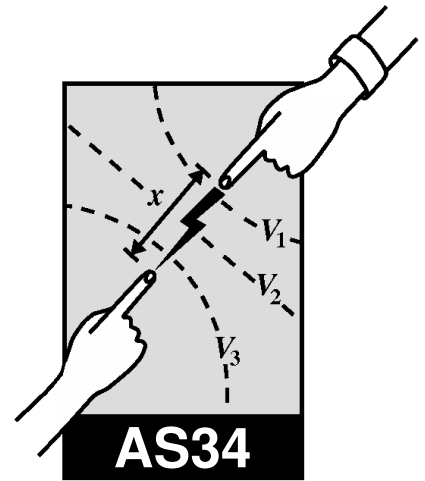


# analysis of a double einstein ring



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## 1 Introduction

Gravitational lensing arises when a massive foreground object (usually referred to as the “lens”) gravitationally deflects light from a background object (usually referred to as the “source”) towards the observer. In the case of perfect alignment where the source is directly behind the lens, the observer sees the source as a ring (the so called “Einstein Ring”) with the lens at its centre. Under rare circumstances, the deflection is sufficiently large to create multiple images of a distant light source; this phenomenon is called strong gravitational lensing. Treu (2010) reviews strong lensing by galaxies and this should be consulted for further details.

One of the most powerful ways of measuring mass distributions in galaxies to have emerged in recent years is gravitational lensing. As a result of recent dedicated efforts, more than a 100 galaxy-scale gravitational lens systems are now known and new discoveries are eagerly anticipated from dedicated surveys planned for the future. The experiment proposed here is intended to illustrate some techniques used in the casual study of astronomical images, and make some very approximate gravitational lensing calculations to determine the mass of a luminous red galaxy (LRG) and its dark matter content.

The Sloan Digital Sky Survey (SDSS) is intended to image roughly one-quarter of the sky in five optical bands and obtain spectroscopic follow-up observations. Approximately 12% of galaxy spectroscopic fibres in the SDSS survey are allocated to LRGs; these are luminous and therefore massive early-type galaxies. Bolton *et al.* (2004) note that LRGs should very effectively lens any background object suitably aligned with them, and such lenses should be comparatively easy to observe because they suffer minimal extinction as LRGs have little dust.

Bolton *et al.* (2008) catalogue SDSS J094656.68+100652.8, and 69 other objects, as showing clear evidence of multiple imaging of a background object by a foreground galaxy. Follow-up observations which Gavazzi *et al.* (2008) make with the Advanced Camera for Surveys (ACS) aboard the Hubble Space Telescope (HST) confirm the nature of SDSS J094656.68+100652.8. Data to be used in the experiment described here are the SDSS spectrum which Bolton *et al.* (2008) use and a Wide Field and Planetary Camera 2 (WFPC2) image obtained on 2007 December 17<sup>th</sup>.

## 2 Initial setup

Begin by creating a `xterm` window in which commands can be typed. The window is created using the mouse to click on the large “X” in the middle of the dock on the bottom of your screen. The large “X” may bob up and down a bit before the window appears.

The steps detailed below provide the setup necessary for experiments carried out in the Astrophysics

Laboratory. The `.profile`, `.bashrc`, `.cups/lpoptions` and `.Xresources` files in your home directory are changed as a result of carrying out steps listed below; if these files have been previously setup to carry out an experiment in another laboratory, please consult a demonstrator before proceeding. If you have already carried out an experiment in the Astrophysics Laboratory and the `.profile`, `.bashrc`, `.cups/lpoptions` and `.Xresources` files in your home directory have remained unchanged since, skip the steps below and move on to the next section.

