Oxford Physics: Part C Major Option Astrophysics

High-Energy Astrophysics

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Today's lecture

- Power-law distribution of high-energy particles from space.
- Second- and first-order Fermi acceleration.
- Fermi acceleration by shock crossing to produce a power-law distribution of particle energies.

Recap of shock results

$$\boxed{\frac{\rho_d}{\rho_u} = 4; \frac{v_u}{v_d} = 4}$$

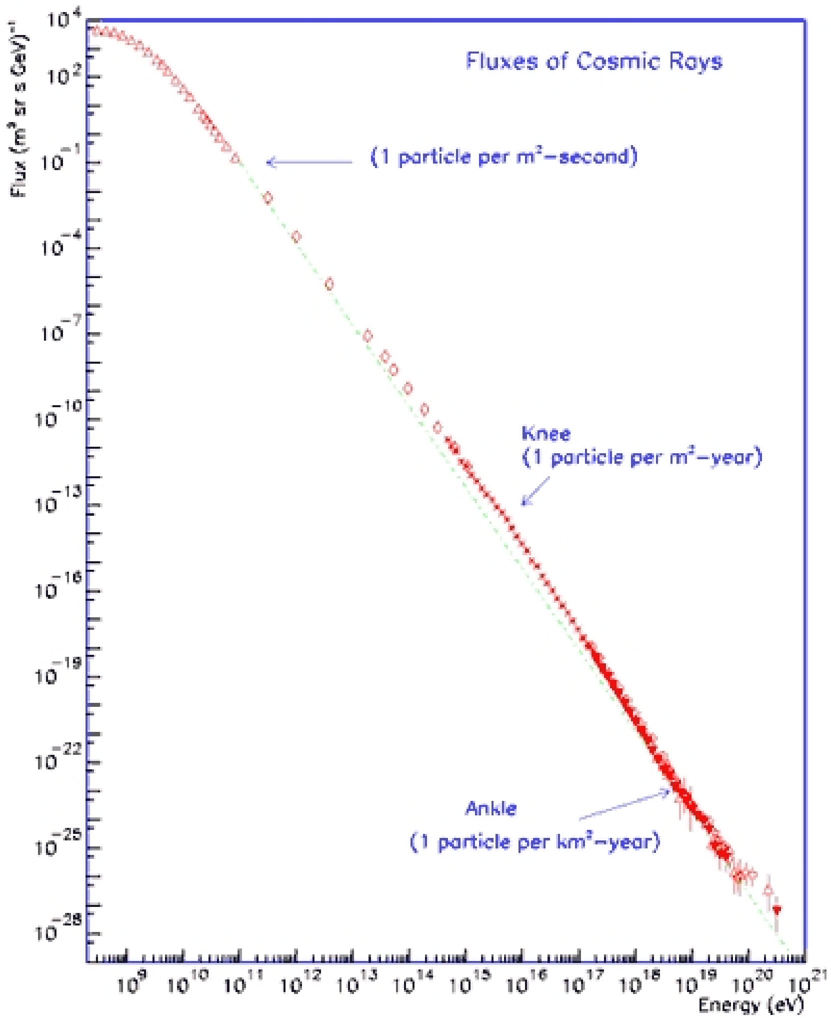
$$\boxed{T_d = \frac{3mv_u^2}{16k_\mathrm{B}}}$$

How to obtain a power-law energy distribution

We are now in a position to investigate mechanisms for producing power-law distributions of particle energies, i.e. those of the form

