Oxford Physics: Part C Major Option Astrophysics

High-Energy Astrophysics

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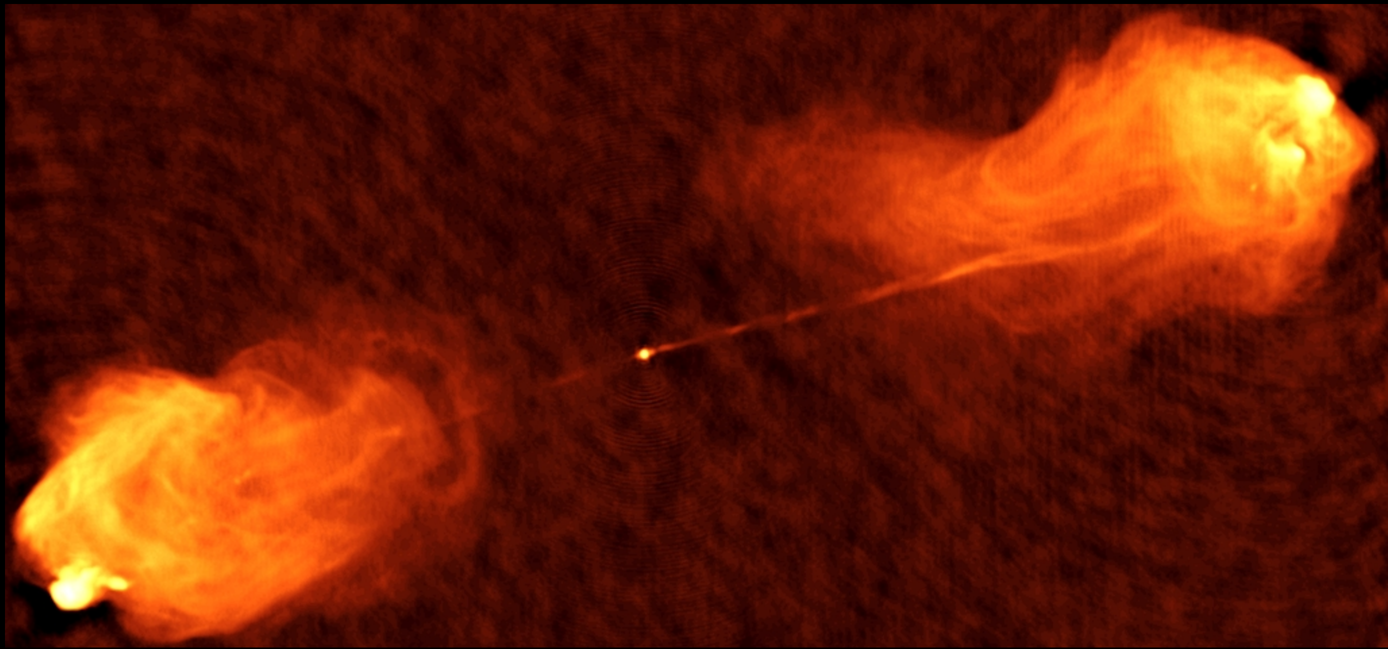
Michaelmas 2011 Lecture 5

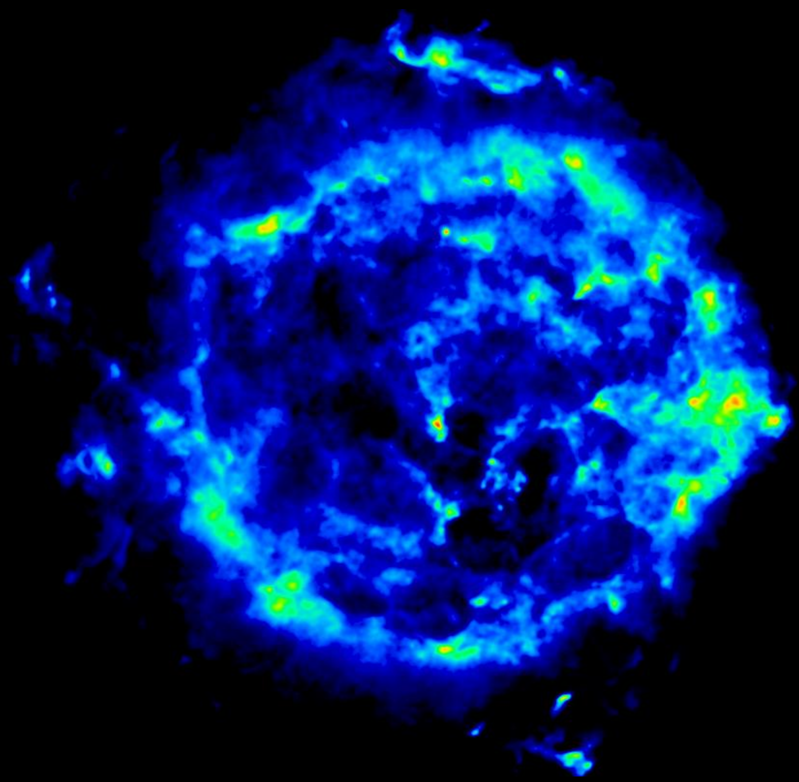
Today's lecture: Synchrotron emission Part II