Provide yourself with copies of the necessary `.profile`, `.bashrc`, `.cups/lpoptions` and `.Xresources` files by executing the commands

```
$ cp ~aelg/astro-lab/profile .profile
$ cp ~aelg/astro-lab/bashrc .bashrc
$ cp ~aelg/astro-lab/Xresources .Xresources
$ cp ~aelg/astro-lab/lpoptions .cups/lpoptions
```

where the leading “\$” represents the operating system command line prompt which by default is some long string. Once the above commands have been typed, logout and login again; this causes the `.profile`, `.bashrc`, `.cups/lpoptions` and `.Xresources` files to be executed and so providing the setup necessary to conduct experiments in the Astrophysics Laboratory. Create a new xterm window, as described above, in which commands can be typed; the first obvious sign of the new setup is the replacement of the default operating system command line prompt with a simple “\$ ”.

It is worth noting some unorthodox inputting methods inherent to Mac keyboards and mice. Some computers in the lab have mice with a right click feature (which is turned on in System Preferences). For mice without this feature (the clear ones), a right click can be emulated by using `ctrl + left click`.

### 3 Objectives

The experiment described in this script is intended to familiarise you with astronomical images, and some of the results which may be extracted from them with comparatively little effort. Tasks proposed are:

- Determine the redshifts of the lensing LRG and sources lensed as inner and outer rings, using a supplied spectrum, and deduce corresponding angular diameter distances.
- Measure the sky background and deduce the number of counts which need to be detected in a pixel if it is to be regarded as a significant signal (with 99% confidence) superimposed on the sky background.
- Determine the “plate scale”, a term which dates from the period when astronomical images were obtained with photographic plates but is still used, which gives the number of arcseconds per pixel.
- Visually classify the ten brightest objects in the supplied field and deduce the point spread function.
- Use gravitational lensing to infer the mass of the LRG believed to be responsible for the lens. Comment on dark matter content as a likely explanation of the higher LRG mass obtained from the outer ring.
- Determine the LRG mass-to-light ratio and deduce that this cannot easily be attributed to the stellar population, given the likely LRG age inferred from its redshift.

## 4 Getting started

All computer commands reproduced below for your guidance are given in typewriter font. The leading “\$” represents the command line prompt. Where a leading “>” is given, this signifies the prompt associated with a program such as `dipso` running under operating system control. Typewriter font is also used to specify file names, file contents and environment variables. Further necessary information is provided in the Third Year Astrophysics Laboratory Practicals page

`http://www-astro.physics.ox.ac.uk/~aelg/Third_Year_Laboratory`

which can be read with a browser such as `safari`.

The first step is to create a new directory in which all files, associated with the experiment described here, will be stored. Issue a command of the form

```
$ mkdir as34
```

to create such a directory. Then type

```
$ cd as34
```

to make it your current working directory. Two files are needed in the working directory and should be copied with the commands

```
$ cp $AELGHOME/einstein_ring/SPECTRUM.STK .
$ cp $AELGHOME/einstein_ring/image_display.dat_00000 .
```

with their purpose being discussed below. Image inspection and measurement in this experiment are carried out with the program `imgdsply` written for the purpose and which is described in the next section. The image to be studied and the associated object catalogue are too large to be stored in your filespace. Instead, environment variables need to be defined

```
$ DIRECTORY=$AELGHOME/einstein_ring
$ FITSDATA=hst_11202_29_wfpc2_f606w_wf_drz.fits
$ CTLGDATA=catalog.xsort
$ export DIRECTORY FITSDATA CTLGDATA
```

to enable `imgdsply` to locate the image and catalogue in the author’s directory.

## 5 `imgdsply`

A rudimentary image display and analysis program, known as `imgdsply` has been developed for carrying out the experiment described in this script. To invoke `imgdsply` type

```
$ imgdsply
```

at the operating system prompt. Image data are expected in the Flexible Image Transport System (FITS) as described by Hanisch *et al.* (2001). On starting up, the program informs you of the image and object catalogue being used. In addition, four files will be created in your working directory; these will have the same name as the image file read in, except the `.fits` is removed and replaced by `.hed_001`, `.hed_002`, `.hed_003` and `.hed_004`. The file with the name extension `.hed_001` contains the primary FITS header and the others headers associated with the three FITS extensions present in the file.

