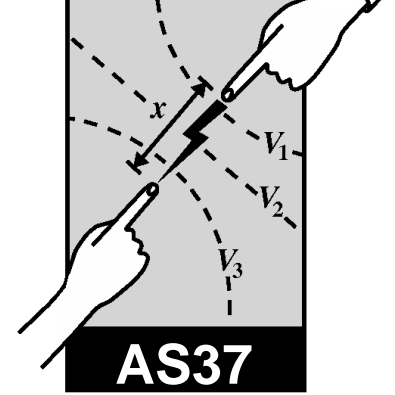


# the Hubble diagram for type Ia supernovae



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text in red indicates html active links, accessible by clicking when viewing electronically

## 1 Aim

The aim of this experiment is to determine constraints on the values of cosmological parameters by statistical analysis of the “Hubble diagram” for a sample of type Ia supernovae. This analysis is one of the key pieces of evidence in favour of a non-zero “cosmological constant” or “dark energy”, which leads to an acceleration of the expansion of the universe. We will use the latest publicly-available data to derive the best-fitting values of the energy density in matter and in “dark energy”, together with the statistical uncertainties on those values.

## 2 Introduction

In the 1920s Hubble discovered that other galaxies in the universe appear to be receding from us, as measured by the Doppler redshift of spectral lines in their optical spectra. Furthermore, the fainter a galaxy, and hence by implication the more distant a galaxy, the larger the recession velocity, with velocity linearly proportional to distance at the low redshifts measured by Hubble. This is the prime evidence for an expanding universe. When we study individual objects (in this experiment, we will look at distant supernovae), those with higher redshift are more distant, and because of the finite speed of light, a view of the universe at high redshift is equivalent to a view back in time to an earlier epoch.

A plot of redshift  $z$  against distance  $d$  or brightness of a population of objects is known as the Hubble diagram. In the local universe, for low values of redshift,  $cz = H_0 d$  where the slope of the relation,  $H_0$ , is known as Hubble’s constant and tells us the current rate of expansion of the universe. In the standard cosmological model this quantity varies with cosmological epoch, and hence the redshift of the objects being used to measure it, so we refer to the redshift-dependent quantity as the Hubble parameter,  $H(z)$ . The manner in which  $H(z)$  varies with redshift depends on the geometry and expansion history of the universe, so in principle by measuring the Hubble diagram at high redshifts we can learn something about what our universe is made of. Nowadays we can probe to much larger redshifts and distances than Hubble could, the aim of this practical is to investigate one of the most important datasets currently available to cosmologists.

## 2.1 Some cosmology

The expansion of the universe is quantified by a “scale factor”  $a$ , which is defined to be a dimensionless parameter with value unity at the present epoch (i.e.  $a = 1$  at  $z = 0$ ). The variation of  $a$  in a homogeneous, isotropic universe is governed by the Friedmann equation,

$$H^2(a) = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} - \frac{kc^2}{a^2} + \frac{\Lambda}{3} \quad (1)$$

where  $H(a)$  is the Hubble parameter, here expressed in terms of  $a$  as the ratio of the time rate of change of  $a$ , denoted by  $\dot{a}$ , to  $a$ . It has units of  $(\text{time})^{-1}$ .  $\rho c^2$  is the total energy density of matter and radiation,  $k$  is a constant that defines the geometry of spacetime and  $\Lambda$  is Einstein’s “cosmological constant”. The effect of  $\Lambda$  is to cause an acceleration in the expansion of the universe, counter to the decelerating effect of gravitating matter. If we divide through by  $H^2$  we find

$$1 = \Omega_M + \Omega_k + \Omega_\Lambda$$

where  $\Omega_M = 8\pi G\rho/3H^2$  etc. Because the  $\Omega$  quantities have differing dependencies on the scale factor  $a$  they are not constants, but change as the universe expands. However, the curvature of the universe is thought to be constant ( $k = \text{constant}$ ). In the relatively recent universe we can neglect the contribution of radiation to  $\rho$  and consider only the contribution of matter (both baryonic “normal” matter and “dark” matter), in which case  $\rho \propto a^{-3}$ . If  $k = 0$  the universe is said to be “flat” and spacetime appears “Euclidean”, so that the angles in a triangle defined by three light rays add up to  $180^\circ$ . Thus the condition for a flat universe is

$$\Omega_{\text{TOT}} \equiv \Omega_M + \Omega_\Lambda = 1.$$

We can also define these quantities explicitly at  $z = 0$ , writing them as  $\Omega_{M,0}, \Omega_{\Lambda,0}$  etc (see Liddle 2003).

