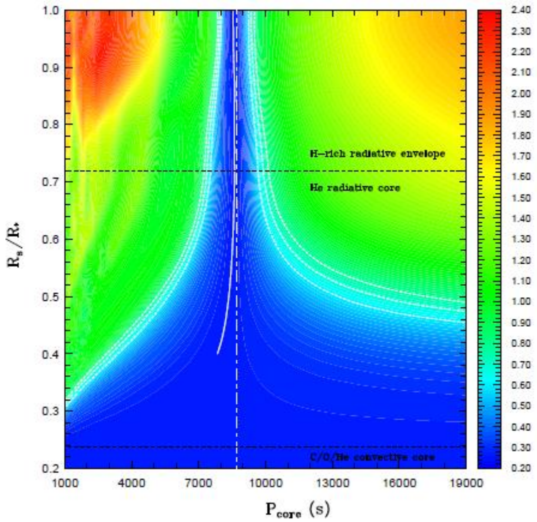
Under the assumption that PG 1159-035 spins slowly **and** rigidly, the rotational data is best explained with a rotation period of 33.671±0.242 h. The supposition of **slow** rotation is easily verified a posteriori, since that period is much larger than the periods of the modes involved (400-840 s). The hypothesis that the star rotates as a **rigid body** is also nicely verified through the results of the three tests shown here. In particular, the lower figure on the right indicates that PG 1159-035 rotates rigidly over more than 90% of its radius, which corresponds to some 99% of its mass.



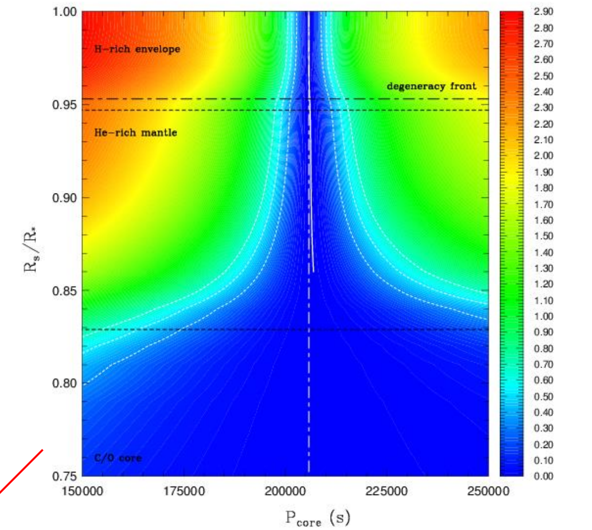
This color bar is a code for log S^2

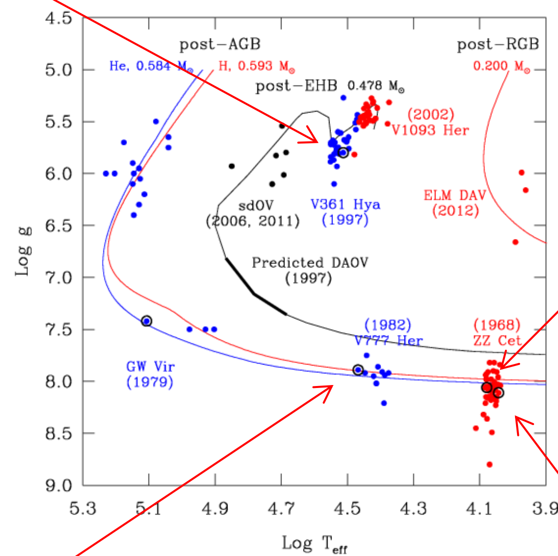
Internal Rotation Profiles: More Recent Seismic Inferences for Other Pulsating Compact Stars

The pulsating sdB component of the synchronized eclipsing close binary system PG 1336-018

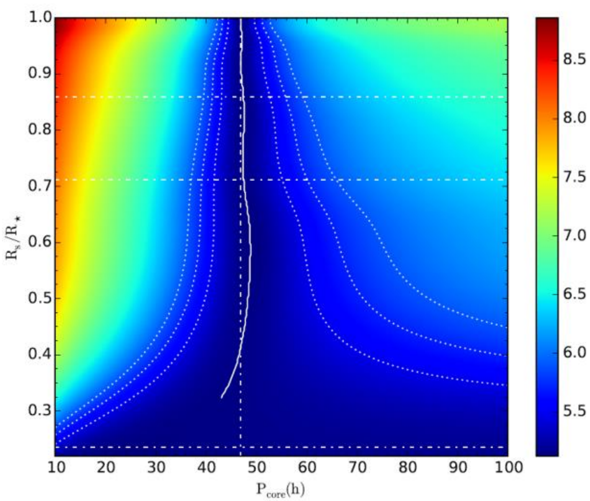


The pulsating ZZ Ceti star GD 165 with trapped modes above the H/He transtion region