- Spectrum of synchrotron radiation from mono-energetic particles.
- Relationship between the power-law synchrotron spectrum and underlying electron energies.
- Synchrotron self-absorption



Radiogalaxy Cygnus A (3C405) imaged at 5 GHz

Cas A supernova remnant, 6 cm radio image



Radio spectra

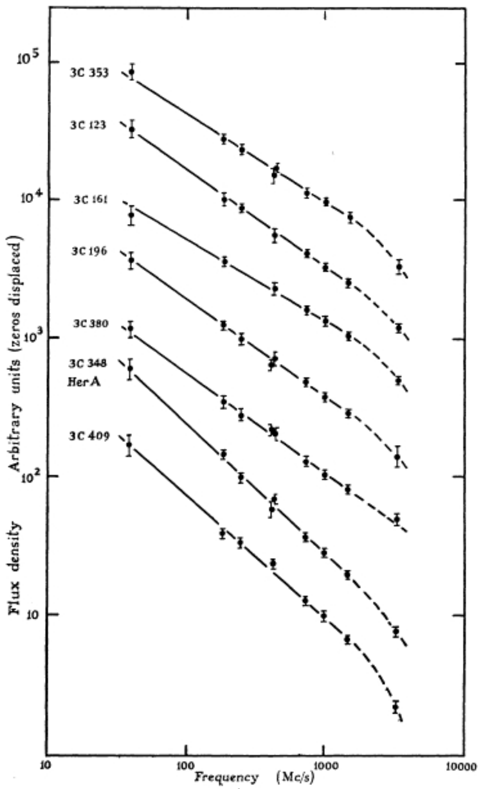
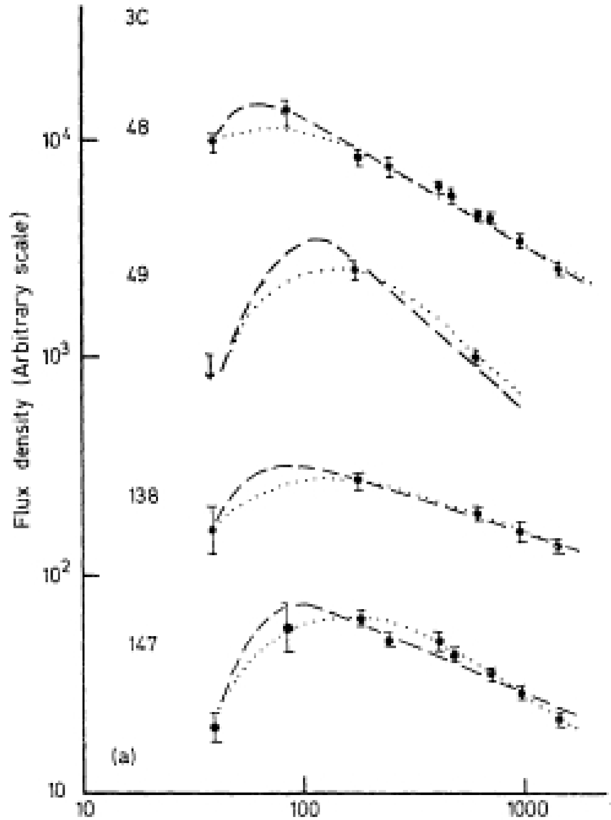


FIG. 1.—*Spectra of the seven calibration sources.*



Recap last lecture's result for the total power radiated in the observer's frame for a synchrotron electron with Lorentz factor γ:

$$P_{\mathrm{rad}} = \frac{e^4 \gamma^2 B^2 v^2 \sin^2\theta}{6\pi\epsilon_0 c^3 m_{\mathrm{e}}^2}$$

More compactly (substitutions non-examinable):

$$P_{\mathrm{rad}} = \frac{4}{3}\sigma_{\mathrm{T}} c U_{\mathrm{mag}} \left(\frac{v}{c}\right)^2 \gamma^2$$

where σ_{T} is the Thomson cross-section and U_{mag} is the magnetic energy density in the observer's frame.

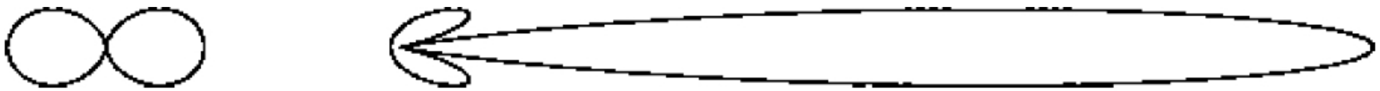
Spectral shape of synchrotron radiation

For a relativistic electron...