$$N(E) \propto E^{\alpha}$$

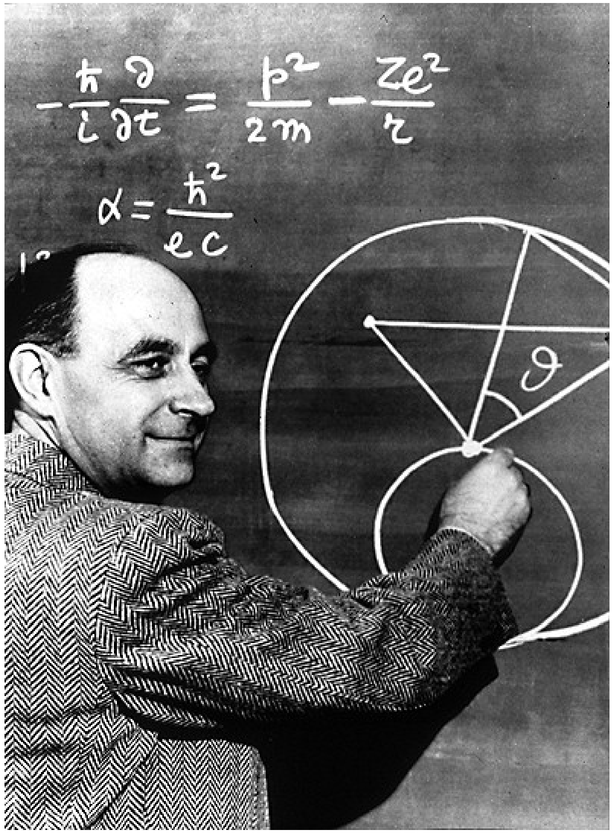
Where α is some constant. Such distributions are sometimes referred to as *non-thermal* because they vary significantly from a Maxwellian distribution.



Spectrum of cosmic rays (mostly protons). Maxiumim collision energy at LHC is 14 TeV.

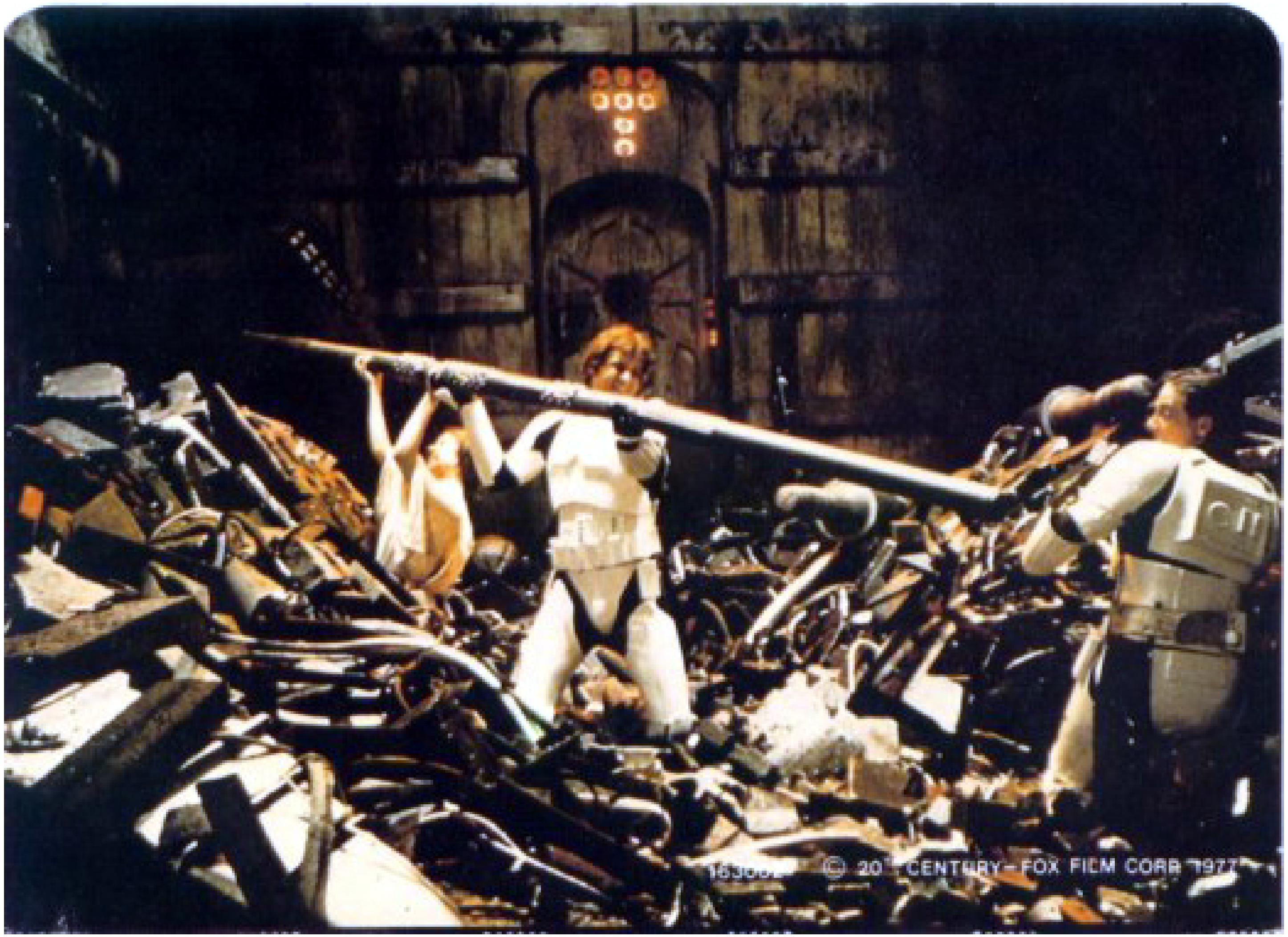
A mechanism by which particles obtain such an energy distribution was originally investigated in 1949 by Fermi as a means of explaining the highest-energy cosmic rays, and in the 1970’s it was demonstrated to be to be an efficient mechanism for the production of synchrotron-emitting electrons in AGN (which will be the main topic of the next few lectures).

