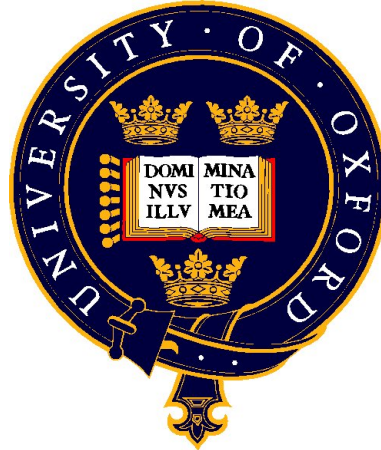


Oxford Physics: Part C Major Option Astrophysics



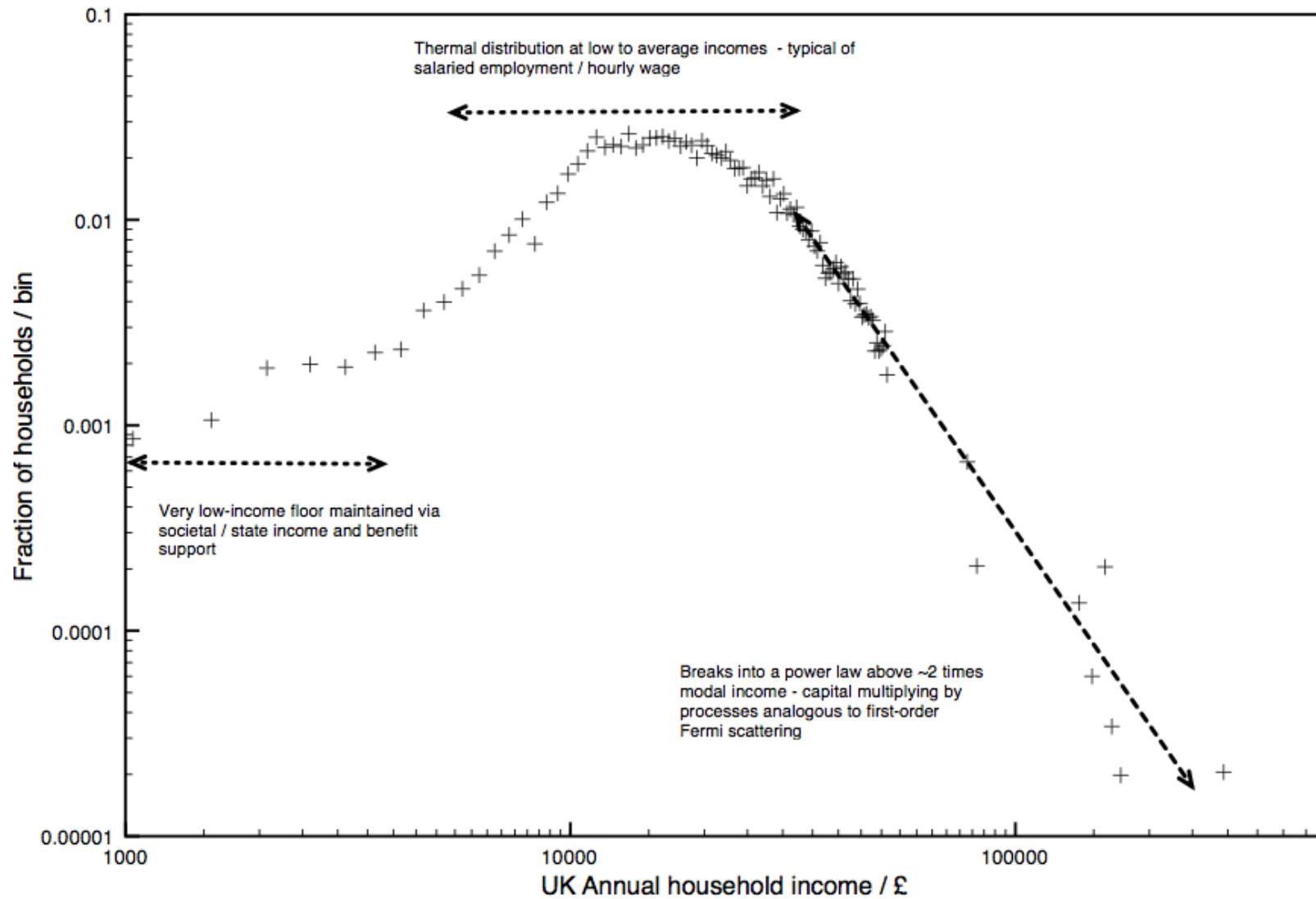
High-Energy Astrophysics

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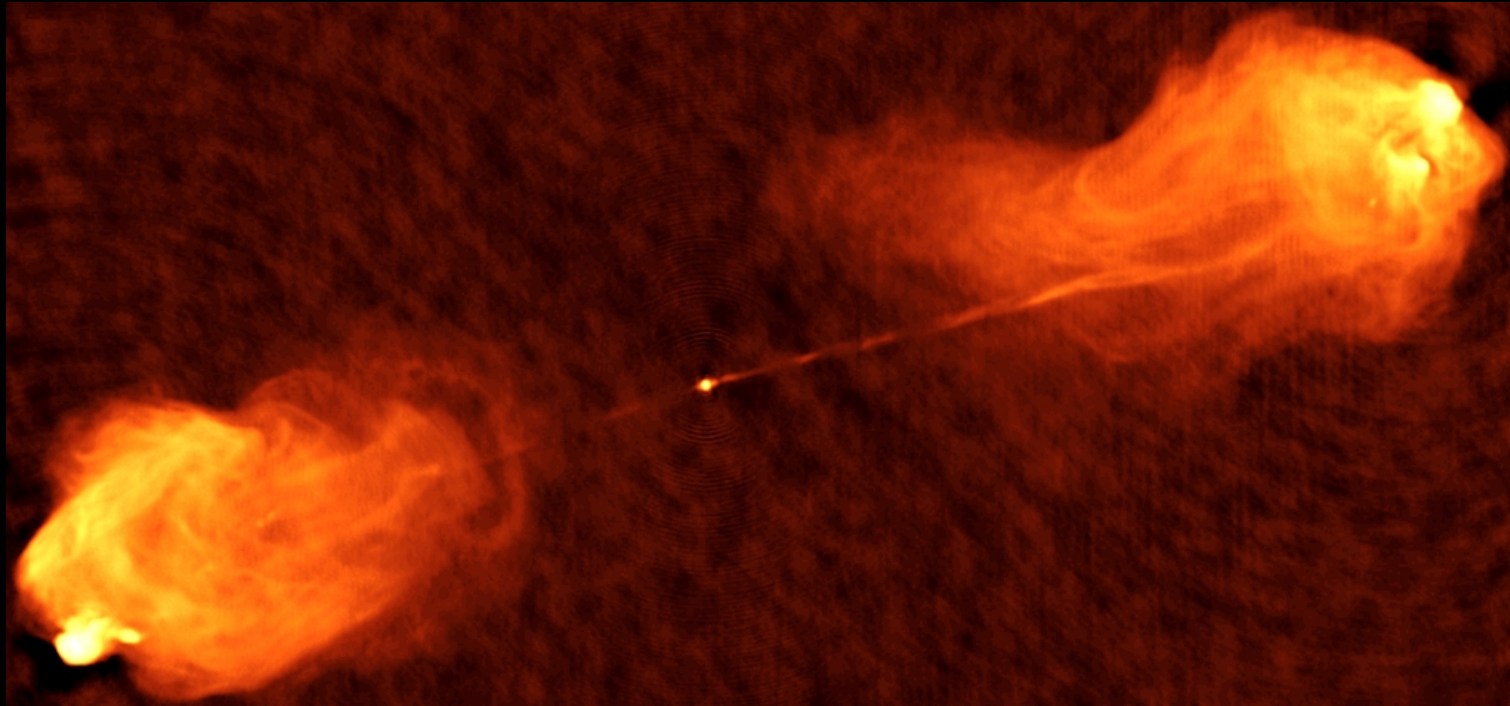
Office 756 DWB

Michaelmas 2011 Lecture 4



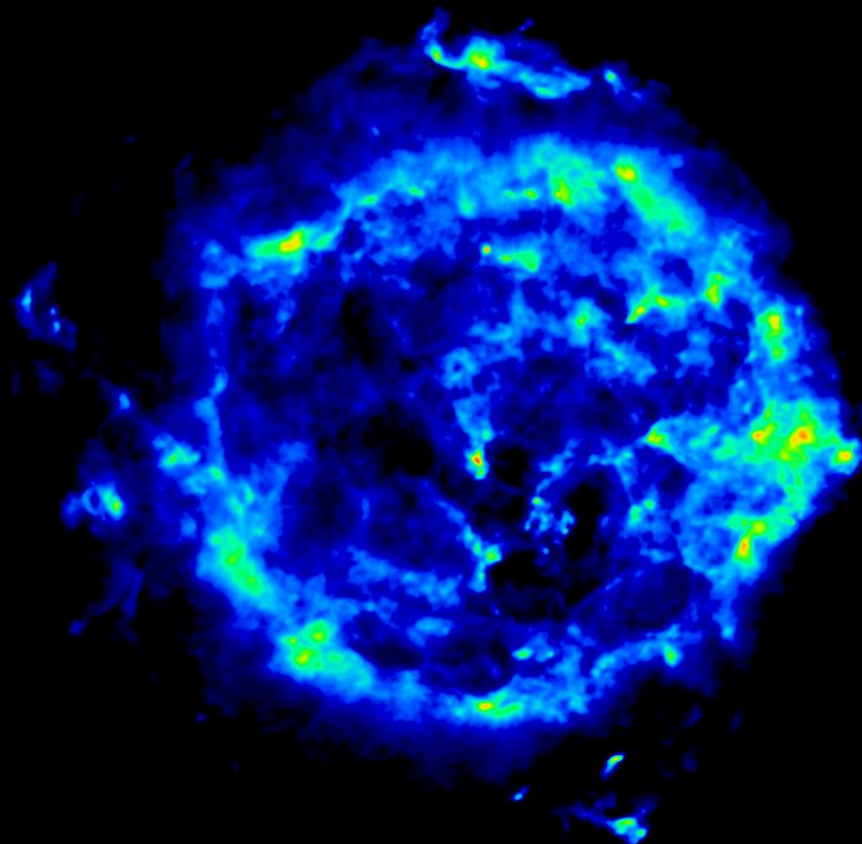
Today's lecture: Synchrotron emission Part I

- Evidence that synchrotron radiation is the emission mechanism in jet-producing objects: polarization, spectrum.
- Total power produced by a synchrotron-emitting particle.
- Minimum energy contained in synchrotron-emitting plasma.
- Spectrum of synchrotron radiation from mono-energetic particles.
- Relationship between the power-law synchrotron spectrum and underlying electron energies.



Radiogalaxy Cygnus A (3C405) imaged at 5 GHz

Cas A supernova remnant, 6 cm radio image



Evidence for synchrotron radiation

Both AGN jets and supernova shocks produce synchrotron radiation via ultra-relativistic electrons spiralling in weak magnetic fields. The extended radio emission seen in AGN (ultimately derived from the jets) and also in ordinary galaxies (ultimately derived from the many supernovae which occur over time in a galaxy) are both synchrotron.

In some cases we can see synchrotron emission extending to optical and X-ray wavelengths (e.g. in very powerful jets and from pulsars).

Several pieces of evidence point towards synchrotron radiation:

- Broad-band smooth spectrum over a vast range of wavelengths with no emission lines.
- The spectral shape can be approximated as a *power law* over large ranges in frequency. Taking S_ν as the flux per unit frequency from a source, we find

$$S_\nu \propto \nu^{-\alpha}$$

- Emission shows strong linear polarization: up to 70%.