Relativistic aberration:

$$\cos\phi = \frac{\cos\phi' + \beta}{1 + \beta\cos\phi'}$$

When γ is large, the rest-frame dipole emission is beamed into a narrow opening angle of $\sim 1/\gamma$



$\gamma = 1$ $\gamma = 1.5$

For a relativistic electron...

The orbital angular frequency changes. A non-relativistic electron in uniform **B** will travel in a helical path such that

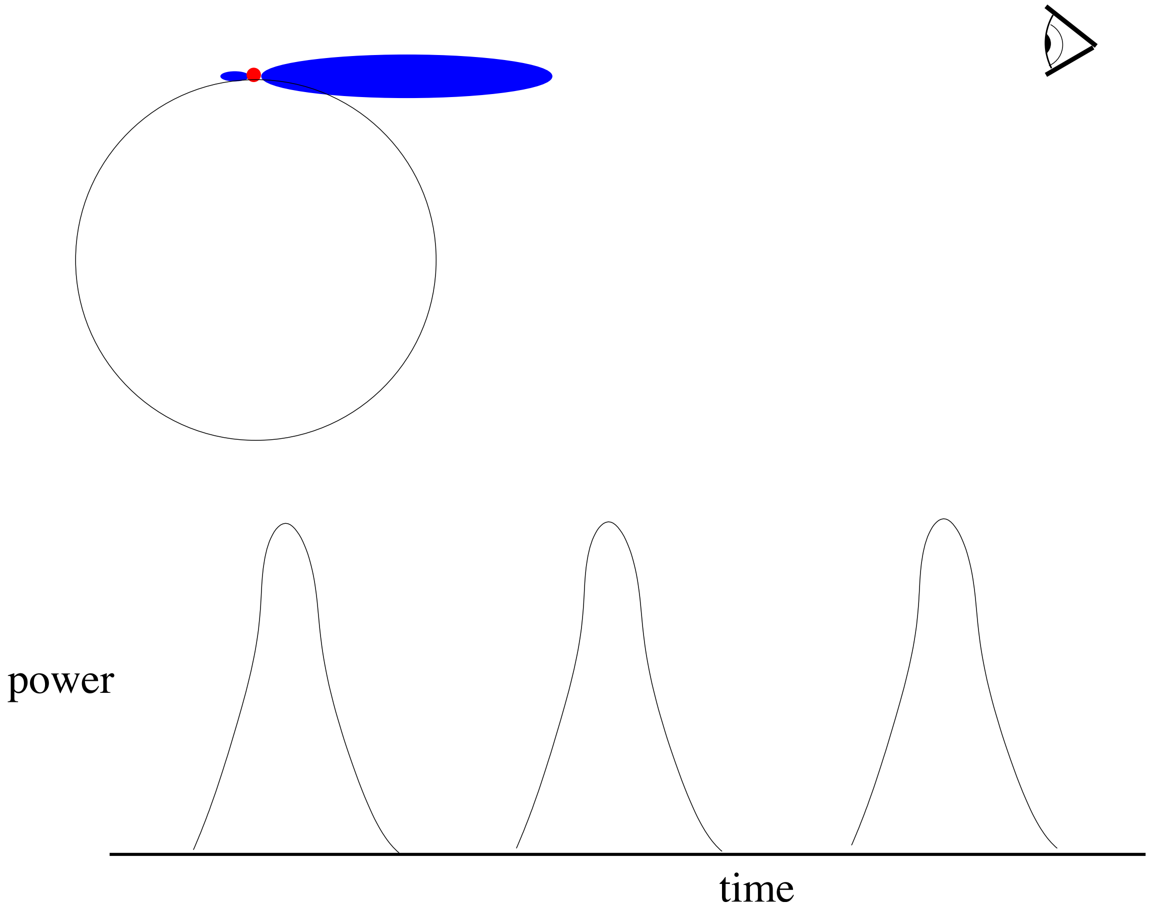
$$\frac{m_{\mathrm{e}} v_{\perp}^2}{r_{\perp}} = e v_{\perp} B$$