Enrico Fermi



Never underestimate the joy people derive from hearing something they already know.

Fermi’s “ping-pong” acceleration process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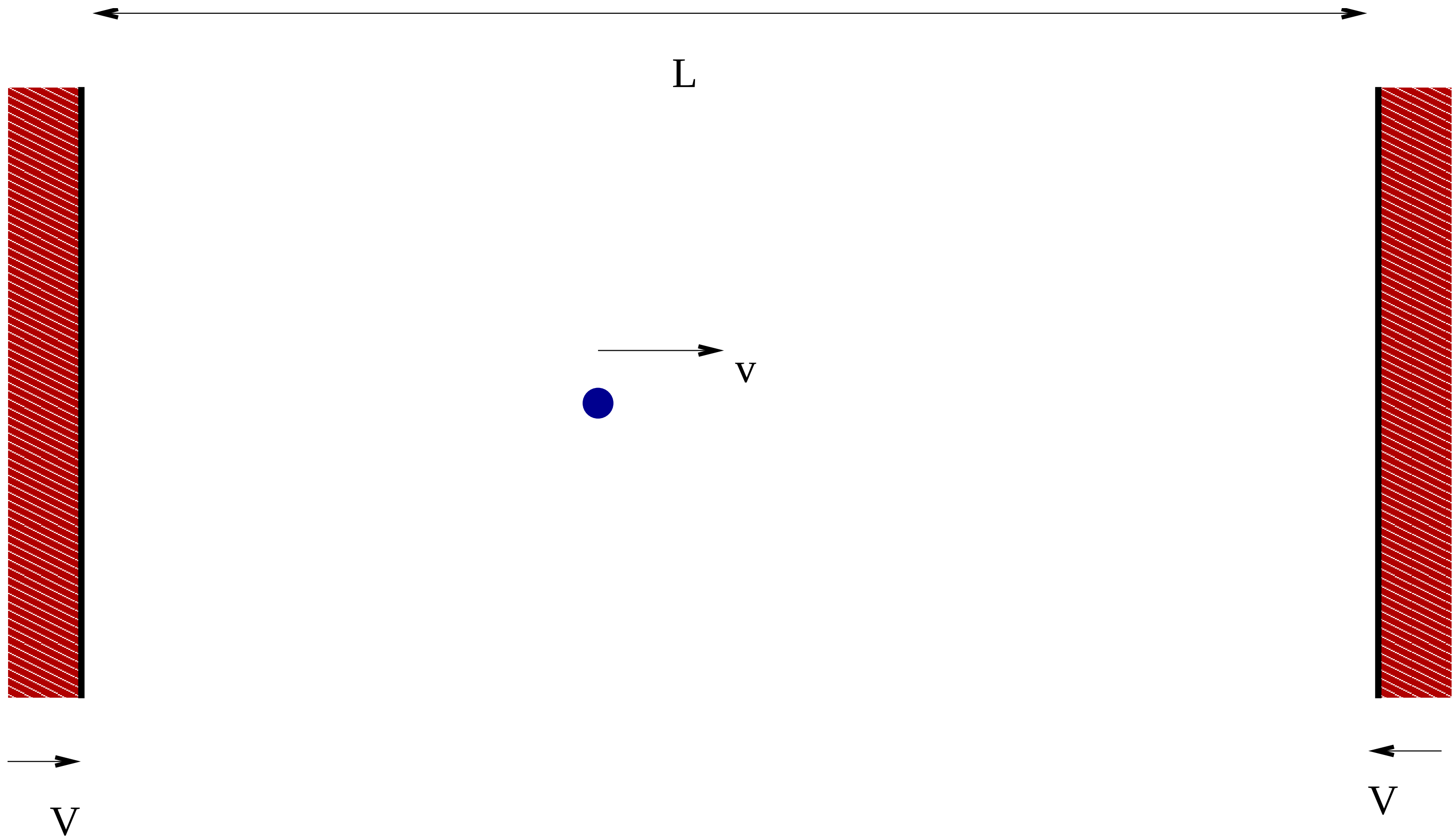
Fermi acceleration: simple first-order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v
V
V