Radio spectra

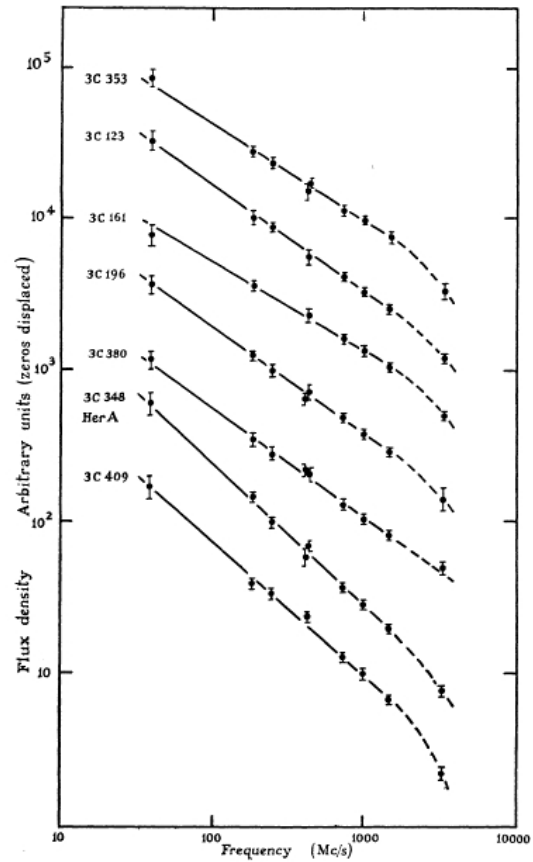
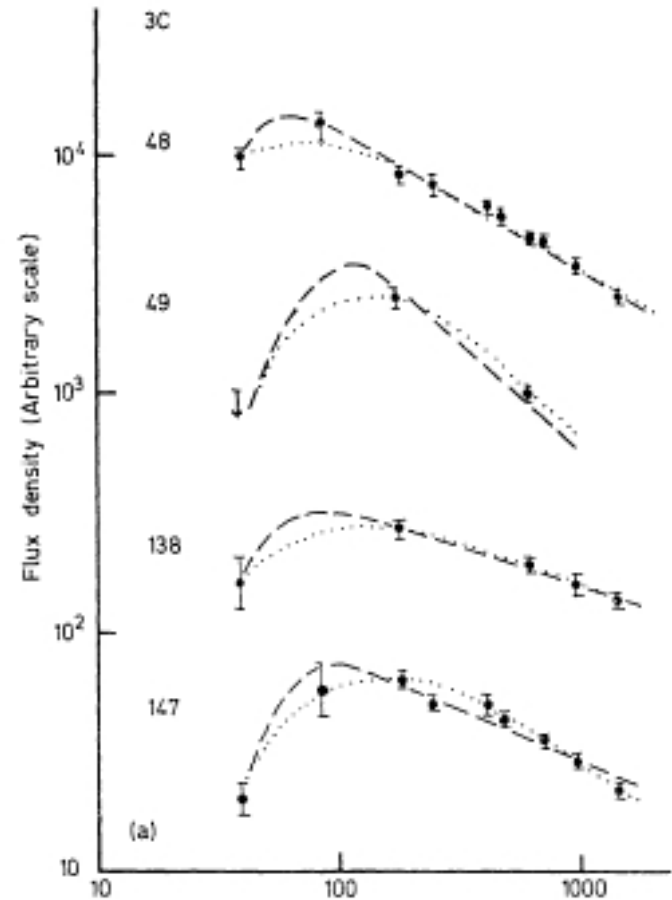


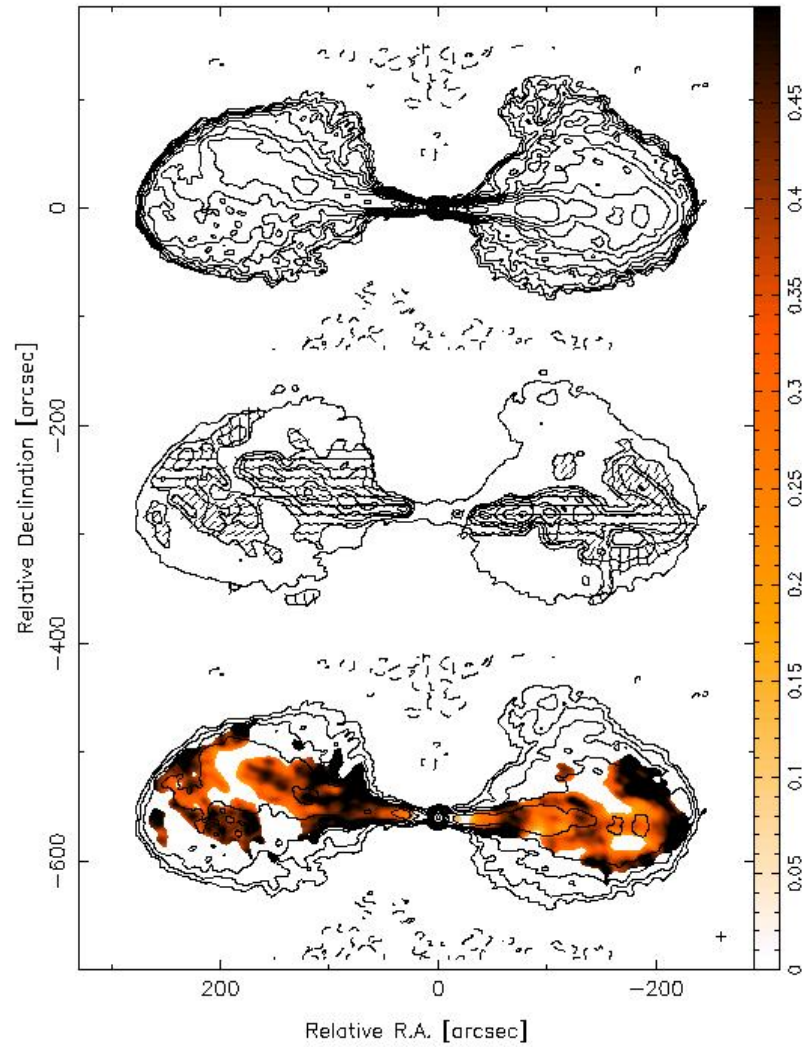
FIG. 1.—Spectra of the seven calibration sources.