Observations of fluctuations in the Cosmic Microwave Background indicate that indeed this condition holds (Liddle 2003, chapter 10 and A5.4; **WMAP**; Spergel et al. 2007) to the accuracy of current measurements. A complementary set of observations however also indicates that  $\Omega_{M,0}$  and  $\Omega_{\Lambda,0}$  are roughly equal - and in particular  $\Omega_{\Lambda,0}$  cannot be zero. Those observations are based on measuring both the brightness and the redshift of distant supernovae and fitting a cosmological model to the resulting Hubble diagram. If we assume that the supernovae all have the same intrinsic brightness (we say they are “standard candles”) then their measured apparent brightness tells us their relative distance. The measured redshift  $z$  is related to the scale factor by  $1 + z = 1/a$  (Liddle 2003, section A2.1), and the relationship between distance and  $z$  can be used to determine the values of  $\Omega_{M,0}$  and  $\Omega_{\Lambda,0}$ .

More generally, the Cosmological Constant can be replaced by a more general class of models known as “dark energy”. The simplest parameterisation of these is through the equation of state of dark energy, that is, the relation between pressure  $P$  and relativistic energy density  $\rho c^2$ , in which  $P = w\rho c^2$ . The Cosmological Constant is a special case of dark energy in which  $w = -1$ . This surprising equation of state is simply a statement that even though the universe is expanding, the energy density of the Cosmological Constant stays constant.

To find the redshift-distance relation for a given cosmology we need the spacetime metric, which for a homogeneous isotropic expanding spacetime is given by the Robertson-Walker metric:

$$ds^2 = -c^2 dt^2 + a^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right],$$

where  $ds$  is a spacetime interval,  $dt$  is a time interval,  $r$  is a radial coordinate,  $\theta$  and  $\phi$  are angular coordinates,  $a(t)$  is the time-dependent scale factor of the universe and  $k$  is the curvature constant already encountered above. If we consider the propagation of a light ray, for which  $ds^2 = 0$ , then

$$\frac{dr}{\sqrt{1 - kr^2}} = \frac{cdt}{a} = \frac{cda}{\dot{a}a} = \frac{cdz}{H(z)} \quad (2)$$

where  $H(z)$  is given by the Friedmann equation (equation 1), and where we have transformed from the scale factor  $a$  to the redshift  $z$  of the object by the relation  $1 + z = 1/a$ . In this experiment we rewrite equation 1 in terms of the parameter values at zero redshift,  $\Omega_{M,0}$  etc:

$$H^2(z) = H_0^2 \left[ \Omega_{M,0}(1+z)^3 + (1 - \Omega_{TOT,0})(1+z)^2 + \Omega_{\Lambda,0} \right]$$

ignoring the contribution of radiation to the energy density. The RHS of equation 2 has to be integrated with respect to  $z$ ; the LHS can be integrated by making the trigonometric substitution  $\sin \theta = k^{1/2}r$  to find

$$r(z) = \begin{cases} \frac{c}{H_0} \frac{1}{(\Omega_{TOT,0}-1)^{1/2}} \sin \left[ (\Omega_{TOT,0}-1)^{1/2} H_0 \int_0^z \frac{dz'}{H(z')} \right] & k > 0 \\ \frac{c}{H_0} \int_0^z \frac{dz'}{H(z')} & k = 0 \\ \frac{c}{H_0} \frac{1}{(1-\Omega_{TOT,0})^{1/2}} \sinh \left[ (1-\Omega_{TOT,0})^{1/2} H_0 \int_0^z \frac{dz'}{H(z')} \right] & k < 0 \end{cases}$$

The RHS integration can be done analytically only for a few special cases, so in general we have to do this numerically. In this experiment the software does the numerical integration for you.

So far we haven't been very specific about what we mean by "distance". The coordinate  $r$  is the comoving distance: because the redshift lowers the photon energy when received by an observer, and because the same time dilation effect lowers the photon arrival rate by the same amount, the bolometric flux  $S$  (that is, the energy received by an observer from a distant supernova with redshift  $z$  per unit time per unit collecting area, integrated over the entire electromagnetic spectrum) is related to the intrinsic bolometric luminosity  $L$  by

$$S = \frac{L}{4\pi(1+z)^2r^2}.$$

This effect is also included in the software provided.