Which gives an angular gyro-frequency of

$$\omega = \frac{v_{\perp}}{r_{\perp}} = \frac{eB}{m_{\mathrm{e}}}$$

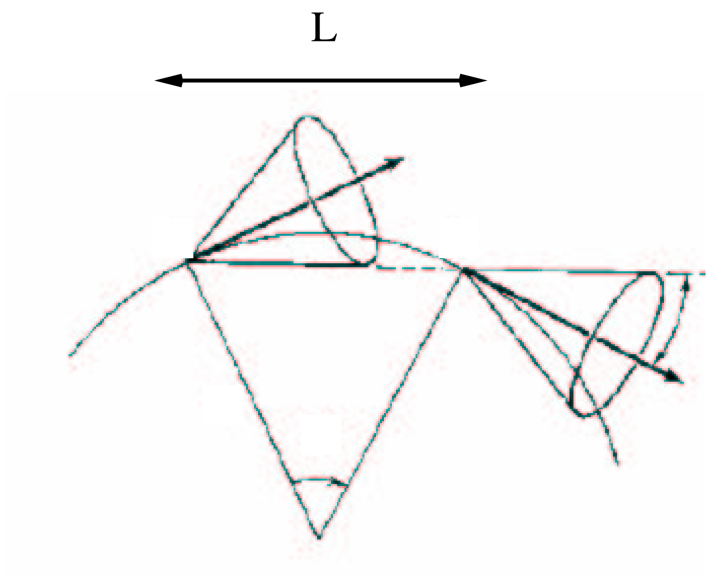
If the motion is relativistic

$$\boxed{\omega_{\mathrm{rel}} = \frac{v_{\perp}}{r_{\perp}} = \frac{eB}{\gamma m_{\mathrm{e}}}}$$


power
time

So a hypothetical observer staring at a single electron would see a series of pulses separated in time by $2\pi/\omega_{\text{rel}}$.

To work out the precise pulse shape we need to relativistically transform the dipole power distribution from the electron's instantaneous rest frame, but a good approximation can be made by using the $\approx 1/\gamma$ opening angle.



- Start of pulse is emitted at $t = 0$ and is seen by observer at

$$t_+ = R/c$$

- End of pulse leaves at time $t = L/v$ and only has to travel a distance $(L - R)$ so it is seen at time

$$t_- = L/v + (R - L)/c$$

- So pulse length seen by observer is

$$\Delta t = \frac{L}{v}\left[1 - \frac{v}{c}\right]$$

- Now note that

$$\frac{L}{v} \sim \frac{1}{\gamma \omega_{\mathrm{rel}}}$$

- and that

$$1 - \frac{v}{c} \sim \frac{1}{2\gamma^2}$$

These give a total pulse length of:

$$\boxed{\Delta t \sim \frac{1}{2\gamma^3 \omega_{\mathrm{rel}}}}$$

Or in terms of the magnetic flux density and electron mass-energy:

$$\boxed{\Delta t \sim \frac{m_{\mathrm{e}}}{2\gamma^2 Be}}$$

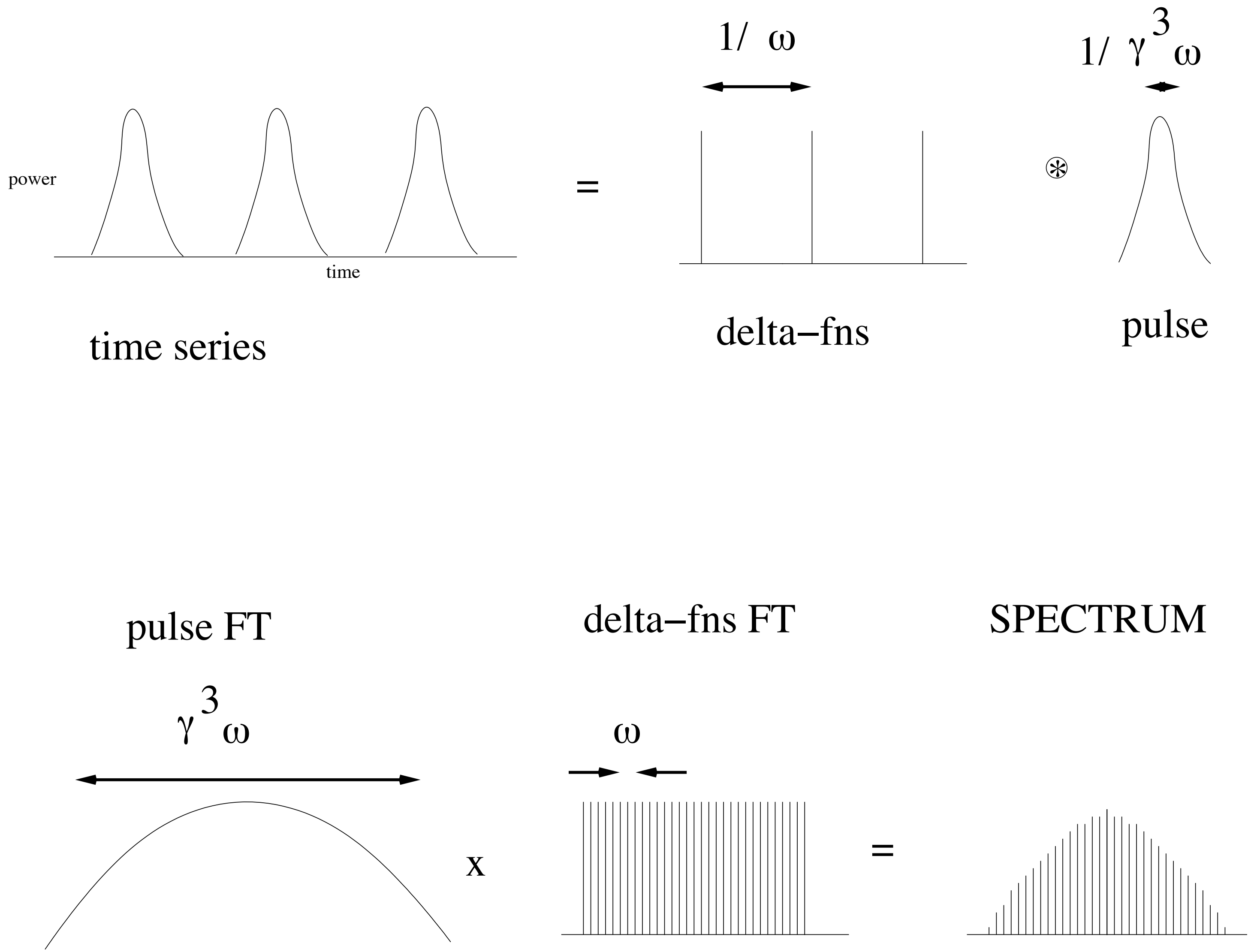
For revision, be wary of the factors of γ! Best to think of the electron having a gyro-frequency of ω_{rel}, which is shortened by one factor γ because of the aberration of the dipole, then by another two factors of γ because you need to correct the light-travel time to *second order*.

