main sequence turn-off age for globular clusters

AS32

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1 Introduction

Much of twentieth century cosmology has involved a tension between the so-called Hubble age of the Universe, based on the Hubble expansion, and the age of individual objects within our own galaxy. Of special interest are globular clusters as they have low metallicity and are therefore believed to be among the oldest objects in our galaxy; they are compact groups of between 10^5 and 10^6 stars with dynamical collapse times of less than 10^6 years and are thought to have coalesced out of a primordial gas cloud which later collapsed, dissipating its energy and settling into the disk of our Milky Way Galaxy. For further information, see Krauss & Chaboyer (2003).

The purpose of this experiment is to determine the distance, reddening, metallicity and age of a globular cluster as a check on results already available in the literature. From globular cluster ages, constraints can be placed on the elapsed time between the Big Bang and their formation; this in turn helps to establish an understanding of how hierarchical structure formed. In carrying out the experiment, the most important thing is to appreciate how the observed colour-magnitude diagram of a globular cluster is dependent on its metallicity. The book by Clayton (1983) is an excellent introduction to stellar evolution and is recommended reading.

Globular cluster ages, metallicities, distances and reddenings are determined using an interactive program `clustage` which overlays theoretical isochrones, once these are adjusted for estimates of the globular cluster distance and reddening, on the observed (V as a function of $(B-V)$) colour-magnitude diagram. Theoretical isochrones are obtained using `yymix2` which is an interpolator provided by Yi *et al.* (2001). It is of course necessary to use `yymix2` and `clustage` together. At the time of writing, only data for M 13 and M 15 are available.

The second part of the experiment is to compute an evolution track for a star of the appropriate mass and metallicity using Morel's (1997) code `cesam`, which for the purpose of the experiment described here has been renamed `etoile`. By noting the age of the star at the Main Sequence Turn-Off and comparing this with the age determined in the first part, an independent check on the age determined by isochrone fitting is obtained. Interpretation of results, as described above, forms the last part of the experiment.

2 Initial setup

Begin by creating a `xterm` window in which commands can be typed. The window is created using your mouse to click on the large "X" in the middle of the dock at the bottom of your screen. The large "X" may bob up and down a bit before the window appears.

The steps detailed below provide the setup necessary for experiments carried out in the Astrophysics Laboratory. The `.profile`, `.bashrc`, `.cups/lpoptions` and `.Xdefaults` files in your home directory are changed as a result of carrying out steps listed below; if these files have been previously setup to carry out an experiment in another laboratory, please consult a demonstrator before proceeding. If you have already carried out an experiment in the Astrophysics Laboratory and the `.profile`, `.bashrc`, `.cups/lpoptions` and `.Xdefaults` files in your home directory have remained unchanged, skip the steps below and move on to the next section.

Provide yourself with the necessary `.profile`, `.bashrc`, `.cups/lpoptions` and `.Xdefaults` files by executing the commands

```
$ cp ~aelg/astro-lab/profile    .profile
$ cp ~aelg/astro-lab/bashrc     .bashrc
$ cp ~aelg/astro-lab/Xdefaults  .Xdefaults
$ cp ~aelg/astro-lab/lpoptions  .cups/lpoptions
```

where the leading "`$`" represents the operating system command line prompt which by default is some long string. Once the above commands have been typed, logout and login again; this causes the `.profile`, `.bashrc` and `.Xdefaults` files to be executed, providing the setup necessary to conduct experiments in the Astrophysics Laboratory. Create a new `xterm` window, as described above, in which commands can be typed; the first obvious sign of the new setup is the replacement of the default operating system command line prompt with a simple "`$ `".

3 Getting started

All computer commands reproduced below for your guidance are given in typewriter font. The leading "`$ `" represents the command line prompt. A leading "`> `" represents a prompt provided by an executing program such as `dipso`. Typewriter font is also used to specify file names, file contents and environment variables. Further necessary information is provided in the Third Year Astrophysics Laboratory Practicals page

```
http://www-astro.physics.ox.ac.uk/~aelg/Third_Year_Laboratory
```

which can be read with a browser such as `safari`.

The first step is to create a new directory in which all files, associated with the experiment described here, will be stored. Issue a command of the form

```
$ mkdir as32
```

to create such directory. Then type

```
$ cd as32
```

to make the new directory your current working directory.

