Oxford Physics: Part C Major Option Astrophysics

High-Energy Astrophysics

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Today's lecture: Accretion Discs Part II

- Recap of last lecture.

- Viscosity in this accretion disc.

- Luminosity and spectrum of thin disc.

- Evidence for accretion onto black holes - will start if time permits.

Recap of accretion concepts and results so far

- The Eddington Luminosity sets the balance between radiation pressure outwards and gravitational force inwards in a system where matter is accreting onto a compact object.

- $L = \frac{4\pi GMc}{\kappa}$ where κ is the opacity of the accreting material.

- for the case of AGN it is often convenient to set $L_{\mathrm{Edd}} = \frac{4\pi GMcm_{\mathrm{p}}}{\sigma_{\mathrm{T}}}$ but generally κ must be calculated precisely depending on the composition and ionisation state of the material involved.

- If the material accreting onto an object radiates away some of its GPE on the way in, $L = \epsilon \dot{M} c^2$, we get a characteristic limiting accretion rate, $\dot{M}_{\mathrm{Edd}} = \frac{4\pi GMm_{\mathrm{p}}}{\epsilon c \sigma_{\mathrm{T}}}$.

- Qualitatively, a cloud of collisional material with some angular momentum must eventually form a disc.
- It is generally hypothesised that tidal interactions between galaxies can force gas with high angular momentum in the outer regions into a state of low angular momentum where is falls towards the nucleus and forms a disc. The details are nowadays the topic of intensive study by numerical simulation.
- In a potential that is Keplerian (or close to it) near the nucleus, a thin disc supported by its own gas pressure must be cold compared to the virial temperature, i.e. it must have a highly supersonic rotation speed.

Viscosity in the disc

For the accreting material to fall into tighter orbits in the disc there must be an outwards flow of angular momentum—a torque acting on the disc. Take the disc viscosity to be η and consider a radius r in the disc with thickness t and angular velocity ω. The tangential force per unit area exerted by the disc inside r on the disc outside r is given by

$$\text{Force per unit area} = \eta r \frac{\mathrm{d}\omega}{\mathrm{d}r}.$$

This force acts over an area $2\pi r t$ so the total torque between adjacent pieces of the disc is

$$\Gamma = 2\pi r^3 t \eta \frac{\mathrm{d}\omega}{\mathrm{d}r}.$$

Remember torque is rate of change of angular momentum...

Again taking the orbits to be Keplerian we have $\omega = \sqrt{\frac{GM}{r^3}}$, so subbing into the previous equation for the torque we have

$$\frac{\mathrm{d}L}{\mathrm{d}t} = -3\pi\eta t(GMr)^{1/2}$$

which is the rate of change of angular momentum of the inner piece of the disc. This must equal the change of angular momentum duw to inflow of disc material, i.e.

$$\frac{\mathrm{d}L}{\mathrm{d}t} = \dot{m}r^2\omega = \dot{m}(GMr)^{1/2}$$

and so we have a relationship between the accretion rate and the disc viscosity,

$$\dot{m} = 3\pi\eta t$$

The problem with viscosity... The problem arises when we consider the *Reynolds number* of the material in the disc—a measure of how turbulent it is.

$$R \sim \frac{VL}{\nu}$$

Here V and L are characteristic speed and length scales and ν is the kinematic viscosity, η/ρ. We find (see e.g. Longair pp 145–146) that $R \sim 10^{12}$. The flow is *highly* turbulent, and so standard kinetic theory dynamic visosity $\eta = \frac{1}{3}\rho c\lambda$ will make a negligible contribution.

We do not yet understand the precise mechanism of viscosity in accretion discs. Highly turbulent flow helps, but precise calculations are difficult. Magnetic fields will be present and will certainly contribute. Much of the progress to date has come from a neat side-step developed by Shakura and Sunyaev (1972). They invented a parameter

$$\alpha = \frac{\nu}{hc_s}$$

which allows detailed models to be made without knowing the exact mechanism for the viscosity.