The section of the image to be displayed, expressed as ranges of pixel numbers in abscissa and ordinate directions, are read from the file `image_display.dat_nnnnn` where `nnnnn` is some integer. Each pixel records a signal at its position in data numbers (DN) per second which is used to determine the shade of grey, between extremes of black and white, to represent a pixel in the image display. Clearly what is seen in a displayed image is to some extent established through the DN per second which are represented by black and white; this information is also read from `image_display.dat_nnnnn`. A new version of `image_display.dat_nnnnn`, with `nnnnn` increased by one, is written on exit from `imgdsply` to reflect any greyscale change and any change to the part of the image displayed. On restarting `imgdsply`, the new `image_display.dat_nnnnn` file is read and the previous display is recovered.

Single key presses and a graphics cursor are used to control `imgdsply`. On startup, the user is presented with the following options:

- C – provides a graphics cursor for abscissa and ordinate measurement from a line plot, or an abscissa, ordinate and pixel measurement from an image. If both an image and line plot are displayed when a cursor is selected, measurements can be made from either window in a single invocation; use “s” or “S” to swap between them and a single mouse click to activate before measurements begin.
- D – displays the selected part of the image using the current greyscale. The user is given a new menu with which to adjust the display as described below.
- H – presents a summary of the function of each key.
- O – provides a graphics cursor with which the user selects an object on the displayed image. The cartesian and equatorial coordinates, referred to the J2000 epoch, are returned. Irrespective of where the cursor is placed, details for the nearest object will be returned.
- P – create a postscript file `pgplot.ps` containing the currently displayed image, which can be subsequently printed.
- X – a graphics cursor is provided with which a cross-section of the image is selected for plotting. Once the window is active, place the cursor at the point where the cross-section is to begin and press “A”. Then move the cursor to the other end of the desired cross-section and press “A” again. A red line will show the selected cross-section; if not satisfied use “D” to remove the previous limit and try again. Once satisfied, press “X” and a plot of the cross-section is sent to a new xwindow and a postscript file.
- Q – leaves `imgdsply` and returns control to the operating system.
- ? – presents a summary of the function of each key.

Except as otherwise stated, a cursor position is indicated with the left hand mouse button. Cursor control is terminated with the right hand mouse button.

When an image is displayed with the “D” key as mentioned above, the user is presented with a sub-menu to control the image display as follows:

- < – returns the image display to the way it was immediately after it was displayed with the “D” key press as described above.

- > – returns control to the main menu described above.
- v – lowers the currently selected greyscale limit.
- ^ – raises the currently selected greyscale limit.
- i – inverts the displayed image so that what was previously white is now black and vice versa.
- x – increases greyscale limit changes, using the “v” or “^” keys, by a factor of ten.
- / – decreases greyscale limit changes, using the “^” or “v” keys, by a factor of ten.
- ℓ – selects the lower greyscale limit for adjustment using the “v” or “^” keys.
- u – selects the upper greyscale limit for adjustment using the “^” or “v” keys.
- z – provides a rectangular graphics cursor with which the user may zoom in on a region of interest. Once the window is active, use the left hand mouse button to select one corner of the rectangle to be displayed. Select the opposite corner of the rectangle and terminate with the right hand mouse button.
- ? – presents a summary of the function of each key.

## 6 The Experiment

### 6.1 Lens and Source Redshifts

Before carrying out any image analysis, it would seem prudent to establish that we are dealing with an extragalactic object by measuring its redshift ( $z$ ). The SDSS lens spectrum is stored in the dipso stack SPECTRUM.STK; restore this stack and plot the spectrum with

```
$ dipso
>restore SPECTRUM
>dev /xw
>pm 1
```

and note the prominent emission lines near  $6000\text{\AA}$  and  $8000\text{\AA}$ . Absorption lines immediately to the left of the prominent emission features are understood to be  $H\beta$  and  $H\alpha$  absorption in the lens rest frame. Measure the observed wavelengths of the two absorption features by continuing the above sequence of dipso commands

```
>pop 1
>xr 5500. 6500.
>pm
>xv
```

and after recording the  $H\beta$  wavelength, return to keyboard control by pressing “s”, and then

```
>xr 7500. 8500.
>pm
>xv
>q
```

to similarly measure the H $\alpha$  wavelength. Proceed to deduce the lens redshift

$$z = (\lambda_{\text{obs}} - \lambda_{\text{lab}}) / \lambda_{\text{lab}}$$

and its standard deviation from the two measurements,  $\lambda_{\text{obs}}$  is the observed wavelength and  $\lambda_{\text{lab}}$  the corresponding laboratory wavelength. There is also a prominent emission line near 7820Å; the absorption immediately to the left is due to H $\beta$  absorption in the source which is lensed to produce the inner ring. Measure the wavelength and deduce the source redshift.