4 Theoretical isochrones

Copies of input data files required by the isochrone interpolator `yymix2` are needed in your working directory, since it will be necessary to change them in accordance with your own requirements. There is a `namelist` input file `YY.nml` which controls `yymix2` and a list of ages `YY.age` to which isochrones are interpolated; they are copied with the commands

```
$ cp ~aelg/globular_clusters/isochrones/YY.nml YY.nml
$ cp ~aelg/globular_clusters/isochrones/YY.age YY.age
```

and it is also worth copying the `README_YYiso_v2` file using an analogous command. Ages in `YY.age` are given in Gyr and there are too many ages specified in the default file to be of much practical use. Ages between 11 and 18 Gyrs should be selected, in steps of 0.5 Gyrs and `YY.age` modified accordingly.

In astronomy a metal is any chemical element other than hydrogen or helium and it is customary to identify metallicity with the iron abundance specified as [Fe/H], which is taken to mean the base-10 logarithm of the iron abundance, as a number fraction of the hydrogen abundance, relative to the solar value. For example, $[\mathrm{Fe/H}] = -1.00$ implies a metallicity of one tenth of the solar metallicity. An alternative is to specify the metal mass-fraction, usually denoted by Z, which could be used interchangeably with [Fe/H] if metal proportions in the atmospheres and envelopes of all stars were the same. In fact metal-poor stars usually have enhanced abundances of "alpha elements" relative to the Sun, expressed logarithmically as $[\alpha/\mathrm{Fe}]$; see the paper by Kim *et al.* (2002), and their Figure 1 in particular, for further details.

Metallicity of interpolated isochrones is specified in `YY.nml` by `AFe`, `targetZ` and `FeH` representing $[\alpha/\mathrm{Fe}]$, Z and [Fe/H] respectively. It is suggested isochrones initially be interpolated assuming no enhancement in abundances of "alpha elements" and adjustments to this be made only as a refinement once a good fit has been obtained; it would then be appropriate to set `AFe=0` in the copied version of `YY.nml` and leave `targetZ` as a small negative number for the time being, so that metallicity is specified completely by the value of [Fe/H]. Retaining `Age=0` in `YY.nml` ensures the reading of ages for which isochrones are required from the file `YY.age`.

Proper selection of the output file name for isochrones is most important since `clustage` is written to make make use of information decoded from the filename. Specifically, the first two characters must be "`yy`" and the fifth character must be "`l`" (that is a lower case "L") or "`g`" depending on whether colour transformations in isochrones upon which the interpolation is based are from Lejeune *et al.* (1998) or Green *et al.* (1987) respectively. In the given `YY.nml` file, the output file is specified as `yy00g.fm15a03o2` implying $[\mathrm{Fe/H}] = -1.5$ and $[\alpha/\mathrm{Fe}] = 0.3$ (since $10^{0.3} \simeq 2$); the colour transformation is according to Green *et al.* (1987).

After preparing the input files `YY.nml` and `YY.age` there is a final step before the isochrone interpolator `yymix2` can be run. The program `yymix2` expects files on which the interpolation is based to reside in a directory `Iso`, located in the current working directory from where it will be run; rather than having a real subdirectory `Iso`, it is better to create a subdirectory `Iso` which is a soft link to the real directory using the command

```
$ ln -s ~aelg/globular_clusters/isochrones/Iso
```

and it can then be seen that there is what appears to be a subdirectory `Iso` (having the required files) in the current working directory. Note that the soft link, once created, remains (even if you log out) until it is explicitly deleted with a

```
$ rm Iso
```

command.

Isochrone interpolation can now be carried out by simply typing

```
$ yymix2
```

which will create a single file in the current directory having isochrones of the specified metallicity and "alpha element" abundance enhancement factor, for each age specified in `YY.age`. Note that it is important to ensure the file you intend to create does not already exist in the working directory; if it does `yymix2` will crash, giving you a core dump, which is fully intended as it is not good practice to overwrite isochrone files already computed without explicitly deleting them first. Core dump files are generally large and, as they are not needed, should be deleted as they are created.