Suppose we inject a particle of mass m is travelling at a *mildly* relativistic speed v between two scattering surfaces separated by distance L and approaching each other with speed $V << c$. If the particle collides alternately head-on with each scatterer, it will gain energy at a rate

$$\frac{dE}{dt} = \text{Rate of collisions} \times \text{energy change per collision}$$

Let us use the fact that the particle is relativistic to our advantage: the momentum increase at each collision is $\gamma m V$ so we can calculate the energy increase via $E = pc$.

$$\begin{aligned}\frac{dE}{dt} &\approx \frac{v}{L} \times \gamma m V c \\ &\approx \frac{\gamma m c^2 V}{L} \\ &\approx \frac{EV}{L}\end{aligned}$$

Where in the last step we are also approximating $v = c$.

Thus we have a situation where a particle gains energy as

$$\frac{dE}{dt} = \frac{E}{\tau}$$

where τ is some timescale, in this case the crossing time between scatterers: the particle's energy will increase exponentially. If a population of particles with slightly different initial energies enter a region where this sort of scattering occurs, the slightly-higher energy particles will be accelerated to much greater energies than the the slightly-lower-energy ones.

This situation is a gross idealisation but it shows how to proceed. If we can find a mechanism for multiple back-and-forth accelerations, we can get a power-law energy distribution. The most significant caveat is that we don't know how to make the particles mildly relativistic to start with, i.e. in order to make use of $E = pc$.