Weaker emission lines are to be seen near 7869Å, 7887Å and 7963Å which also have weak absorption features on their short wavelength sides; these can be understood as the Lyman  $\delta$ , Lyman  $\gamma$  and Lyman  $\beta$  absorption in the source producing the outer ring. Measure the wavelengths and deduce the outer ring source redshift. Note that the Lyman  $\gamma$  absorption is blended and gives a slightly lower redshift.

From redshifts, it is necessary to obtain angular diameter distances as seen by an observer on Earth. Begin by obtaining a luminosity distances ( $D_L$ ) using analytic formulae by Wickramasinghe & Ukwatta (2010) or Pen (1999), assuming the Universe is flat so that  $\Omega_\Lambda = 0.74$ . Corresponding angular diameter distances ( $d_A$ ) then follow from

$$d_A = D_L / (1 + z)^2.$$

As an alternative, there is an on-line tool

<http://www.icosmo.org>

which may be used to calculate  $d_A$  directly, as well as useful tutorials on cosmology and gravitational lensing. It should be noted that an uncertain Hubble Constant is an important source of error when making  $d_A$  estimates. It is suggested that the Hubble Constant used, and its associated error limits, be those which Suyu *et al.* (2010) adopt.

## 6.2 Sky Background

The next step is to use `imgdspy` to display the image provided and measure the number of counts per second per pixel. In order to do so, it will be necessary to adjust the greyscale so that everything other than the sky background is displayed as black and the noise in the sky background is clearly visible. Identify a corner of the image where no objects are seen and obtain a few cross-sections here, plots of these may be examined with the graphics cursor to judge the mean DN per second per pixel in the sky background.

To get the mean number of counts observed in the sky background, it is necessary to multiply the mean DN by the gain and the exposure time, both of which may obtained from the FITS header; look for the keywords `ATODGAIN` and `EXPTIME`. In the absence of any bias in the sky background and other sources of error, the observed number of counts would be expected (given an infinite number of measurements) to follow a Poisson distribution and have a standard deviation which is the square root of the number of counts observed. Deduce the number of counts that must be collected in a single pixel for it to be considered a “ $3\sigma$  detection”; that is, the signal in a pixel is three times its standard deviation. If we have an object covering  $n$  pixels, what must the average count in those pixels be for a “ $3\sigma$  detection”?

## 6.3 Plate Scale

Objects in the sky are considered to lie on the Celestial Sphere, a sphere which is concentric with the Earth; it may be envisaged as that sphere of objects that would be seen if the Earth were entirely transparent and the observer’s eye located at its centre. The positions of objects on the Celestial sphere are

often specified in terms of right ascension ( $\alpha$ ) and declination ( $\delta$ ). If we consider a right handed cartesian coordinate system ( $x$ ,  $y$  and  $z$ ) whose origin lies at the centre of the Celestial Sphere with the  $z$  axis pointing towards the North Celestial Pole, and the  $x$  axis pointing towards the Vernal Equinox (the point where the Sun crosses the Celestial Equator on September 21<sup>st</sup> each year) the position vector of an object on the Celestial Sphere is

$$\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$$

where  $\mathbf{i}$ ,  $\mathbf{j}$  and  $\mathbf{k}$  are unit vectors in the  $x$ ,  $y$  and  $z$  directions respectively.