A less compact description of the above is given by Liddle (2003) and is recommended reading for this practical and for the third-year astrophysics course.

## 2.2 Type Ia supernovae

Supernovae are short-lived (days/weeks) explosions at the end points of the lives of stars. At their peak brightness an individual supernova can outshine an entire galaxy of stars, and so we can see them at vast cosmological distances. In fact, there are now known to be several different types of supernovae, that have different causes. Each type has a different optical spectrum and a different decay in brightness with time, so by measuring the spectra and following the brightness with time (known as the "light curve") we can determine the type of the supernova. The most massive stars undergo core collapse when their nuclear fuel is exhausted, and these result in "type II" supernovae. Type Ia supernovae are thought to be formed when a white dwarf is pushed over the 1.4 solar

mass Chandrasekhar mass limit by new material being added from a companion star, resulting in a thermonuclear explosion of the white dwarf. As these stars all have the same mass (more or less, rotation can cause the actual mass limit to differ between stars) they all have very similar brightnesses when they explode. They are also extremely luminous, around  $10^{10}$  times the brightness of our sun, so they are good to use as standard candles in cosmology.

## 2.3 The statistical method

### 2.3.1 The $\chi^2$ statistic

The values of the cosmological parameters will be obtained by fitting the supernova Hubble diagram by models for the relationship between distance (and hence relative apparent brightness) and redshift. Each model has some unknown free parameters which are: the zero-point of the supernovae brightness scale;  $\Omega_{M,0}$ ;  $\Omega_{\Lambda,0}$ ; and  $w$ . How well each model agrees with the data is measured using the  $\chi^2$  goodness-of-fit statistic, where  $\chi^2$  (“chi-squared”) is defined as

$$\chi^2 = \sum_i^n \left( \frac{y_i - \hat{y}_i}{\sigma_i} \right)^2 \quad (3)$$

and where  $y_i$  is the  $i^{\text{th}}$  data value,  $\hat{y}_i$  is the corresponding model value, and  $\sigma_i$  is the measurement error for that data value. Here we assume the redshifts can all be measured arbitrarily accurately, and that the measurement uncertainty is entirely due to the error on the apparent brightness. If there are  $n$  data points (here there are 115) and  $p$  free parameters, then for  $p < n$  the probability of obtaining a particular value of  $\chi^2$  is known (see [Wolfram](#)). The best-fitting model is the one with the lowest  $\chi^2$ , and in general we expect the best-fitting model to have  $\chi^2 \simeq n - p$ . ( $n - p$ ) is often known as the number of “degrees of freedom” (if  $n = p$  there are no degrees of freedom and we expect the model and data to agree perfectly, usually an absurd outcome since there are measurement errors which mean that even the “correct” model should not be a perfect fit to the data). So to use this statistic, we shall vary the model parameters until we obtain the minimum  $\chi^2$  value.

### 2.3.2 Confidence regions

Having done that, we can also estimate uncertainties on those parameters. For **one** free parameter, the uncertainty is obtained by varying the parameter until  $\chi^2$  has increased by unity from its minimum value. The range of the parameter value is then known as the “68 percent confidence interval” since for gaussian errors this range will encompass the true value 68 percent of the time.

However, in reality we might have a number of free parameters, which might be correlated with each other, so estimation of confidence regions becomes more tricky. First, we have to decide **which** of the parameters that exist in the problem are ones that are “interesting”, in the sense of wanting to know the best-fit value and its uncertainty. For example, here we have up to four free parameters (the supernova absolute brightness, the matter density parameter, the cosmological-constant density parameter and the dark energy equation of state parameter) but we are only interested in some of these. e.g. we don’t really care what the value of the supernova absolute brightness is - we need