5 Globular cluster ages

Globular cluster ages are determined using the interactive program `clustage` which plots Johnson V magnitudes for globular cluster stars as a function of their colour, in this case expressed as $(B - V)$. Data are taken from Piotto *et al.* (2002) and in order for `clustage` to find the data it is necessary to define environment variables using the commands

```
$ DIRECTORY=~aelg/globular_clusters/data/M_13/ ; export DIRECTORY
$ CMD_V_BMV=n6205.cmd ; export CMD_V_BMV
```

if you have elected to work on M 13; if you have decided to work on M 15 then replace `M_13` with `M_15` and `n6205.cmd` with `n7078.cmd` in the above commands. In order to successfully invoke `clustage` it is necessary to have a set of isochrones in a file called `yy00g.fm15a00o2` in your current working directory; as explained above, the isochrones in this file should be for $[\mathrm{Fe/H}] = -1.5$ and $[\alpha/\mathrm{Fe}] = 0$. Once tasks described in this paragraph are accomplished, `clustage` is invoked by issuing the command

```
$ clustage
```

and the colour-magnitude diagram for the selected globular cluster should be plotted in a separate window with the isochrones overlaid. In making the first plot, isochrones are shifted by a distance modulus (D) of 14.5 magnitudes and an interstellar reddening of $E(B - V) = 0.02$.

Control of `clustage` is affected by means of single key presses, The function of each key is case insensitive; in other words, it does not matter whether upper or lower case is used, the effect is the same. Mouse operation on Machintosh computers is aided using the keyboard. Hold the `ctrl` key down and click the mouse to get a left-hand mouse button click. Similarly, a right-hand mouse button click is achieved by clicking the mouse while holding down the "apple" key (two keys to the right of the `ctrl` key). A description of each key now follows:

- A - ("a") selects the age range of isochrones plotted superimposed on the colour-magnitude diagram. On pressing, a request for a new isochrone age range is presented, with the current range in parentheses; the new range needs to be entered as two floating point numbers (include a decimal point) separated by at least one space. By default all isochrones read in are plotted. By selecting just one isochrone for plotting, it is easier to assess the quality of the fit achieved.
- D - ("d") changes the distance modulus by which isochrones are shifted. On pressing, a request for a new value of the distance modulus is given with the current value shown in parentheses. A value needs to be given in response, even if no change is required, and the format should be the same as that used to specify the current distance modulus.
- E - ("e") changes the interstellar reddening by which isochrones are shifted. On pressing, a request for a new value of the interstellar reddening is given with the current value shown in parentheses. A value needs to be given in response, even if no change is required, and the format should be the same as that used to specify the current interstellar reddening. Note that changing the interstellar reddening shifts the isochrones in both coordinates; the visual magnitude change being given by $A_V = 3.2E(B-V)$.
- I - ("i") reads in a new set of isochrones. On pressing, a request for the name of a file containing a new set of isochrones is made with the name of the current file shown in parentheses. If it is decided to continue with the existing set then simply press return; otherwise give the new file name in exactly the format stipulated in the previous section.
- H - ("h" or "?") prints a single line-description of each key.
- M - ("m") provides a graphics cursor in the form of a rectangle which can be used to mark stars which are to be excluded when calculating "goodness-of-fit" statistics. Any corner of the rectangle is anchored using the left hand mouse button and the rectangle dragged across the plotting surface with the mouse. The left hand mouse button can be used repeatedly to anchor one corner of the rectangle. It is only when the right hand mouse button is used is the final rectangle chosen and the unmarked stars within it marked; the colour magnitude diagram is redrawn with marked stars coloured in blue. If some already marked stars are included in the final rectangle, this is of no consequence as they remain marked.
- Q - ("q") signals that use of `clustage` is to be ended. Termination causes two things to happen. In the first place, if some stars have been marked, a binary file is written to the current directory so the details of marked stars are saved; when `clustage` invoked again, this file is read automatically and avoids the need to mark the same stars again. Secondly, a file called `pgplot.ps` is written to the current directory, overwriting any existing file with the same name; it contains a plot of the colour magnitude diagram and a fitting isochrone for the currently selected distance modulus and interstellar reddening. If an isochrone for a specific age had been previously selected, this is used for generating `pgplot.ps`; otherwise the best fitting isochrone is used.
- R - ("r") simply restores the original plot scale if the "Z/z" command has previously been used to zoom in on a region of the colour magnitude diagram of interest.
- U - ("u") provides a graphics cursor in the form of a rectangle which can be used to unmark stars previously marked with the "M/m" command. Any corner of the rectangle is anchored using the left hand mouse button and the rectangle dragged across the plotting surface with the mouse. The left hand mouse button can be used repeatedly to anchor one corner of the rectangle. It is only when the right hand mouse button is used is the final rectangle chosen and any marked stars within it unmarked; the colour magnitude diagram is redrawn with unmarked stars coloured in white. If some unmarked stars are included in the final rectangle, this is of no consequence as they remain unmarked.