Real Fermi acceleration at shocks

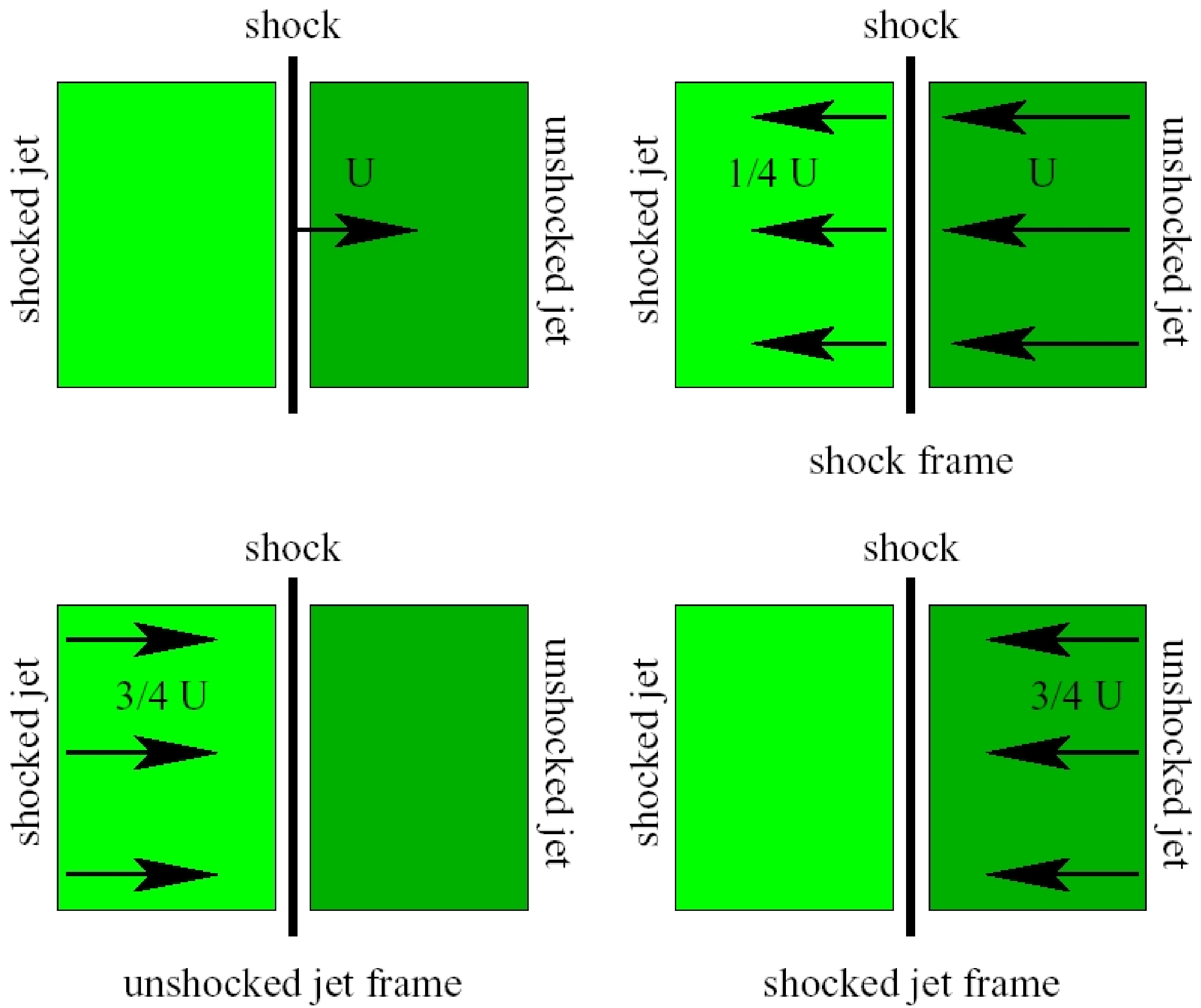
The way to do this in practice is to make use of shocks: in the jets and at the hotspots of radiosources, and at the blast-wave shock in a supernova.

Take the case of a shock propagating into “cold” gas at a speed v_u. In the frame of the shock, we have our familiar results: we see unshocked gas ahead of us approaching at speed v_u, and hot shocked gas streaming away behind us at $v_d = \frac{1}{4}v_u$

Now consider electrons initially at rest in the unshocked gas frame. They see the shock approaching at v_u but they also see the hot shocked gas approaching at $\frac{3}{4}v_u$. As they cross the shock they are accelerated to a mean speed of $\frac{3}{4}v_u$, as viewed from the frame of the unshocked gas, and are also thermalised to a high temperature.

The clever part is next: consider what would happen if, say as a result of its thermal motion, or tangled magnetic field, an electron is carried back over the shock front. With respect to the frame it has just come from—the shocked gas frame—it is *once again accelerated by* $\frac{3}{4}v_u$.