And so to the spectrum:

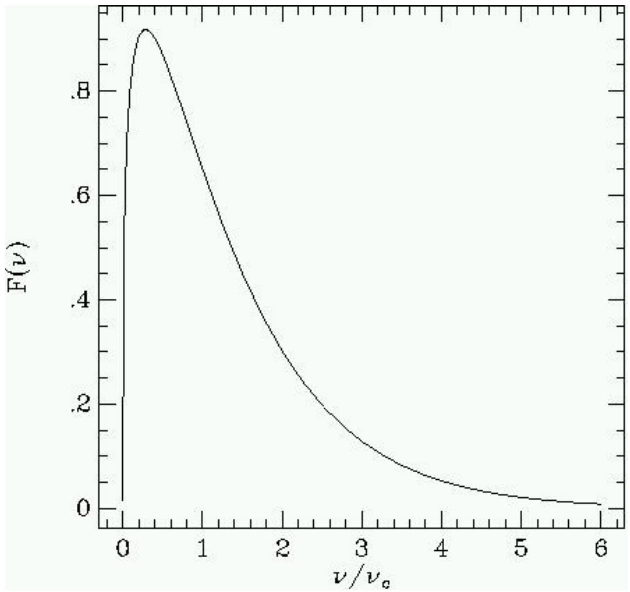
For a realistic case, we will not be observing the single pulses of any electron, rather, the superposition of pulses from the population of electrons in the plasma. And even if they *were* all mono-energetic, it would be supernaturally unlikely for them to be in phase, so we consider the *spectrum* which we will observe.

To calculate the spectrum we take the Fourier transform of the power time series:

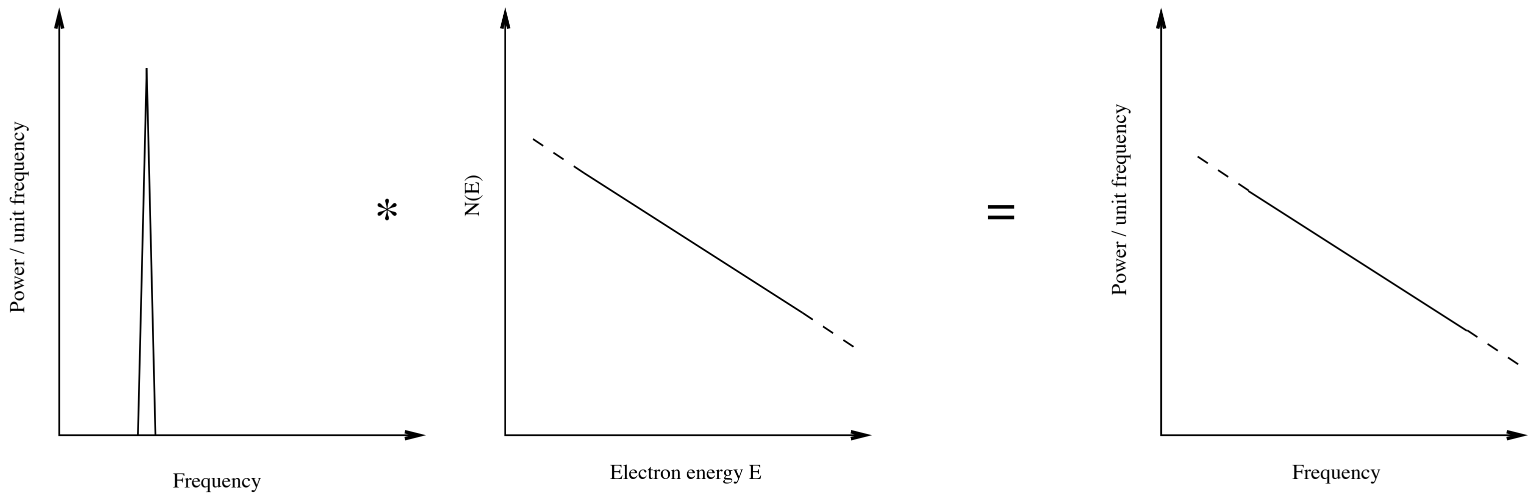
(power,time) $\leftarrow$ FT $\rightarrow$ (power per unit frequency, frequency)