Luminosity of a thin accretion disc

Neglecting the energy transport due to viscosity, we can calculate the rate at which accreting material in the disc must lose gravitational potential energy if it is to fall closer to the accreting object. For an annulus between r and $r + \mathrm{d}r$, the energy which must be dissipated will be

$$L(r) = -\left(\frac{\mathrm{d}E}{\mathrm{d}t}\right) = \frac{G\dot{M}M}{2r^2}\mathrm{d}r$$

where $\dot{M}$ is the accretion rate and M is the mass of the central object. Including viscous energy transport we gain a total luminosity three times this value *(non-examinable—see Longair pp 149-150)*.

Temperature structure of a physically thin, optically thick disc

If the disc is optically thick, each annulus between r and $r + \mathrm{d}r$ will radiate as a blackbody with the luminosity derived above. Hence via Stefan's Law (remembering the disc has two surfaces), the annulus at at r will radiate with $2\sigma T^4 \times 2\pi r \mathrm{d}r$. Thus

$$\sigma T^4 = \frac{3GM\dot{M}}{8\pi r^3}$$

and so

$$\boxed{T(r) = \left(\frac{3G\dot{M}M}{8\pi r^3 \sigma}\right)^{1/4}}$$

Spectrum of the thin disc

We are now in a position to describe the form of the overall spectrum of the disc, i.e. the sum of all the black-body contributions at different radii.

$$I_\nu \propto \int_{r_{\mathrm{inner}}}^{r_{\mathrm{outer}}} 2\pi r B_\nu \left\{T(r)\right\} \mathrm{d}r$$

where from lecture 1 we have

$$B_\nu \propto \nu^3 \left(\mathrm{e}^{h\nu/kT} - 1\right)^{-1}$$

From before we have $T \propto r^{-3/4}$ so d$r \propto (1/T)^{1/3}$d$(1/T)$ and we can integrate dT instead of dr. Including B_ν explicitly we therefore have

$$I_\nu \propto \int_{r_{\text{inner}}}^{r_{\text{outer}}} \left(\frac{1}{T}\right)^{4/3} \nu^3 \left(\mathrm{e}^{h\nu/kT} - 1\right)^{-1} \left(\frac{1}{T}\right)^{1/3} \mathrm{d}\left(\frac{1}{T}\right)$$

We can proceed by changing variable $x = (h\nu/kT)$—recall from the second-year thermo problem set where you used the same substitution to derive the functional form $u(T) \propto T^4$. This yields

$$I_\nu \propto \frac{\nu^3}{\nu^{8/3}} \int_{x_{\text{inner}}}^{x_{\text{outer}}} x^{4/3} \left(\mathrm{e}^{x} - 1\right)^{-1} x^{1/3} \mathrm{d}x$$