- Z - (“z”) provides a graphics cursor in the form of a rectangle which can be used to select an area of the colour magnitude diagram to be plotted at an expanded scale. Any corner of the rectangle is anchored using the left hand mouse button and the rectangle dragged across the plotting surface with the mouse. The left hand mouse button can be used repeatedly to anchor one corner of the rectangle. It is only when the right hand mouse button is used is the final rectangle chosen and the selected region of the colour magnitude diagram redrawn at an expanded scale.

Care needs to be taken with the middle mouse button if that is pressed by mistake and the mouse has previously been used to cut and paste text; if this button is pressed then all characters in the buffer are queued as key presses and so it is good practice to cut and paste a single space character before `clustage` is invoked. With the exception of the middle mouse button, pressing any key other than those listed above simply causes a request for another key press to be issued.

When `clustage` is successfully started for the first time, a plot appears in which each star is plotted as a white point and isochrones as red lines, shifted according to the default reddening ($E(B-V) = 0.02$) and distance modulus ($D = 14.5$ magnitudes). In the text window, a statistic measuring the “goodness of fit” for each isochrone is given together with the standard error in this quantity. The statistic is calculated for each isochrone by calculating the root-mean-square (rms) distance of each star from every discrete point which represents that isochrone and selecting the minimum, the weighted mean of all such minima for all stars gives a rough measure of “goodness of fit” and this quantity is referred to here as $\bar{s^2}$. In calculating the weighted mean, weights are assigned to each star in proportion to their V-band flux; in other words, a star having $V = 14$ is given 2.5 times the weight of star having $V = 15$.

Before attempting to fit an isochrone to the Main Sequence Turn-Off and Giant Branch in the colour magnitude diagram, it is suggested that trial changes to the distance modulus and reddening be made to get some feel for how the value of $\bar{s^2}$ is affected. Also notice that there are many stars in the colour magnitude diagram for which there is no equivalent theoretical representation in the isochrone, this is particularly important in the case of the Horizontal Branch where the stars are comparatively bright. It is therefore a good idea to begin by marking those stars to be excluded from the fit using the `M` command as described above. Once stars have been marked, the plot is redrawn with marked stars now plotted in blue and the value of $\bar{s^2}$ updated.

Also notice that the Giant Branch has a finite width which is due to stars having slightly different metallicities and proportions of metals. The presence of binary companions is less important on the Giant Branch as these will generally have a much lower luminosity than the red giant. Binary companions will have luminosities comparable with Main Sequence stars and this probably explains why the Main Sequence appears to be considerably wider than the Giant Branch. A contribution to the Giant Branch and Main Sequence widths also comes from uncertainties in the V and $(B-V)$ measurements but these are comparatively small. The astrophysical origin of the Giant Branch and Main Sequence widths, and the calculation of isochrones assuming all stars are single and have the same composition, means that the standard error in $\bar{s^2}$ is not subject to the usual statistical interpretation.

Start with the default isochrone and attempt to establish a reddening and distance modulus which give the best fit as judged by eye when looking at the plot and by attempting to reduce the value of $\bar{s^2}$. The vertical part of the Giant Branch is a good guide when it comes to establishing $E(B-V)$, and the position of the Main Sequence and upper part of the Giant Branch similarly good guides when determining the distance modulus. Material submitted for marking must include a table listing values of $\bar{s^2}$, $E(B-V)$, D, [Fe/H], [α/Fe] and age (of the isochrone giving the best fit) for each set of values tried; the point is that you can then easily return to any previous fit if you wish. Once a satisfactory fit has been obtained with the default isochrone set, leave `clustage`; in doing so your current best fit is written to a file `pgplot.ps` in the current directory and details of marked stars are saved in a file which is automatically read when `clustage` is restarted so that marking of stars does not need to be done again. The current best fit should be printed using the command

```
$ lpr pgplot.ps
```

and annotated with the corresponding values of $E(B-V)$, the distance modulus, [Fe/H], [α/Fe]. the age and the mass of stars turning off the Main Sequence; this last quantity is obtained by inspecting the computed isochrone file with a text editor. The annotated plot needs to be included with material submitted for marking.