to let this be a free parameter in the fitting because we don't know its actual value very well, but we don't actually care what that value is. So, out of the four parameters, we can deem the supernova absolute brightness to be “uninteresting”, the two cosmological density parameters to be “interesting” with the fourth equation-of-state parameter needing to be fixed and not a variable parameter. Then, to work out the confidence region for those two “interesting” parameters, we again find the best-fit (minimum)  $\chi^2$  value, then vary the interesting parameters until  $\chi^2$  has increased by some amount,  $\Delta\chi^2$ . If there are  $q$  “interesting” parameters, then we must increase  $\chi^2$  by an amount that corresponds to the value of  $\chi^2$  which yields the required confidence region probability (e.g. 68 percent) **for  $q$  degrees of freedom**. NB in our problem  $q$  is a lot less than the number of data points  $n$ , so we're using the  $\chi^2$  distribution in a rather different way from in the basic consideration of whether we've got a decent best-fit or not. This is where the value of  $\Delta\chi^2 = 1$  in the previous paragraph came from, when we were describing the case of one free parameter. For **two** free parameters the 68 percent confidence interval for the joint distribution is found by defining the range over which  $\chi^2$  has increased by  $\Delta\chi^2 = 2.3$ . To find this value yourself, use the handy table at <http://faculty.vassar.edu/lowry/tabs.html#csq>: enter a test value for  $\Delta\chi^2$  (e.g. 2.3) into the left-hand box, enter the number of interesting parameters  $q$  (e.g. 2) into the “number of degrees of freedom” box, the tool will then display the tail probability, the probability of the true value lying *outside* this region by chance. The corresponding confidence region probability (the probability that the true value lies *inside* this region) is one minus this value.

In the procedures described below we'll do this in practice by stepping through combinations of the two “interesting” parameters and simply allowing the other parameters either to be free but “uninteresting” or to be fixed (in the case of  $w$ ). This works OK if the error distributions are “normal” (i.e. have a Gaussian distribution function). In more advanced cases we need to average the results over all possible values of the “uninteresting” parameters, a process known as “marginalisation”, which is considerably more compute-intensive but which is not needed here.

### 3 The data

The data provided is from the “Supernovae Legacy Survey” ( **SNLS** ) published by Astier et al. (2006). The SNLS is a large survey to find 500 type Ia supernovae (SNe) at cosmological distances, and use them to measure values of cosmological parameters. The presently-available dataset comprises 115 SNe. The data file **SNLS.dat** contains for each observed type Ia supernova the measured redshift ( $\log_{10}$  value) and the measured brightness and its error, expressed logarithmically in “magnitudes” (see Appendix A).

### 4 Initial setup

Begin by starting X11 and creating a **xterm** window in which commands can be typed. The window is created using the mouse to click on the large “X” in the middle of the dock on the bottom or at the side of your screen. The large “X” may bob up and down a bit before the window appears.

The steps detailed below provide the setup necessary for experiments carried out in the Astrophysics Laboratory. The **.profile**, **.bashrc**, **.lpoptions** and **.Xdefaults** files in your home directory are changed as a result of carrying out steps listed below; if these files have been previously setup to carry out an experiment in another laboratory, please consult a demonstrator before proceeding. If you have already carried out the immediately following steps for another astrophysics

practical in the current year then you can skip these next steps until the specific extra steps needed for AS37.

Provide yourself with copies of the necessary `.profile`, `.bashrc`, `.lpoptions` and `.Xdefaults` files by executing the commands

```
$ cp ~aelg/astro-lab/profile .profile
$ cp ~aelg/astro-lab/bashrc .bashrc
$ cp ~aelg/astro-lab/Xdefaults .Xdefaults
$ cp ~aelg/astro-lab/lpoptions .lpoptions
```

where the leading "\$" represents the operating system command line prompt which by default is some long string (if you have previously carried out one of the third-year astrophysics lab experiments in this current year then you don't need to redo the above step). Once the above commands have been typed, logout and login again; this causes the `.profile`, `.bashrc`, `.lpoptions` and `.Xdefaults` files to be executed and so providing the setup necessary to conduct experiments in the Astrophysics Laboratory. Create a new `xterm` window, as described above, in which commands can be typed; the first obvious sign of the new setup is the replacement of the default operating system command line prompt with a simple "\$".

It is worth noting some unorthodox inputting methods inherent to Mac keyboards and mice. Some computers in the Lab have mice with a right click feature (which is turned on in "System Preferences."), For mice without this feature (the clear ones), a right click can be emulated by using `ctrl + left` click.

All computer commands reproduced above and below for your guidance are given in typewriter font. The leading "\$ " represents the command line prompt. A leading "> " represents a prompt provided by an executing program such as `qdp`. Typewriter font is also used to specify file names, file contents and environment variables. Further necessary information is provided in the Third Year Astrophysics Laboratory Practicals page

[http://www-astro.physics.ox.ac.uk/~aelg/Third\\_Year\\_Laboratory](http://www-astro.physics.ox.ac.uk/~aelg/Third_Year_Laboratory)

which can be read with a browser such as `safari`.