Broad-band radio spectra of radiosources from the 3C catalogue (log-log scale). A power-law of the form $S_\nu \propto \nu^{-\alpha}$ with $\alpha = 0.5$ — 0.8 is a reasonable fit over most of the wavelength range. Note, however, that the spectrum usually “turns over” at low and at high frequencies: we shall return to these later.

Polarization



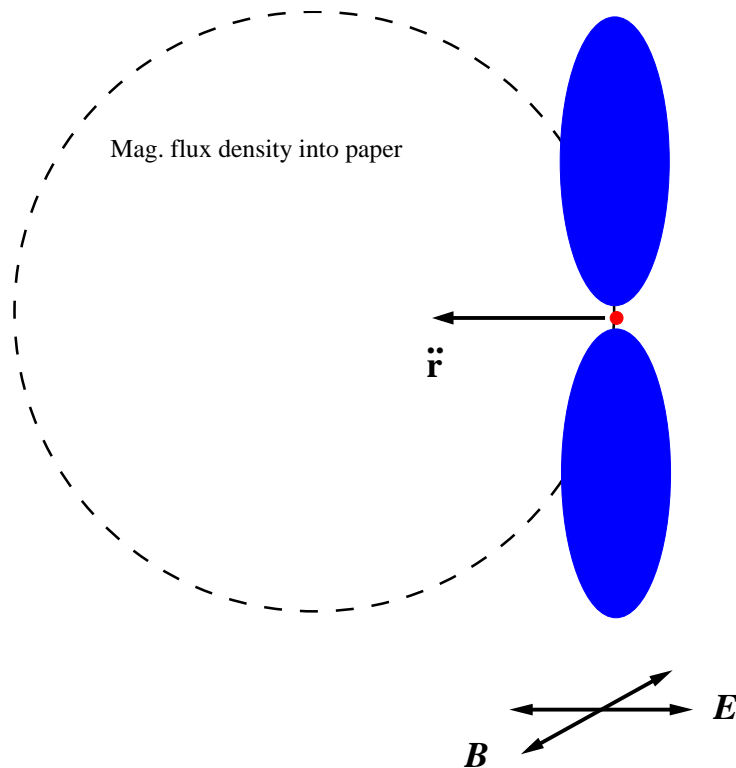


Total intensity

Polarization direction

Percentage polarization

Synchrotron total power



- Recap for a non-relativistic electron...
- Power radiated by an electron in its rest frame is obtained via Larmor's equation:

$$P_{\text{rad}} = \frac{e^2 |\ddot{\mathbf{r}}|^2}{6\pi\epsilon_0 c^3}$$

- The polar diagram of the emitted power varies as $\sin^2\theta$.
- The radiation is polarised with the \mathbf{E} vector lying in the plane of the acceleration.

For a relativistic electron:

Calculating the power emitted in the observer's frame via transformation of the Larmor equation is difficult but we can use a trick: the power is Lorentz invariant, so we can calculate it in the electron's instantaneous rest frame (exercise: convince yourself this is the case!)

Suppose the electron is spiralling round a magnetic field \mathbf{B} at a pitch angle θ (and there is no \mathbf{E} field in the observer's frame). In the electron's instantaneous rest frame, the force on it will be given by the \mathbf{E}' field, which we obtain by Lorentz-transforming \mathbf{B} :

$$E'_x = E_x = 0$$

$$E'_y = \gamma(E_y - vB_z) = -v\gamma B_z = -v\gamma B \sin\theta$$

$$E'_z = \gamma(E_z - vB_y) = 0$$

So the force on the electron in its instantaneous rest frame is

$$F' = eE' = -ev\gamma B \sin\theta$$

And the acceleration which we can now insert into Larmor's equation is

$$\ddot{r}' = \frac{ev\gamma B \sin\theta}{m_e}$$

Which gives our final result for the total power radiated in the observer's frame for a synchrotron electron with Lorentz factor γ :

$$P_{\text{rad}} = \frac{e^4 \gamma^2 B^2 v^2 \sin^2 \theta}{6\pi \epsilon_0 c^3 m_e^2}$$

This can be conveniently re-written (see Longair pp 231–232 for substitutions) as:

$$P_{\text{rad}} = \frac{4}{3} \sigma_{\text{T}} c U_{\text{mag}} \left(\frac{v}{c}\right)^2 \gamma^2$$

where σ_{T} is the Thomson cross-section and U_{mag} is the magnetic energy density in the observer's frame.

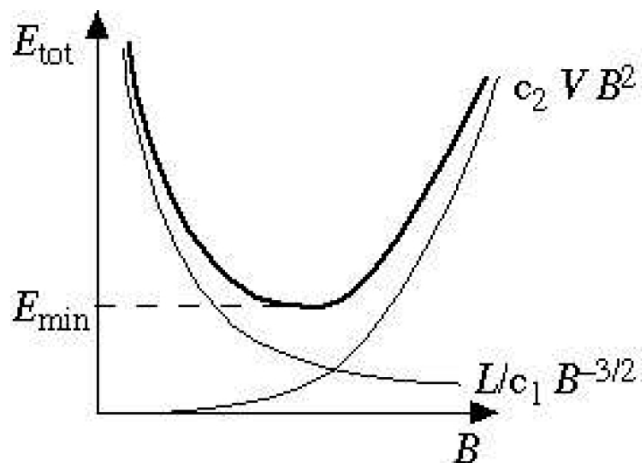
Minimum energy density in synchrotron plasma

Given a box of synchrotron-emitting plasma radiating with some total power, we cannot immediately determine either the magnetic energy density or the energy density in electrons. There is a degeneracy: the same synchrotron power output can be obtained by a low energy density in the electrons and strong \mathbf{B} , or by high energy density in electrons and weak \mathbf{B} ; or anywhere in between. However, a very useful limit for studying the energetics of astrophysical synchrotron sources can be calculated. This was first shown by Geoffrey Burbidge in 1956.