A meridian is that plane passing through an object whose position vector is  $\mathbf{r}$  and the North Celestial pole; in the Celestial Equator it makes an angle  $\alpha$  with the  $x$  axis, where  $\alpha$  increases in a counter-clockwise direction about the  $z$  axis. In the meridian plane, the vector  $\mathbf{r}$  makes an angle  $\delta$  with the Celestial Equator. Two objects on the Celestial Sphere, with position vectors  $\mathbf{r}_1$  and  $\mathbf{r}_2$  and equatorial coordinates are  $(\alpha_1, \delta_1)$  and  $(\alpha_2, \delta_2)$ , are separated on the Celestial Sphere by an angle  $\theta$  given by

$$\cos \theta = \cos \delta_1 \cos \delta_2 \cos(\alpha_1 - \alpha_2) + \sin \delta_1 \sin \delta_2 \quad (1)$$

as can be shown by taking the vector scalar product  $(\mathbf{r}_1 \cdot \mathbf{r}_2)$ . Equation (1) does not give an accurate result when the difference between  $\alpha_1$  and  $\alpha_2$  is very small. An expression which is accurate in all cases may be obtained by rewriting Equation (1) as

$$\sin^2(\theta/2) = \cos \delta_1 \cos \delta_2 \sin^2(\Delta\alpha/2) + \sin^2(\Delta\delta/2) \quad (2)$$

with  $\Delta\alpha = (\alpha_1 - \alpha_2)$  and  $\Delta\delta = (\delta_1 - \delta_2)$

In order to measure the angular sizes of the rings due to gravitational lensing of a background source by SDSS J094656.68+100652.8, we need to know how many arcseconds on the sky corresponds to the distance between the centres of two pixels. The so called ‘‘plate scale’’ is determined from the known positions of other objects in the field. Use Equation (2) above to determine the angular separation of enough pairs of objects to satisfy yourself that you know how the plate scale varies across the field. Note that because a tiny part of the Celestial Sphere has been projected on to a planar charge coupled device (CCD), a small variation in plate-scale across the field is to be expected. If the WFPC2 pixel size is  $15\mu$ , what is the focal length of the telescope-camera combination?

## 6.4 Point Spread Function

It is difficult to classify objects in an astronomical image. The resolution is always less than desired as is the signal-to-noise (S/N). Select the ten brightest objects in the field, excluding the lens and associated rings, and plot cross-sections through them in two orthogonal directions. Stars should appear as Gaussians with the same full width at half maximum (FWHM) irrespective of the cross-section chosen. Asymmetric objects should be regarded as possible galaxies. Note also that cosmic rays can be detected in object pixels and consequently missed during the removal process; these may be identified from cross-sections which show a small number of pixels having a significantly stronger signal than those surrounding them.

Material submitted for marking should include a plot of the image with objects classified marked with some form of identification. From stars identified in the field, deduce the FWHM of the point spread function for the WFPC2 and HST combination. How does the derived point spread function compare with that expected for a diffraction limited 2.4 metre telescope observing at  $6000\text{\AA}$ ?

## 6.5 Dark Matter Content of the Lensing Luminous Red Galaxy

Note that the rings produced by SDSS J094656.68+100652.8 are not quite complete and so we do not have perfect alignment; however, for the purpose of this experiment we assume that the alignment is

perfect and the angular radius of a ring is given by

$$\theta_E = \sqrt{\frac{4GM}{c^2} \frac{d_{LS}}{d_L d_S}} \quad (3)$$

where  $G$  is the Gravitational Constant,  $M$  the mass of the lensing LRG,  $c$  the velocity of light,  $d_{LS}$  is the angular diameter distance from the lens to the source,  $d_L$  is the angular diameter distance from the observer to the lens and  $d_S$  the angular diameter distance from the observer to the source. Note that  $d_{LS} \neq d_S - d_L$ ; its derivation is dependent on an adopted cosmological model.