The next step is to compute new isochrone sets with different values of [Fe/H], holding [α/Fe] $= 0$ in the first instance to see if better fits can be achieved using the procedure described above; a plot, annotated as described above, and corresponding to the best fit obtained for each [Fe/H] tried needs to be submitted for marking. It is suggested that the process be continued until an approximate [Fe/H] giving the best fit is established.

In practice, it is expected that almost equally good fits (as indicated by very similar local minima in the values of $\bar{s^2}$) will be found for a number of [Fe/H] values on either side of that giving the best fit; these will each correspond to different ages. The age should be plotted as a function of [Fe/H] using a suitable plotting program (DIPSO for example) and included with material submitted for marking. Should you elect to use DIPSO, there is a script for making the required diagram which may need modification depending on your selected values of [Fe/H]; it can be copied into your working directory using

```
$ cp ~aelg/globular_clusters/scripts/C.CMD .
```

where the final "." is an abbreviation meaning the copied file has the same name in the current directory and the assumption made in `C.CMD` is that your [Fe/H] values and ages are written into a two-column file ([Fe/H] on the left) in your current directory called `cov.dat`. In preparing `cov.dat`, it is important to separate the columns with spaces and not tabs. Review the contents of `C.CMD`, making any necessary changes, and then produce a screen plot with

```
$ dipso
>@C
```

and if the plot is acceptable, generate postscript output by editing `C.CMD` and running the script again from within DIPSO. Postscript will then be in a file called `pgplot.ps` in the current directory and the command

```
$ lpr pgplot.ps
```

will send it to the printer. Examine the resulting plot and comment on the dependence of the derived age on assumed [Fe/H]; in particular note how this dependence limits the accuracy with which the age can be determined when [α/Fe] $= 0$ is adopted.

Once the best [Fe/H] has been identified, try [α/Fe] $= 0.3$. Set `AFe` to 0.3 in `YY.nml` and note the metal mass-fraction (Z) in the best fit [α/Fe] $= 0.0$ isochrone file, use this to set the quantity `TargetZ` in the file `YY.nml`; this results, when `yymix2` is run again, in a new set of isochrones with [Fe/H] adjusted so the best-fitting metallicity is retained. Further guidance on selecting [Fe/H] and [α/Fe] for a particular Z is given by Kim *et al.* (2002) in their table 2. Run `clustage` as before to find the age and mass of stars turning off the Main Sequence. In all probability, further isochrone sets with different [Fe/H] and [α/Fe] will be needed in order to obtain the best fit in the sense of $\bar{s^2}$ being minimised.

Carefully examine the plot resulting from the fit which gives the global minimum $\bar{s^2}$. In particular, note whether the Main Sequence is well-fitted at the expense of having a good fit to the Giant Branch; this would be inappropriate because the Main Sequence is almost certainly contaminated by the presence of companions in binary stars which make a contribution to the observed V and $(B-V)$ but which is not accounted for in the computed isochrones. The Giant Branch is contaminated in a similar way but the increased luminosity of red giants makes this less important.

Be particularly suspicious of your "best-fit" if the derived age is surprisingly large when compared to the 13.7 ± 0.2 Gyr age of the Universe obtained by Spergel *et al.* (2003). If you have a good fit to the Main Sequence and a not so good fit to the Giant Branch, with a derived age which appears to be surprisingly high, decrease [Fe/H] and (possibly) increase $[\alpha/\mathrm{Fe}]$ to see if a better fit to the Giant Branch can be obtained for a lower age.

Once optimum values of [Fe/H] and $[\alpha/\mathrm{Fe}]$ are determined, calculate a new set of isochrones using these parameters and having a smaller age range and age spacing but centred about the age that gives the adopted fit; this will enable the turnoff age to be refined. Looking back through your table of $\bar{s^2}$, $E(B-V)$, D, [Fe/H], $[\alpha/\mathrm{Fe}]$ it will be possible to assign errors to each quantity; an appropriate value of the error is that perturbation which has no significant affect on the fit or $\bar{s^2}$.