Scan ©American Institute of Physics

Burbidge, Burbidge, Fowler & Hoyle, a.k.a. B^2FH



The total energy stored in the plasma is divided into that stored in particles and that stored in \mathbf{B} . To produce a given synchrotron power, one may have strong \mathbf{B} and less energy in particles, or vice-versa. The lower limit on the total energy required is near to the equipartition value.

The equipartition value is theoretically appealing if the plasma is energised by some stochastic process. The only solid supporting evidence comes from X-ray observations of inverse-Compton scattered CMB radiation, which gives field/particle strengths perhaps a factor of three to four away from the minimum value. We also can't be completely sure that the plasma doesn't contain a significant quantity of protons.

However, this argument shows that the energy *stored* in the plasma is huge: a lower limit of some 10^{54} J for a source like Cygnus A.

Adopting the minimum energy values, we typically find \mathbf{B} ranging from $\sim 10^{-5}$ T down to $\sim 10^{-10}$ T. We can then begin to estimate the typical Lorentz factors of the electrons; these range from a few tens up to $\sim 10^5$ or more for bright regions. We know that some cosmic rays have energies of $\sim 10^{19}$ eV (i.e. a Lorentz factor of 10^{13} for an electron or 10^{10} for a proton) and these are likely to be the very high-energy tail of the synchrotron particles.

Derivation of the minimum energy is for you to try on the problem sheet.

Spectral shape of synchrotron radiation

For a relativistic electron...

Relativistic aberration:

$$\cos\phi = \frac{\cos\phi' + \beta}{1 + \beta\cos\phi'}$$

When γ is large, the rest-frame dipole emission is beamed into a narrow opening angle of $\sim 1/\gamma$



$$\gamma = 1$$



$$\gamma = 1.5$$

For a relativistic electron...

The orbital angular frequency changes. A non-relativistic electron in uniform \mathbf{B} will travel in a helical path such that

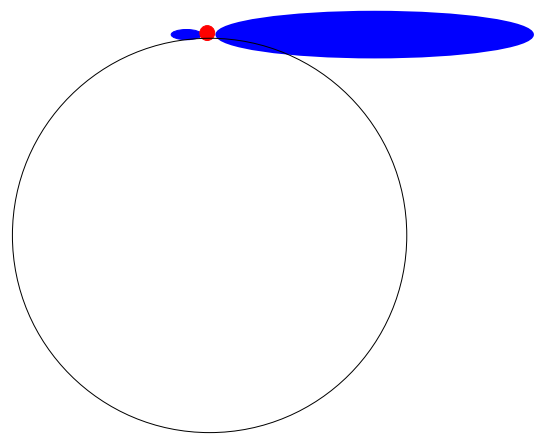
$$\frac{m_e v_{\perp}^2}{r_{\perp}} = e v_{\perp} B$$

Which gives an angular gyro-frequency of

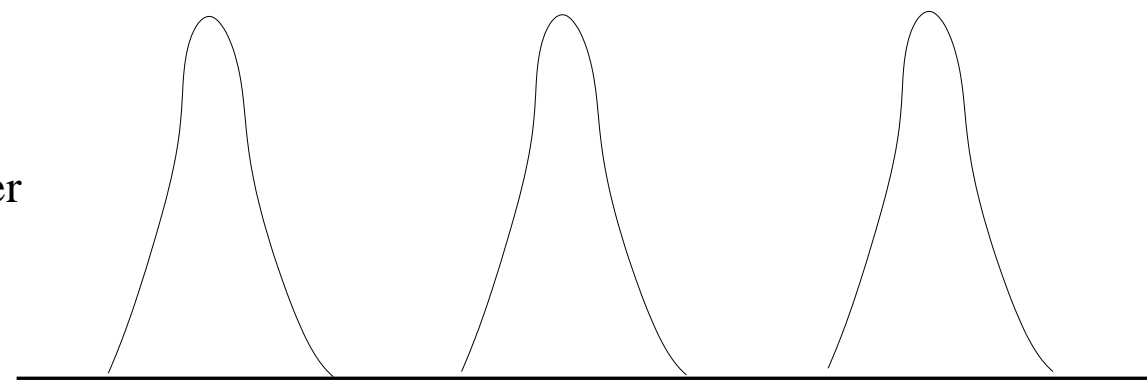
$$\omega = \frac{v_{\perp}}{r_{\perp}} = \frac{eB}{m_e}$$

If the motion is relativistic

$$\omega_{\text{rel}} = \frac{v_{\perp}}{r_{\perp}} = \frac{eB}{\gamma m_e}$$



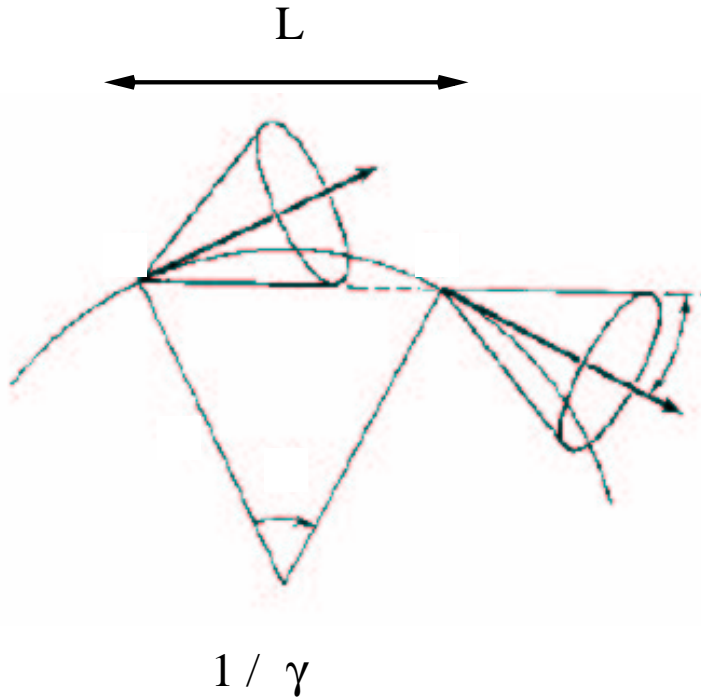
power



time

So a hypothetical observer staring at a single electron would see a series of pulses separated in time by $2\pi/\omega_{\text{rel}}$.