From Peacock (1999 - his equation 3.91)

$$d_A = (1+z)^{-1} R_0 S_k(r)$$

where  $R_0$  is the current scale-length and, assuming the Universe is flat with  $k = 0$ ,  $S_k(r) = r$  where  $r$  is the comoving distance. And so for the lens

$$d_L = (1+z_L)^{-1} R_0 r_L \quad (4)$$

and background source

$$d_S = (1+z_S)^{-1} R_0 r_S \quad (5)$$

where  $z_L$ ,  $z_S$ ,  $r_L$  and  $r_S$  are respectively the lens redshift, the source redshift, the lens comoving distance and source comoving distance respectively. Peacock (1999) further shows (his equation 4.4) that the angular diameter distance of the source as measured by an observer in the lensing LRG is

$$d_{LS} = R_0 S_k(r_S - r_L) / (1+z_S)$$

and so since  $k = 0$  when we assume the Universe is flat

$$d_{LS} = R_0 (r_S - r_L) / (1+z_S).$$

Eliminating  $R_0 r_S$  and  $R_0 r_L$  using Equations (4) and (5) gives

$$d_{LS} = d_S - d_L (1+z_L) / (1+z_S). \quad (6)$$

Make several measurements of the inner ring angular radius and deduce the corresponding standard deviation in the mean. With estimates of  $d_L$  and  $d_S$  already obtained,  $d_{LS}$  from Equation (6), proceed to estimate the mass of lensing LRG and the standard deviation in this quantity using Equation (3). In a similar way, make measurements of the outer ring and deduce the corresponding quantities. Compare lens masses obtained from both the inner and outer rings and attempt to account for the difference. Note that Barber *et al.* (2007) find the stellar mass of LRGs to be in a narrow range near  $\sim 3 \times 10^{11} M_\odot$ .

## 6.6 Mass to Light Ratio

The flux density received from SDSS J094656.68+100652.8 has not been measured at all wavelengths; from the SDSS spectrum it can be seen that there are no data for  $\lambda \leq 3700\text{\AA}$  and  $\lambda \geq 9200\text{\AA}$ . A crude estimate for the non-observed part has been provided; it was obtained assuming the lensing LRG is not too different from post-starburst galaxies which Kriek *et al.* (2010) consider. The model by Kriek *et al.* (their figure 4 – left hand panel –  $\Delta T = 0$ ) for the unobserved wavelength regions has been scaled to match the observed spectrum.

Examine the resulting energy distribution using



```

$ dipso
> restore SPECTRUM.STK
> dev /xw
> xr 1000. 63000.
> yr 0. 15.
> pm 1
> nb
> crot
> mark
> pm 2
> pm 3

```

and if satisfied, proceed to estimate the bolometric flux received from the LRG lens at the top of the Earth's atmosphere. Integrations are carried out for the three parts of the spectrum separately by continuing the above sequence

```

> pop 1, integrate
> pop 2, integrate
> pop 3, integrate
> q

```

the bolometric flux ( $S$ ) being the sum of the three results. Note that ( $S$ ) is in units of  $10^{-17}$  ergs/cm<sup>2</sup>/sec.

Bolometric flux observed at the top of the Earth's atmosphere is related to luminosity ( $L$ ) by

$$S = \frac{L}{4\pi D_L^2}$$

where  $D_L$  is the LRG lens luminosity distance already determined. Proceed to deduce the luminosity and mass-to-light ratio, expressing the result in solar units. If the derived mass-to-light ratio were attributable to Main Sequence stars, for which

$$\frac{L}{L_\odot} = \left( \frac{M}{M_\odot} \right)^{3.3}$$

may be assumed, what would be their mass?

Peacock (1999 - his equation 3.46) gives

$$t(z) = H(z)^{-1} \left[ 1 + \frac{1}{2} \Omega(z)^{0.6} \right]^{-1}$$

as a good approximation to the time-redshift relation where

$$H(z) = H_0(1+z)\sqrt{(1+\Omega z)}$$

and

$$\Omega(z) = \frac{\Omega(z+1)}{1+\Omega z}.$$

Assume the Universe is flat with  $\Omega(z) = 1$  for all  $z$ ; estimate the age of the Universe which corresponds to the LRG redshift; in view of this, can the mass-to-light ratio be attributed to a Main Sequence stellar population, and if not how may it be explained?

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