It is important to compare results obtained with those already in the literature and comment on the comparison. Harris (1996) publishes a database of parameters for globular clusters and Salaris & Weiss (2002) present homogeneous age determinations for fifty-five Galactic globular clusters; data for M 13 and M 15 can be found in both papers. If you have chosen to work on M 13, look also at the paper by Sneden *et al.* (2004); they find a spread in metallicities among M 13 giants ranging from $-1.77 \leq [\mathrm{Fe/H}] \leq -1.4$ and relative abundances of metals which only approximately follows the pattern depicted in Figure 1 of Kim *et al.* (2002). Comment on how the Sneden *et al.* results, if correct, would have compromised your determination of age and metallicity for M 13. Should you have decided to work on M 15, then look at the paper by McNamara *et al.* (2004) who determine a dynamical distance to M 15 and use this to determine its age; how well do your results compare with those in this paper?

6 Stellar evolution calculation

The idea behind carrying out a stellar evolution calculation is to verify the age of the globular cluster using an independently written code; that is, a code which is not the stellar evolution code used to compute the isochrones. A second reason is that the code `cesam` selected for carrying out the evolution calculation has an elegant graphical display which enables you to watch what is going and hopefully learn something in the process. The calculation is made for a star whose mass is such that at the age of the globular cluster it is just turning off the Main Sequence; that mass is identified by looking at the file containing the isochrone which gave the best fit using an editor, noting which one corresponds to the bluest colour. When stars turn off the Main Sequence, in the isochrone sets used for this experiment, they are as blue as they are going to get.

A lot of files are needed and created by `cesam` and so it is suggested that a new working directory, different from the one used for `yymix2` and `clustage` runs, be created. Make the new directory your default working directory and copy the files you need using the commands

```
$ mkdir cesam
$ cd cesam
$ cp ~aelg/globular_clusters/cesam/* .
```

and at the same time note that a substantial fraction of your disk space allocation is now used up so it is important to remember to delete these files once you are finished with them.

Parameters controlling the `cesam` run are in the file `evol_inp.don`; the only ones that need be changed are `MTOT`, `X0` and `Y0` which represent the total stellar mass, hydrogen mass fraction and helium mass fraction respectively. The metal mass fraction `ZSX0` is set to 0.0; in this case `cesam` ensures that the metal mass fraction is determined by subtracting the sum of the hydrogen and helium mass fractions from unity and so the required metal mass fraction is achieved by adjusting the hydrogen mass fraction. Obtain required values for `MTOT`. `X0` and `Y0` from the isochrone adopted as representing the observed colour magnitude diagram; as explained above, this is not necessarily the one giving the best fit.

To start `cesam` running, issue the command

```
$ etoile
```

and you will be asked (in French) whether you wish to continue an existing evolution calculation, initialise a Zero Age Main Sequence model or initialise a pre-main sequence model; you need the last option so type `3` and press return. Then you are asked whether the pre-main sequence model is in binary; it is not so type `n` for "non" and press return. In response to the request for the name of the ASCII file containing the pre-main sequence model, type `pms.dat` and press return. The next piece of information sought is the contraction constant used to determine how pre-main sequence evolution is to proceed; it is mass-dependent and determines among other things the radius of the initial gas cloud from which the star is to form. Stars turning-off the Main Sequence when their ages are the same as that of a typical globular cluster, have a mass of about 0.7 $M_\odot$ and for these an appropriate contraction constant is `1.8D-2`; enter this value and press return. The penultimate request is for an identification of the model; since the input data are in `evol_inp.don` the identification is `evol_inp`, type this and press return. Finally you are asked whether plots are to be presented during the run; you are strongly recommended to respond with `o`, for "oui" and press return. Two or three further questions merely check whether everything is "ok"; if so, respond with `o` and press return in each case. Output in the text screen can be ignored; the important bits will be stored in `evol_inp.lis`.

Of particular interest are the plots which show how stellar evolution is progressing; all four are continually updated. In the top left hand corner there is a plot showing the variation of temperature and pressure with distance from the centre of the star expressed in solar radii; the same plot shows the cumulative contribution of each part of the star to the total mass and luminosity. The top right hand plot shows which parts of the star are convective. In the bottom left hand corner there is a plot showing the variation in abundance of various species with position in the star; this serves as a fascinating "window" on the nuclear reactions taking place. The bottom right hand corner shows a plot of the star's progress in the Hertzsprung-Russell (HR) Diagram as it evolves.