To work out the precise pulse shape we need to relativistically transform the dipole power distribution from the electron's instantaneous rest frame, but a good approximation can be made by using the $\approx 1/\gamma$ opening angle.



distance R to observer

- Start of pulse is emitted at $t = 0$ and is seen by observer at

$$t_+ = R/c$$

- End of pulse leaves at time $t = L/v$ and only has to travel a distance $(L - R)$ so it is seen at time

$$t_- = L/v + (R - L)/c$$

- So pulse length seen by observer is

$$\Delta t = \frac{L}{v} \left[1 - \frac{v}{c} \right]$$

- Now note that

$$\frac{L}{v} \sim \frac{1}{\gamma \omega_{\text{rel}}}$$

- and that

$$1 - \frac{v}{c} \sim \frac{1}{2\gamma^2}$$

These give a total pulse length of:

$$\Delta t \sim \frac{1}{2\gamma^3\omega_{\text{rel}}}$$

Or in terms of the magnetic flux density and electron mass-energy:

$$\Delta t \sim \frac{m_e}{2\gamma^2 B e}$$

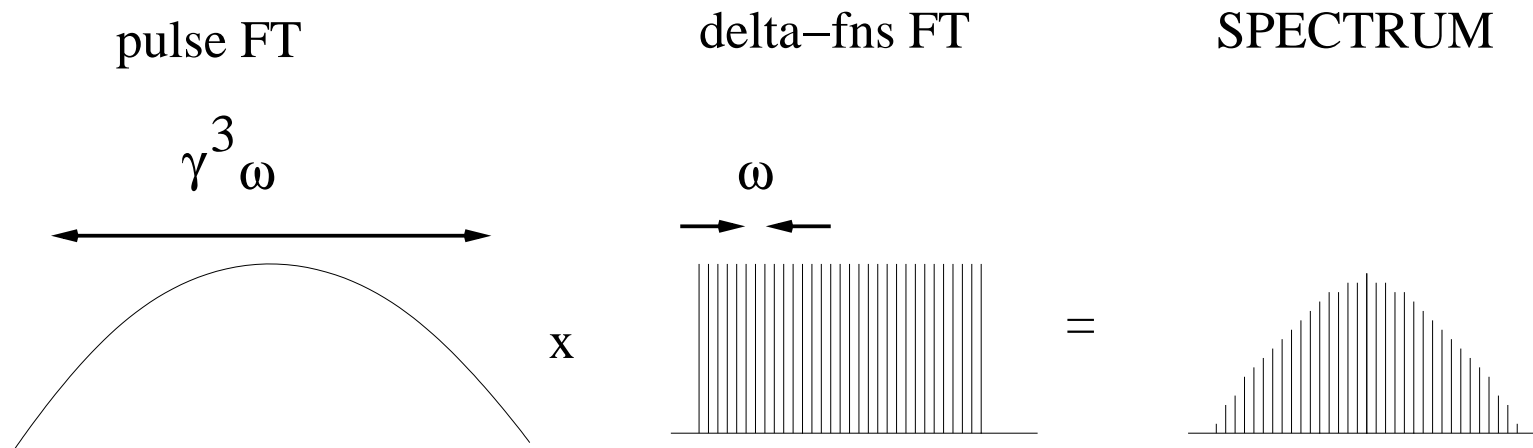
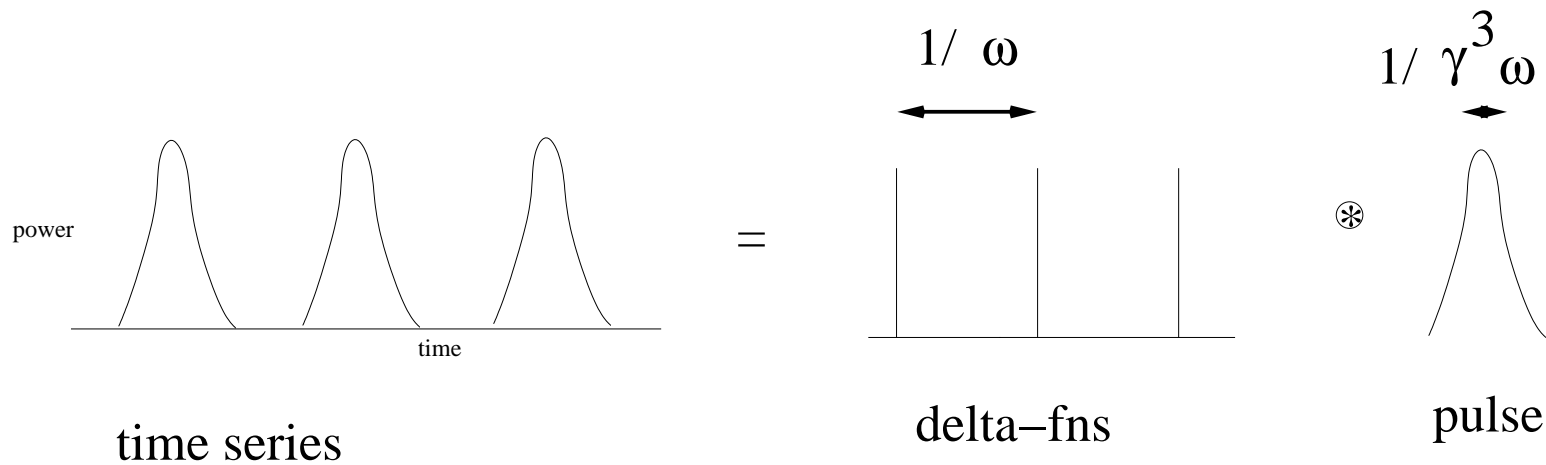
For revision, be wary of the factors of γ ! Best to think of the electron having a gyro-frequency of ω_{rel} , which is shortened by one factor γ because of the aberration of the dipole, then by another two factors of γ because you need to correct the light-travel time to *second order*.