The first step is to create a new directory in which all files, associated with the experiment described here, will be stored. Issue a command of the form

```
$ mkdir as37
```

to create such a directory. Then type

```
$ cd as37
```

to make the new directory your current working directory. Every time you log in to work on AS37 you should `cd as37`.

AS37-6

The system then needs to be set up so that it can find the various programs and scripts that you need to run, and also to copy the data into your filespace. To do this simply type

```
$ AS37
```

This should tell you whether you are logged in to an Intel or a PowerPC Mac and if no error message is generated then the setup has been successful. Check that the data file and script that you need now exist in your current working directory by using the directory list command:

```
$ ls
```

This should show the files named `SNLS.dat`, `qdprun.pl` and `ras37.pdf`. The latter file is the PDF of this script that you are reading. If you open the script on your terminal, either using `open ras37.pdf` or by double-clicking on the filename in a Finder window, then you can access references in the text by clicking on the red highlighted links.

For the analysis we use a modified version of the fitting and plotting package “`qdp/plt`”. For an online help manual for this package, see <http://wwwastro.msfc.nasa.gov/qdp/help/plt.html>. To start the program with the SNe data, type `qdp SNLS.dat` in an x11 window (filenames are case-sensitive). At the prompt, choose the plotting device: normally this should be `/xwindow` or `/xw` for short, where the `/` is important - this starts up an xwindow plot on your screen. You should see a plot of the data. You can change the appearance of the plot using commands such as `label`, and you can change the displayed data range using `rescale` - see the online help for more information and the full set of available commands. Most commands can be abbreviated to their first two letters.

## 5 The analysis

### 5.1 The Hubble diagram

You can see that the relation between magnitude and  $\log(\text{redshift})$  is almost linear with a slope about 5. This is what we would expect from Hubble’s original relation: if the cosmological recession velocity  $v \propto d$ , where  $d$  is distance, then there will be a Doppler shift such that  $z \simeq v/c \propto d$ . But apparent brightness of a standard candle  $S \propto d^{-2}$  in a Euclidean universe, so we expect  $S \propto z^2$ . But using the magnitude system (see Appendix A),  $m = -2.5 \log_{10} S + \text{constant}$ , so a graph of  $m$  against  $\log_{10} z$  should have a slope of 5 (note that we are plotting log quantities, so the slope now is not Hubble’s constant, but tells us about the inverse-square law: Hubble’s constant would be measured on this log plot as the offset in the relation, since at low redshift we would have  $\log cz = \log H_0 + \log d$ ).

### 5.2 Do we live in a Euclidean universe?

Start investigating the Hubble diagram by fitting a linear relation to  $m$  v.  $\log_{10} z$ . In `qdp`, first set the model as being a constant term plus a linear dependence,  $\hat{y} = a + bx$ , where  $a$ ,  $b$  are free parameters. This is done using the commands: `mo` (to see a list of available models); `mo cons linr`

(to set the model as comprising a constant component plus a linear term). Just hit “carriage return” at each prompt, to set default starting values for each parameter in turn. When the `PLT >` prompt returns, start the iterative fit process by typing `fit`. Iterations proceed until  $\chi^2$  has converged to a minimum value, at which point the fitting stops and reports the best-fit values of the parameters. Note the best-fit value of the slope and the value of  $\chi^2$  (“chi-squared”). Type `plot` after the fit to refresh the graph, you will see the values of the model parameters, the  $\chi^2$  values and the number of data points written up the side of the plot.

Now find errors on the slope as follows. We need to find the range of the slope parameter over which  $\chi^2$  increases by unity from its best-fit value (refer back to the subsection on confidence regions). We will let the `cons` component be a freely-varying “uninteresting” parameter. There is one “interesting” parameter in this problem, the slope, being the `linr` parameter. To find its confidence interval, rather than allowing the slope parameter to vary freely within `qdp`, we need to fix it and vary its value manually, because we need to record how the value of  $\chi^2$  varies as this parameter varies. Freeze the parameter using the command `freeze 2` to freeze the second parameter (if this corresponds to the slope in the model you are fitting). Then vary its value slightly with the command `new 2 5.4`, which changes the value of the second parameter to a new value of 5.4. `fit` again - and note the new value of  $\chi^2$  - it should be worse than the best-fit value. Vary the values for the slope until you obtain the range, both higher than and lower than the best-fit value, over which  $\chi^2$  varies by unity. This yields the 68 percent confidence interval for the slope. Quote the best-fit value and range, and hence decide whether a simple “Euclidean, inverse-square-law” universe adequately describes the data.