There are supplementary programs `plot_ZCm`, `plot_ZCr`, `plot_abund_m`, `plot_abund_r`, `plot_hr` and `plot_osc` which assist in the presentation of the results of the stellar evolution calculation once it is complete (answer "o" to the final question). Simply type the name of the program required as listed above and supply the input file name as requested; this will be `evol_inp` if your cesam input file was `evol_inp.don`. It is also necessary to supply the graphics device name, select `/xs` initially to get a screen plot. When satisfied select `/ps` for a monochrome plot, or `/cps` for a colour plot, in a postscript file which can then be sent to a printer in the usual way; they should be previewed first using the "`open`" command if working in the Laboratory and "`gv`" if working remotely. Access to a colour printer can be provided on request.

In the material presented for marking, include a description of the essential processes taking place as discerned from watching the four plots change as the evolution calculation proceeds. For example, note the collapse of the gas cloud and what happens to the central temperature. Note also the points on the HR Diagram at which deuterium and hydrogen burning begin and how abundances of ^3He and ^4He vary with time and position in the star; how does the star move in the HR Diagram during these two critical burning phases? How does the star move in the HR Diagram once all the hydrogen in the core has been converted into helium and where does burning then take place? Comment also on the different stages at which the envelope and core are convective.

Once the `cesam` run is complete, find the age of the star at the Main Sequence Turn-Off for comparison with the result obtained by isochrone fitting. The command

```
$ grep "age" evol_inp.lis | more
```

will extract the relevant information from the output file and present to you a screen full at a time. Again the trick is to identify the age at which the star is as blue as it is going to get, at least in the `cesam` calculation carried out here.

7 Interpretation

The most important part of any experiment is the interpretation of results; there is little point in determining the age of a globular cluster unless some advantage is gained by doing so. To begin with, compare the ages derived from the stellar evolution calculation and isochrone fitting. Then look at papers describing `cesam` (Morel 1997) and the isochrone calculation (Yi *et al.* 2001) to see if there are clues there as to why the two results should be the same or different as the case may be.

The final step is to look at the papers by Krauss & Chaboyer (2003) and Krauss (2003). Krauss & Chaboyer (2003) in their Figure 3 give an age distribution of the oldest 10000 globular clusters; how many standard deviations away from their mean age is your age determination? Krauss (2003) then uses the results of Krauss & Chaboyer (2003) to estimate an upper limit to the age of the Universe (that is how long after the Big Bang) when the oldest globular clusters were formed. Explain in the material presented for marking how Krauss estimates his upper limit for the age of the Universe at the epoch of oldest globular cluster formation and whether your age determination is consistent with this estimate. Why is it so important to know the age of the Universe at the time when the oldest globular clusters formed?

REFERENCES

Clayton DD, 1983, Principles of Stellar Evolution and Nucleosynthesis, University of Chicago Press.

Green EM, Demarque P, King CR, 1987, The Revised Yale Isochrones and Luminosity Functions (New Haven: Yale University Observatory)

Harris WE, 1996, AJ 112, 1487

Kim Y-C, Demarque P, Yi SK, Alexander DR, 2002, ApJS 143, 499

Krauss LM, 2003, ApJ 596, L1

Krauss LM, Chaboyer B, 2003, Science 299, 65

Lejeune Th, Cuisinier F, Buser R, 1998, AA 130, 65

McNamara BJ, Harrison TE, Baumgardt H, 2004, ApJ 602, 264

Morel P, 1997, AAS 124, 597

Piotto G, King LR, Djorgovski SG, Sosin C, Zoccali M, Saviane I, De Angeli F. Riella M, Recio Blanco A, Rich RM, Meylan G, Renzini A, 2002 AA 391, 945

Salaris M, Weiss A, 2002, AA 388, 492

Sneden C, Kraft RP, Guhathakurta P, Peterson RC, Fulbright JP, 2004, AJ 127, 2162

Spergel DN, Verde L, Peiris HV, Komatsu E, Nolta MR, Bennett CL, Halpern M, Hinshaw G, Jarosik N, Kogut A, Limon M, Meyer SS, Page L, Tucker GS, Weiland JL, Wollack E, Wright EL, 2003 ApJS 148, 175

Yi SK, Demarque P, Kim Y-C, Lee Y-W, Ree CH, Lejeune Th & Barnes S, 2001, ApJS 136, 417