And so to the spectrum:

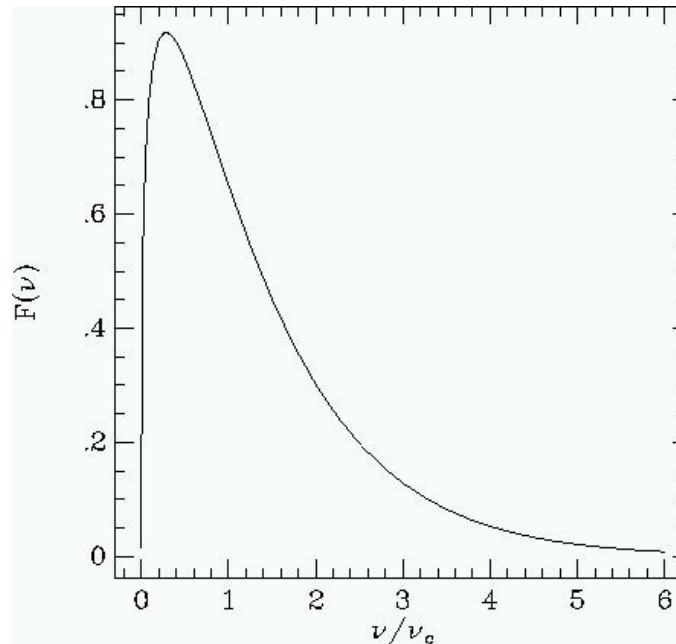
For a realistic case, we will not be observing the single pulses of any electron, rather, the superposition of pulses from the population of electrons in the plasma. And even if they *were* all mono-energetic, it would be supernaturally unlikely for them to be in phase, so we consider the *spectrum* which we will observe.

To calculate the spectrum we take the Fourier transform of the power time series:

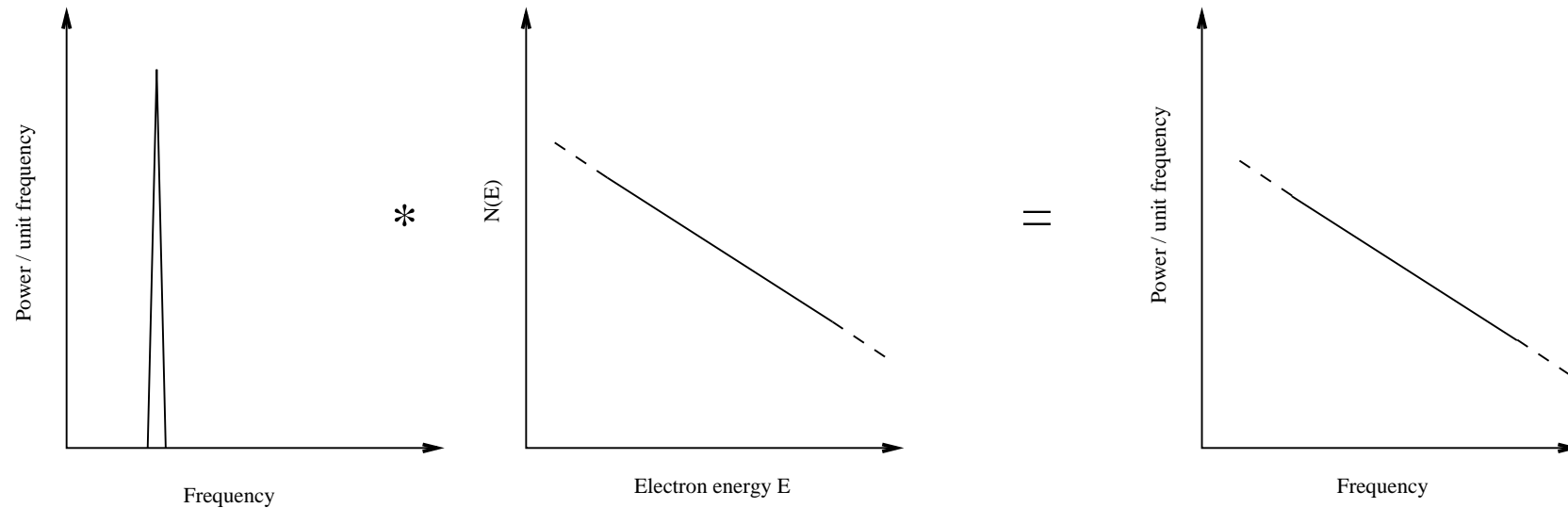
(power,time) \leftarrow FT \rightarrow (power per unit frequency, frequency)



And so we have the synchrotron spectrum from a population of mono-energetic electrons with Lorentz factor γ . For the purposes of this course—and virtually all applications in high energy astrophysics—we treat this spectrum as being sharply peaked at $\gamma^3 \omega_{\text{rel}}$



Relationship between single-energy spectrum and typical radiosource power-law spectrum



The frequency range over which each electron energy emits is very narrow compared with the total range of frequencies observed.

We infer that the power-law spectrum is the result of an underlying power-law distribution in *electron energy*, with the higher energy electrons causing the high-frequency emission.

The observed spectrum is the *convolution* of the electron energy distribution with the spectrum from a single electron.

Aside: how do we know that **B is uniform?**

Of course there are small-scale variation in the magnetic flux density in any plasma, and we see clear evidence for turbulent flows in high-resolution maps of radiosources. But we are making an assumption here that **B** is on average homogeneous, in order to convolve the mono-energetic electron spectrum with the electron energies. Are we justified in doing this?