So we have a system of symmetric head-on collisions that we can use to generate a power-law energy distribution.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Much of the pioneering work in calulating the details of first-order Fermi acceleration in astrophysical situation was done by our very own Prof Tony Bell *(although like many of us in astro he was a Tab at the time...)*

Fermi acceleration at shocks: calculation

Now we will demonstrate explicitly that this gives a power-law in electron energies.

Let us say the fractional change in kinetic energy at each crossing is β. After n crossings, a particle with initial energy E_0 will have energy $E = E_0\beta^n$.

The particles will not continue crossing the shock indefinitely: the net momentum flux of the shocked gas downstream will carry them away in due course. So let us call the probability of of remaining in the shock-crossing region after each crossing P. Then after n crossings there will be $N = N_0 P^n$ of the original N_0 electrons left.

We can eliminate n from these expressions to find:

$$\frac{\log(N/N_0)}{\log(E/E_0)} = \frac{\log P}{\log \beta}$$

Which gives:

$$\frac{N}{N_0} = \left(\frac{E}{E_0}\right)^{\frac{\log P}{\log \beta}}$$

If we take the differential form of this we get

$$N(E)dE = \text{constants} \times E^{\left(-1+\frac{\log P}{\log \beta}\right)}$$

So we have recovered our power-law energy distribution with its form

$$N(E)dE \propto E^{-k}dE$$

Fermi acceleration at shocks: the power-law index

The detailed derivation of the power-law index is not examinable.

The result, which you *are* required to remember, is that it can be shown that

$$\frac{\ln P}{\ln \beta} = -1$$

Therefore the power-law index is -2, i.e.

$$\boxed{N(E)\mathrm{d}E \propto E^{-2}\mathrm{d}E}$$

Which is close to - but not quite - the spectral index observed in cosmic rays, and as we will see over the course of the next few lectures it closely underlies the spectrum of synchrotron radiation in AGN.

Much work is still done to build models in which the index is slightly steeper than 2, as is observed in the real cosmic ray energy spectrum.

Supplementary

Fermi acceleration at shocks: derivation of the power-law index ***non-examinable***

(see Longair Vol 2 pp 354–355 for full details)

First consider the particles crossing from the upstream to the downstream side of the shock. As in our cartoon example we will take particles which are at least mildly relativistic, and we will use a non-relativistic shock speed U. Using our jump conditions, we find that a particle upstream of the shock sees the shocked gas coming towards it at speed $V = \frac{3}{4}U$.

We calculate the energy of the particle when it crosses into the downstream region via the Lorentz transform:

$$E' = \gamma_V(E + p_x V)$$

Where p_x is the component of momentum perpendicular to the shock. Now $\gamma_V \approx 1$ and $p_x = (E/c)\cos\theta$, so for each particle

$$\frac{\Delta E}{E} = \frac{V}{c}\cos\theta$$

Now we need to use a standard result from kinetic theory. The number of particles crossing the shock between θ and $\theta + \mathrm{d}\theta$ is proportional to $\sin\theta$, and the rate at which they approach the shock is proportional to the x component of their velocities, $c\cos\theta$.

So to normalise our energy gain per shock crossing we need to integrate over

$$p(\theta) = 2\mathrm{sin}\theta\mathrm{cos}\theta\mathrm{d}\theta$$

Giving an average energy gain per crossing of $\frac{2}{3}\frac{V}{c}$. A round trip produces twice this, so the fractional energy gain per round trip is

$$\beta = \frac{E}{E_0} = 1 + \frac{4V}{3c}$$

Using $V << c$ we get

$$\ln\beta = \ln(1 + \frac{4V}{3c}) \approx \frac{4V}{3c} = \frac{U}{c}$$

Next we need to calculate P; this is simpler, because we can see that the rate at which particles are being swept away from the shock is $\frac{U}{c}$ per crossing. So

$$P = 1 - \frac{U}{c}$$

Again, take $U << c$, and we have

$$\ln P \approx -\frac{U}{c}$$

So we have our result:

$$\frac{\ln P}{\ln \beta} = -1$$

Therefore the power-law index is -2, i.e.

$$\boxed{N(E)\mathrm{d}E \propto E^{-2}\mathrm{d}E}$$