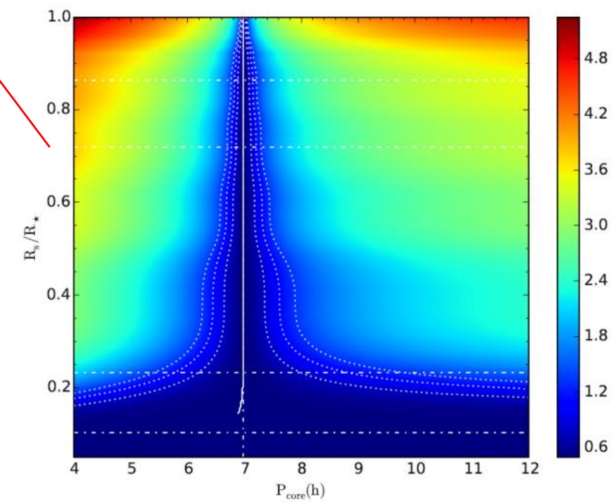


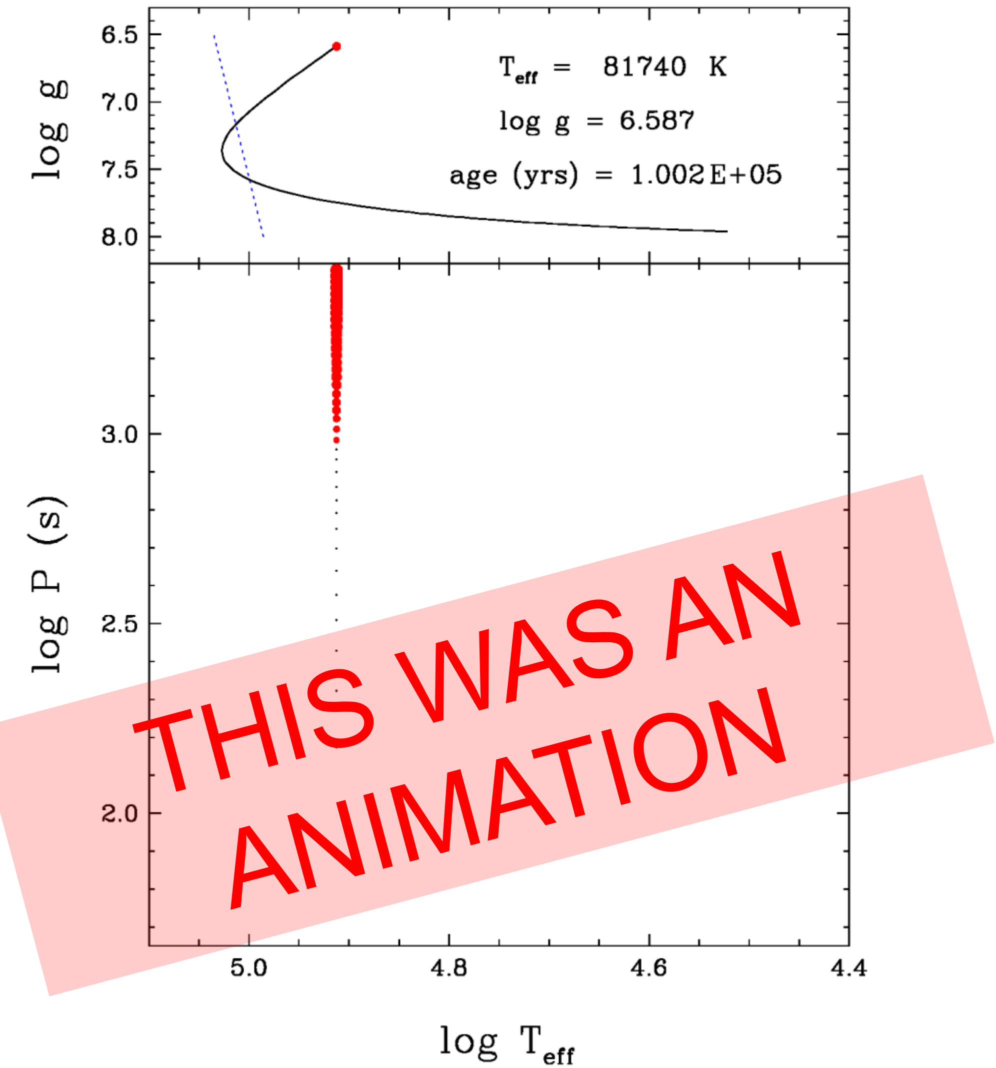


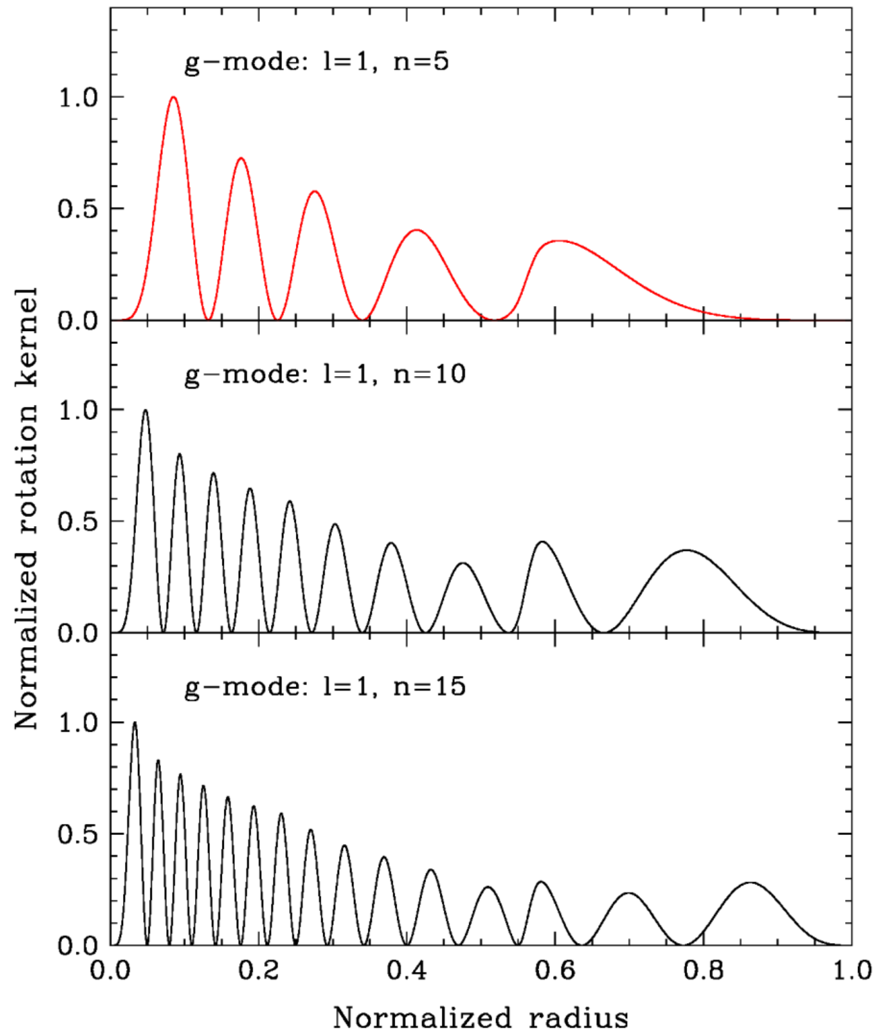
The pulsating DB white dwarf in the Kepler FOV, J1929+4447, with some confined modes at depth



The pulsating ZZ Ceti star GD 1212 observed with Kepler 2, with several confined, deep modes




log g
6.5
7.0
7.5
8.0
T_{eff} = 81740 K
log g = 6.587
age (yrs) = 1.002E+05
log P (s)
3.0
2.5
2.0
5.0
4.8
4.6
4.4
log T_{eff}
THIS WAS AN
ANIMATION


g−mode: l=1, n=5
g−mode: l=1, n=10
g−mode: l=1, n=15
Normalized rotation kernel
1.0
0.5
0.0
0.0
0.2
0.4
0.6
0.8
1.0
Normalized radius