One argument in favour of homogeneous **B** is that the sound speed in relativistic plasma is very high: $c/\sqrt{3}$, in fact. Any large variations in **B** would cause variations in internal energy density, i.e. pressure, and these would quickly be smoothed out by pressure waves.

There may, however, be reasonably smooth bulk motions of plasma inside the radio lobes, *which could support gradual pressure changes across the lobes on the largest scales*. In this case we'd still locally see the spectrum as a convolution of mono-energetic synchrotron spectrum with the electron energy distribution; but from point to point within the source, radiation of a particular frequency might be emitted by electrons of different energies.

Electron energy distribution

So, we infer that there is a power-law distribution of electron energies over a wide range in energy E , which we will describe as

$$N(E)dE \propto E^{-k}dE$$

We can approximate the convolution by taking the single-electron spectrum to be a delta-function and transforming energy into frequency.

The power emitted by a single electron is proportional to B^2 and to the square of the electron energy (yesterday's lecture).

$$P(E) \propto E^2 B^2$$

And we have just seen that the characteristic frequency emitted by an electron must scale as the square of the electron energy,

$$E \propto \nu^{1/2} \quad \text{and} \quad dE \propto \nu^{-1/2}d\nu$$

Electron energy distribution contd.

Now, the power radiated between ν and $\nu + d\nu$ will be equal to the power radiated by electrons with energies between the corresponding E and $E + dE$. Hence we can substitute ν for E and we have a form for the spectrum...

$$\begin{aligned} S(\nu)d\nu &= P(E)N(E)dE \\ &\propto E^2 B^2 E^{-k} dE \\ &\propto E^{2-k} dE \\ &\propto \nu^{(1-k)/2} d\nu \end{aligned}$$

Note that, because the power-law is smooth, this form is exactly what would be achieved by doing the convolution properly over the smooth region.

For our observed spectrum with a form $S(\nu) \propto \nu^{-\alpha}$, we can infer the energy spectral index is given via

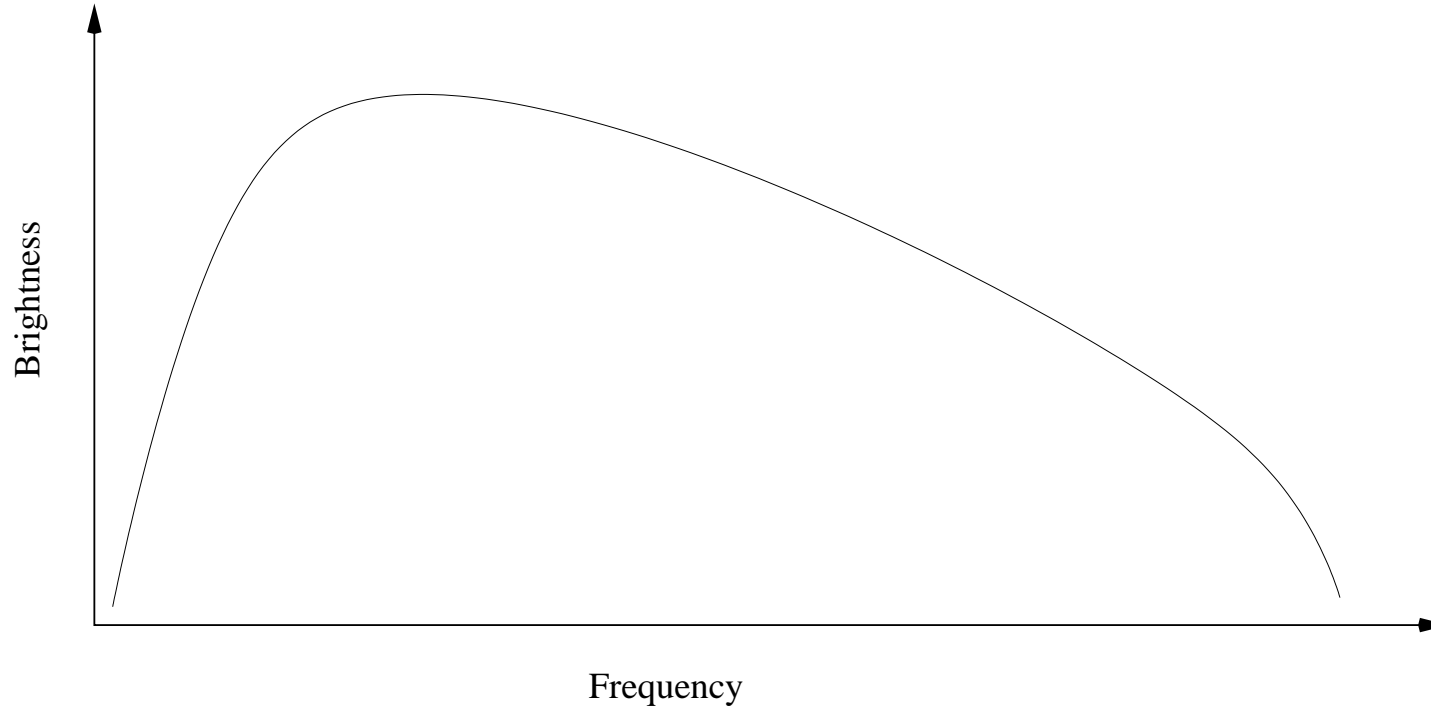
$$\alpha = (k - 1)/2$$

For a synchrotron index $\alpha = 0.5$, which is about the shallowest we see in optically-thin synchrotron emission, we find $k = 2$, i.e. the underlying electron energy distribution has the form

$$N(E)dE \propto E^{-2}dE$$

So we have almost completed the circle! But we still have the niggle that it is more common to see the synchrotron index a bit steeper than 0.5, just as our measurements of the cosmic ray energy spectrum have an index a bit steeper than 2.

Next time...



How to deal with the turn-overs we see at the low- and high-frequency ends of the spectrum: synchrotron self-Compton and spectral ageing.