A fair bit of the departure from the expected value of 5 arises because of the high-redshift data points. Try restricting the range of redshift using the `rescale x MIN MAX` command (`replot` will reset the plot limits). Hubble’s original measurements only probed to low redshift - at what redshift do the supernova data start to show statistically-significant evidence for a departure from a value of 5? There are a number of possible ways of carrying out this statistical test. The easiest here is to limit the plot to some redshift range, find the best-fit slope and its corresponding  $\chi^2$  value, then freeze the slope to be exactly 5.0, re-fit to find the minimum  $\chi^2$  value when the slope is fixed, and hence get a value  $\Delta\chi^2$  for how much worse a fit is obtained when slope=5.0. As described in section 2.3.2 we can then evaluate the level of statistical significance by asking if  $\Delta\chi^2$  exceeds some value, where that value has a low probability of arising by chance. You can use tables of  $\chi^2$  to decide what value to choose, or use a  $\chi^2$  tool such as <http://faculty.vassar.edu/lowry/tabs.html#csq>. Make sure you specify the correct number of degrees of freedom in this test, as explained in section 2.3.2. Before each iteration you will need to type `plot` to clear the previous fit from the plot, and also make sure that you have parameters “frozen” using `freeze` or “free” using `thaw` (e.g. `thaw 2` to allow the second parameter to be a free parameter in the fit) as appropriate.

You should make hard copies of plots by creating a postscript file, `hard plotname.ps/cps` which you can then print on a postscript printer. You can then return to the xwindows plot by entering `plot`. For producing publication-quality plots make sure you label the axes appropriately using `label` and also change the line widths, colours etc to clear values using the command options listed at <http://wwwastro.msfc.nasa.gov/qdp/help/plt.html>

### 5.3 Fitting cosmological models



The model `mo cosm` allows us to fit a Friedmann model to the data (this model has been added in specially for this experiment and so doesn't appear in the `qdp` help pages). There are four parameters available in this model, although it is not possible to allow them all to be simultaneously free parameters in the fit. The parameters are

1	<b>Abs Mag</b>	The Absolute Magnitude of Type 1a SNe (defines the zero-point of the relation)
2	<b>Omega(M)</b>	$\Omega_{M,0}$
3	<b>Omega(T)</b>	$\Omega_{TOT,0} = \Omega_{M,0} + \Omega_{\Lambda,0}$
4	<b>w</b>	dark energy equation of state, $w$

The model is parameterised in this way so that we can fix the geometry of the universe through the value of  $\Omega_{TOT,0}$  if we wish. Note that if we wish to investigate dependences on  $\Omega_{\Lambda}$  we must cast this in terms of  $\Omega_{TOT}$ .

Now let's test various cosmological models against the data to see which cosmological parameters the data are sensitive to, and what constraints on those parameters we can obtain.

1. Start by considering a flat universe with a possible non-zero cosmological constant. For this we need to fix the value of  $w = -1$  and  $\Omega_{TOT,0} = 1$  using the `new` and `freeze` commands: `new 3 1.0, freeze 3, new 4 -1.0, freeze 4`. Make sure your plot is covering the full redshift range of the data: if you still have a restricted range, issue the command `rescale` (with no arguments) to reset it.
  - (a) The command `fit` will then find the best-fit values of the supernova absolute magnitude and  $\Omega_{M,0}$ .
  - (b) Using the principle outlined above, evaluate the 68 percent confidence interval on  $\Omega_{M,0}$  by `freezing` this parameter, and stepping through a set of values using `new 2` to see how  $\chi^2$  varies with  $\Omega_{M,0}$  (remember that the **Abs Mag** parameter must be floating, not frozen, and that  $\Omega_{TOT,0}$  and  $w$  must be fixed). Use a text editor to make a table that has two columns, the first containing values of  $\Omega_{M,0}$ , the second containing the corresponding values of  $\chi^2$ . This table may then be plotted using `qdp`. From your plot, read off the 68 percent confidence region for  $\Omega_{M,0}$ .
  - (c) If the uncertainties on the Hubble diagram are normally-distributed, the plot of  $\chi^2$  v. parameter value should have a quadratic form - is this the case? To test it, try fitting a quadratic function to the plot using the standard model options in `qdp` (type `mo` to see the names of the model components that you can use, hint: you will need to use three components simultaneously to fit a quadratic function).
  - (d) Assuming we do indeed live in a flat universe, use the above result to evaluate the 68 percent confidence interval on  $\Omega_{\Lambda,0}$ .
  - (e) What worsening in  $\chi^2$  results if you also fix  $\Omega_{M,0} = 1$  (and hence  $\Omega_{\Lambda,0} = 0$ )? Use tables of  $\chi^2$  for one degree of freedom to evaluate the probability that  $\Lambda$  could be zero, given this data, provided we assume (a) the SNe are standard candles and (b) the universe is flat.