power
time
time series
=
1/ ω
delta−fns
1/ γ³ω
⊛
pulse
pulse FT
γ³ω
x
delta−fns FT
ω
=
SPECTRUM

And so we have the synchrotron spectrum from a population of mono-energetic electrons with Lorentz factor γ. For the purposes of this course—and virtually all applications in high energy astrophysics—we treat this spectrum as being sharply peaked at $\gamma^3 \omega_{\rm rel}$



Relationship between single-energy spectrum and typical radiosource power-law spectrum



The frequency range over which each electron energy emits is very narrow compared with the total range of frequencies observed.

We infer that the power-law spectrum is the result of an underlying power-law distribution in *electron energy*, with the higher energy electrons causing the high-frequency emission.

The observed spectrum is the *convolution* of the electron energy distribution with the spectrum from a single electron.

Aside: how do we know that B is uniform?

Of course there are small-scale variation in the magnetic flux density in any plasma, and we see clear evidence for turbulent flows in high-resolution maps of radiosources. But we are making an assumption here that **B** is on average homogeneous, in order to convolve the mono-energetic electron spectrum with the electron energies. Are we justified in doing this?

One argument in favour of homogeneous **B** is that the sound speed in relativistic plasma is very high: $c/\sqrt{3}$, in fact. Any large variations in **B** would cause variations in internal energy density, i.e. pressure, and these would quickly be smoothed out by pressure waves.

There may, however, be reasonably smooth bulk motions of plasma inside the radio lobes, *which could support gradual pressure changes across the lobes on the largest scales.* In this case we'd still locally see the spectrum as a convolution of mono-energetic synchrotron spectrum with the electron energy distribution; but from point to point within the source, radiation of a particular frequency might be emitted by electrons of different energies.

Electron energy distribution

So, we infer that there is a power-law distribution of electron energies over a wide range in energy E, which we will describe as

$$N(E)dE \propto E^{-k}dE$$

We can approximate the convolution by taking the single-electron spectrum to be a delta-function and transforming energy into frequency.

The power emitted by a single electron is proportional to B^2 and to the square of the electron energy (yesterday's lecture).

$$P(E) \propto E^2B^2$$

And we have just seen that the characteristic frequency emitted by an electron must scale as the square of the electron energy,

$$E \propto \nu^{1/2} \quad \text{and} \quad dE \propto \nu^{-1/2}d\nu$$

Electron energy distribution contd.

Now, the power radiated between ν and $\nu + d\nu$ will be equal to the power radiated by electrons with energies between the corresponding E and $E + dE$. Hence we can substitute ν for E and we have a form for the spectrum...

$$\begin{aligned} S(\nu)d\nu &= P(E)N(E)dE \\ &\propto E^2 B^2 E^{-k} dE \\ &\propto E^{2-k} dE \\ &\propto \nu^{(1-k)/2} d\nu \end{aligned}$$

Note that, because the power-law is smooth, this form is exactly what would be achieved by doing the convolution properly over the smooth region.