Use command `plot` in between successive fits in order to clear the previous fit from your graph.

2. Now consider a non-flat universe by allowing the  $\Omega_{\text{TOT},0}$  to be an “interesting” parameter. You will need to keep  $w = -1$  or no best-fit will be found (the data are completely insensitive to a joint variation in  $\Omega_{\text{TOT},0}$  and  $w$ ). Now there are two freely-varying cosmological parameters of interest, so we need to derive the uncertainties on the joint distribution of parameters 2 and 3.
  - (a) First, find the best-fit by allowing parameters 1, 2, 3 to be free.
  - (b) Then to derive the confidence region we need to **freeze 2, 3** and step through over some range and make a two-dimensional grid of  $\chi^2$  values. It is quite tedious to do this by hand, so there exists a perl script which does this for you, the file `qdprun.pl`. To run it, type `qdprun` (or, equivalently, `perl qdprun.pl`). This script will prompt you for the range of each of the two parameters, and also the sampling interval to be used. It will then step through a 2D grid of parameter values, running `qdp` without the graphics, and returning a 2D table of  $\chi^2$ . This table is named `chisqtable.dat`. If you rerun the script you must first either delete or rename any existing `chisqtable.dat`. When the script has finished you can look at its contents by typing `more chisqtable.dat` or you can edit it with a text editor. You will see that there are some header commands at the start of this file followed by the grid of  $\chi^2$  values. The header commands allow this table to be immediately plotted by `qdp` as a set of contours of  $\chi^2$  on a 2D plot whose axes are the two cosmological parameters. When starting to run the script, give some thought to a sensible choice of sampling of the parameter values, and to the range of values. Not all combinations of parameter values are allowed, in the sense that some combinations mean that the expansion had a turning point in the past, i.e. no Big Bang - `qdp` will either hang or crash if you ask it to calculate such values. To look up what range is reasonable, see [Carroll, Press & Turner \(1992, pages 506-7\)](#). You will also need to decide on a sensible choice of contour level that reflects some choice of confidence region - for example, plot contours that enclose the 68 percent and 90 percent confidence regions using the appropriate  $\Delta\chi^2$  above the minimum value (the minimum value is reported at the end of the script when it runs). You can put in the correct contour levels by editing the file `chisqtable.dat` and changing the line that starts `CONTOUR LEVELS`.
  - (c) What do you learn about the constraints that the supernova data alone provide us with? Why are your conclusions different from the case where you assumed a flat universe?
3.
  - (a) Investigate dark energy models by returning to the flat-universe case, running `qdp` as you did originally (not the perl script) and allowing  $w$  to be a free parameter instead of  $\Omega_{\text{TOT},0}$ . Possible values for  $w$  are  $w = 0$  (pressure-less matter, or dust),  $w = 1/3$  (radiation),  $w = -1/3$  (“topological defects” in the early universe),  $w = -1$  (Cosmological Constant): given our lack of knowledge we should probably assume that any value in the range  $-2 < w < 1/3$  is feasible. What is the 68 percent confidence region for  $w$  from your SNe data?
  - (b) Add constraints from the microwave background observations. Spergel et al. (2007) use the latest WMAP data to show that  $\Omega_{k,0} = -0.014 \pm 0.017$ . You can include this result with yours by working out the value of  $\Omega_{k,0}$  for each point in turn in your grid of  $\Omega_{M,0}, \Omega_{\Lambda,0}$ , then evaluating an additional amount of  $\chi^2$  to be added to your SNe  $\chi^2$  value at that point using equation 3, e.g., the model point  $\Omega_{M,0} = 0.8, \Omega_{\Lambda,0} = 0.4$  has  $\Omega_{\text{TOT},0} = 1.2$  and hence  $\Omega_{k,0} = -0.2$ , so we need to add  $\Delta\chi^2 = ((-0.2 + 0.014)/0.017)^2 = 119.7$  to the SNe-derived  $\chi^2$  at that model point. The most effective way of doing this in fact is for you to modify the perl script `qdprun.pl`: using a text editor, find the line where `$deltachisq = 0` - change the right-hand side of this to make `$deltachisq` equal to the

additional  $\Delta\chi^2$  from WMAP. The value of `$deltachisq` of course has to be a function of  $\Omega_{M,0}, \Omega_{\Lambda,0}$  so you need a little piece of code on the relevant line in order to calculate the value as a function of the parameters. Looking through the perl script, you will see variables already declared and calculated that you will need: in particular `$omegatotal` is the current value of  $\Omega_{TOT,0}$  (variables are denoted by the \$ sign as their first character). If you mess up you can copy the original perl script into your file space by invoking `AS37` again. When running the script, because the constraints on the values of cosmological parameters will now be much tighter, you will need to create a more finely-sampled grid of parameter values. If your contour plot looks fragmented then you need finer sampling.

- (c) Comment on the effect of adding the CMB constraint, and compare with the results you got when you *assumed* the universe is flat in case (1).
  - (d) Differentiate equation (1) with respect to time (remembering the  $a$  dependence of  $\rho$ ) and use your best-fit parameter values to decide whether the expansion of the universe is currently accelerating ( $\ddot{a} > 0$ ) or decelerating. What combinations of the parameters  $\Omega_{M,0}$  and  $\Omega_{\Lambda,0}$  would result in a universe with zero acceleration at the present epoch, and at what value of probability can such a model be excluded by your WMAP+SNLS analysis?
4. Finally, investigate the effect of adding new data. We would like to obtain higher-precision measures of cosmological parameters, for example to see whether  $w$  really is consistent with being a Cosmological Constant. To do this we need to make new observations, to discover more SNe. We can simulate what effect adding more SNe would have on the size of our confidence regions if we artificially double the sample size, by duplicating each entry in the table `SNLS.dat` and rerunning the analysis (don't redo it all, just the flat-universe case should be enough). How do you expect the size of the confidence region to depend on the number of SNe? Test your prediction by artificially increasing the size of the data by larger factors. To make the test, you might want to plot  $\log(\text{number of datapoints})$  versus  $\log(\text{size of confidence region})$  and use the plotted relation to either derive or improve your model for the dependence of accuracy on sample size. How large a survey would be needed if we wanted a precision on  $w$  ten times better than the current observations?

Once you have completed these tasks, make sure you have hard-copy of all relevant plots and tables and summarise what we can learn about the make-up of the universe from these types of observation, particularly when combined with other observations. What is the *disadvantage* of combining observations to obtain improved measurements of cosmological parameters, rather than simply analysing them independently?

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## A The astronomical magnitude scale

In astronomy, brightness is measured on the logarithmic “magnitude” scale, somewhat akin to the decibel scale. The *apparent magnitude*  $m$  is defined as

$$m = -2.5 \log_{10}(S/S_0)$$

where  $S$  is the measured flux in some waveband and  $S_0$  is a normalisation that varies with the waveband being considered. Note the factor 2.5 (used for historical reasons to map the scale onto earlier eye estimates of brightness) and the minus sign convention, which means that fainter objects have larger apparent magnitudes. The brightest stars in the night sky have  $m \simeq 0$ , the faintest visible to the human eye have  $m \simeq 6$ . The faintest SNe in this experiment have  $m \simeq 25$ , nearly a factor  $10^8$  times fainter than detectable by the human eye.

We often want to refer to the intrinsic brightness, or luminosity, of an astronomical object, independent of its distance. This is commonly done by defining the *Absolute Magnitude*, which is the value of  $m$  that an object would have if viewed at a distance of 10 parsecs (pc). The parsec is the most commonly used astronomical distance unit and is equivalent to 3.26 light years, or  $3.0856 \times 10^{16}$ m. The peak Absolute Magnitude of Type 1A SNe is about -19, which may be compared with the Absolute Magnitude of our Sun which is about 4.6. Thus the SNe are a factor  $10^{(4.6+19)/2.5} \simeq 3 \times 10^9$  more luminous than our Sun at their peak.