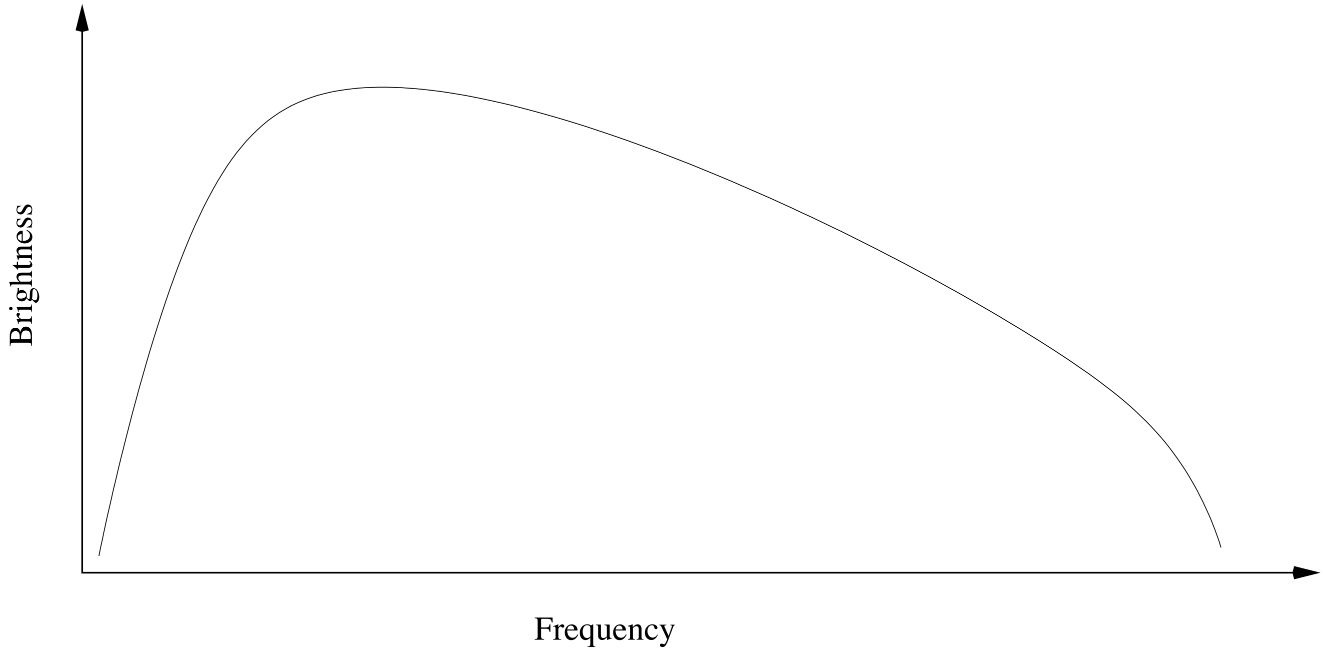
For our observed spectrum with a form $S(\nu) \propto \nu^{-\alpha}$, we can infer the energy spectral index is given via

$$\alpha = (k-1)/2$$

For a synchrotron index $\alpha = 0.5$, which is about the shallowest we see in optically-thin synchrotron emission, we find $k = 2$, i.e. the underlying electron energy distribution has the form

$$N(E)dE \propto E^{-2}dE$$

So we have almost completed the circle! But we still have the niggle that it is more common to see the synchrotron index a bit steeper than 0.5, just as our measurements of the cosmic ray energy spectrum have an index a bit steeper than 2.



So the bulk of the broad-band spectrum from e.g. the lobes of powerful radio sources is explained reasonably well.

Now we must consider the rising spectrum at low frequency and the cut-off at high frequency.

Low-frequency cutoff: synchrotron self-absorption

Scattering within synchrotron plasma

In calculating the synchrotron spectrum so far, we have implicitly assumed that all of the synchrotron radiation emitted by each electron reaches the observer. However this is not necessarily the case: as a photon propagates through the plasma on its way out of the source, there is a chance that it will scatter off one of the synchrotron electrons. This is known as *synchrotron self-absorption*.

If such scattering occurs many times many times before the photon can get out of the source, the result is that an outside observer only “sees” emission from a thin layer near the surface of the source. Beneath this, the synchrotron electrons are simply exchanging photons, in a quasi-thermal-equilibrium fashion, and so the total flux the observer sees will be much smaller than if all the synchrotron photons escaped the source. This is analogous to looking at the surface of a dense object in thermal equilibrium—e.g. the surface of the Sun.

For a thermal distribution of particle energies, the calculation of the black-body spectrum observed is a fairly straightforward set piece. However we are dealing with a relativistic power-law distribution of particle energies, and each particle emits and receives radiation only at its characteristic wavelength. The detailed calculation of the self-absorption cross-section as a function of wavelength is difficult (Longair pp258–260) and we will not discuss it here; instead we will calculate the spectral index.

The absorption cross-section for a synchrotron electron and a low-energy photon is greater at longer wavelengths. So for a source of a given size, we see very-long wavelength emission only from a very thin shell at the surface of the source. As our observing frequency increases, we see photons coming from regions of the source that are progressively deeper and deeper, and as we do so the total flux density increases. Eventually we reach a point where we can “see all the way through” the plasma, and above this frequency we recover the underlying power-law distribution.

By analogy with the terms used in optical astronomy, we call the self-absorbed region of the spectrum, where the photon mean free path is much smaller than the source size, *optically thick*; and we call the remaining part of the spectrum, where the photon mean free path is greater than the source size, *optically thin*.

Synchrotron self-absorption: calculation

Without doing the detailed calculation we can however recover the spectral shape of the optically-thick spectrum, via a very simple (but in fact strictly correct) trick.

Let us assign an *effective temperature*, $T_{\rm Eff}$, to the electrons in the synchrotron plasma. Although the particle energy distribution is not thermal, we can still define such an effective temperature according to

$$k_{\rm B} T_{\rm Eff} \sim \gamma m_{\rm e} c^2$$

We are now familiar with the result that the electron emits at a characteristic frequency, so we have

$$k_{\mathrm{B}}T_{\mathrm{Eff}} \propto E \propto \nu^{1/2}$$

Now here is the trick: even though the electrons are not in a thermal distribution, at any frequency they cannot emit radiation more effectively than a black-body. So we use the above functional form for $T_{\mathrm{Eff}}(\nu)$ in the Planck formula! This gives us the maximum brightness that a self-absorbed synchrotron plasma can have.

Here is the full form for the brightness from a black-body (recall from lecture 1):

$$B_\nu = \frac{2h\nu^3}{c^2}\left\{\exp\left(\frac{h\nu}{k_\mathrm{B}T}\right) - 1\right\}^{-1}$$

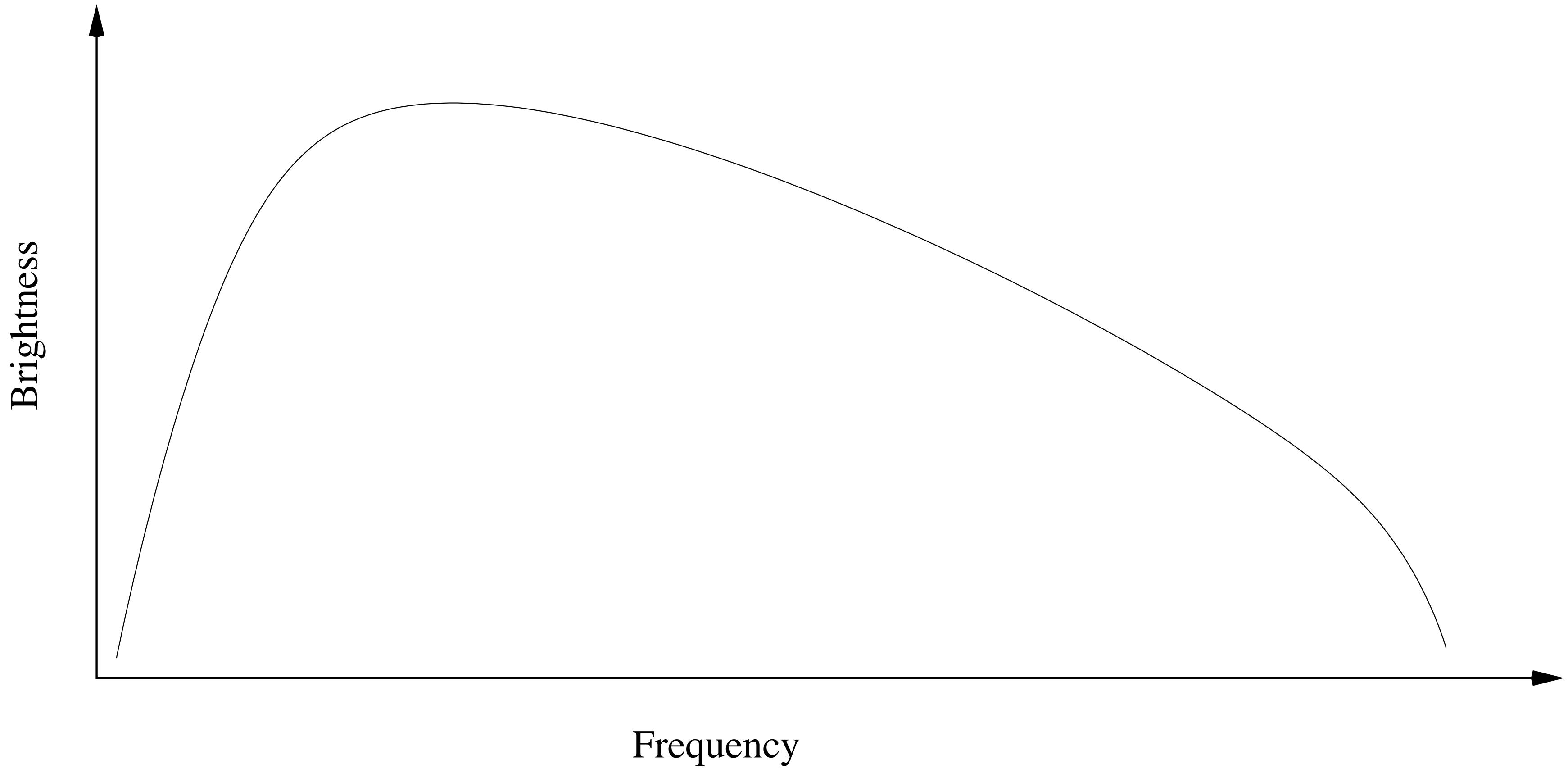
We will make a simplification and use the Rayleigh-Jeans limiting form (as we are explicitly on the low-frequency tail here...)

$$B_\nu = \frac{2k_\mathrm{B}T\nu^2}{c^2}$$

Which, using $T_\mathrm{Eff} \propto \nu^{1/2}$, gives us

$$B_\nu \propto \nu^{5/2}$$

So the spectrum *rises* steeply at low frequencies, until the mean free path between scatterings reaches the size of the source.


Brightness
Frequency