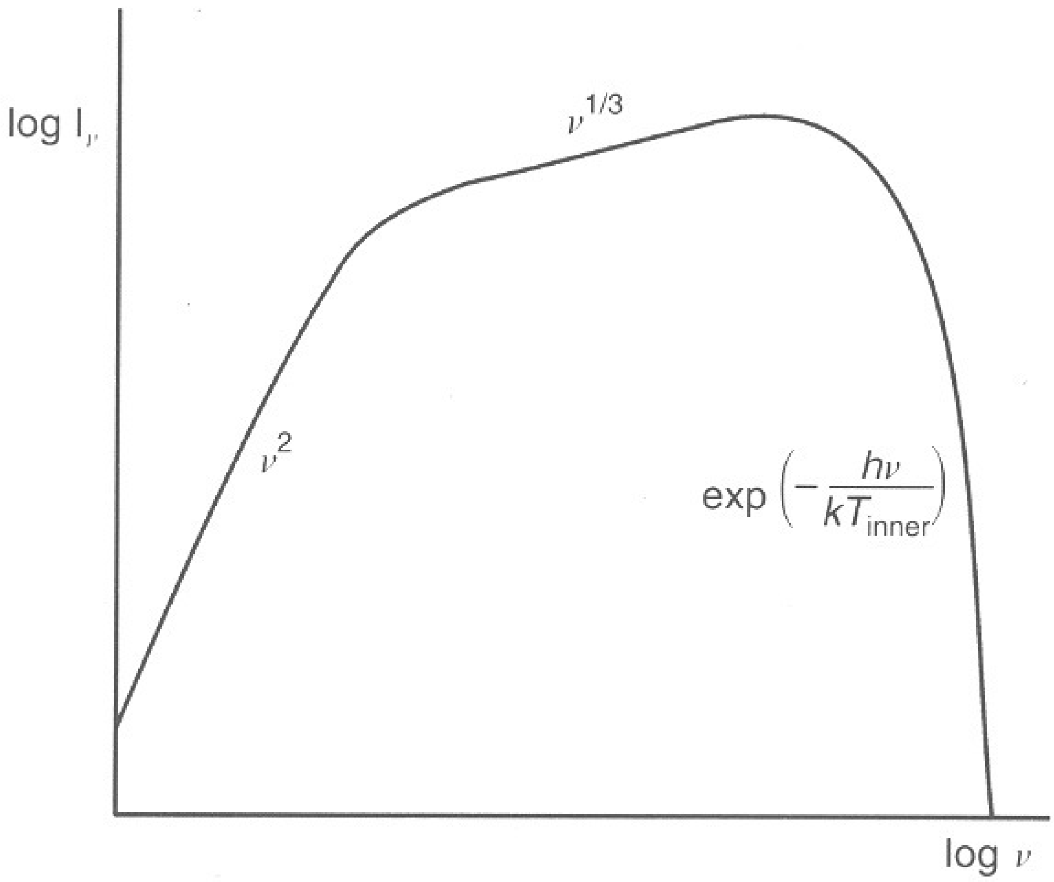
The integral dx is just a numerical constant so we now have the shape of the spectrum over most of its range:

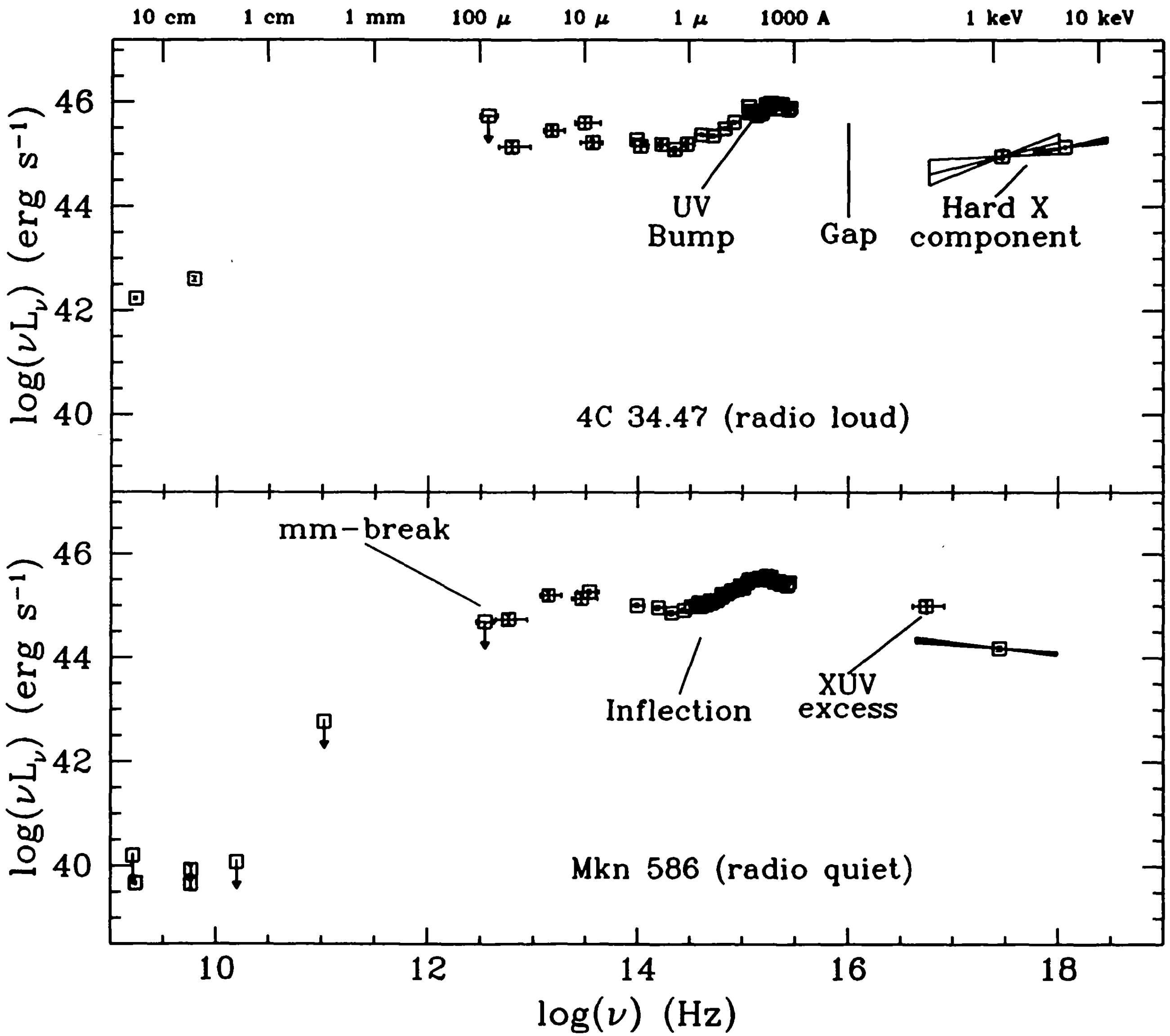
$$I_\nu \propto \nu^{1/3}.$$

Note, though, that the low- and high-frequency ends will have a different form:

- From the outer edge of the disc we will see the Rayleigh-Jeans tail of T_{outer}, $I_\nu \propto \nu^2$.
- From the inner edge, an exponential cut-off $I_\nu \propto \left(\mathrm{e}^{-h\nu/kT_{\mathsf{inner}}}\right)$.

Theoretical spectrum of thin accretion disc.





10 cm
1 cm
1 mm
100 μ
10 μ
1 μ
1000 A
1 keV
10 keV
log(νL_ν) (erg s^-1)
46
44
42
40
UV
Bump
Gap
Hard X
component
4C 34.47 (radio loud)
mm-break
Inflection
XUV
excess
Mkn 586 (radio quiet)
10
12
14
16
18
log(ν) (Hz)

Total luminosity of the thin disc

Let's now estimate ϵ. If we approximate with a Newtonian potential, take the particle to have started its trip at $r = \infty$ and total energy zero, and calculate the total energy it has on the last stable orbit. The amount of GPE which must be lost (by radiation) is equivalent to 1/12 of the rest-mass energy of the particle.

For the best possible case—the closest orbit around a rapidly-rotating black hole—the efficiency rises to a whopping 0.42. Compare this to nuclear fusion in stars, which has an efficiency of only 0.7 percent!

Hence in practice, astronomers usually adopt an approximate value of $\epsilon = 0.1$ for accretion onto black holes.

Example: estimating a quasar accretion rate

Suppose we observe a quasar to have a total power output of 10^{40} W. We are now in a position to estimate the mass of the central black hole and the rate at which its mass is increasing.

First let us assume that the accretion is Eddington limited. From our equation for the Eddington luminosity we have

$$L_{\mathrm{Edd}} = \frac{4\pi G M c m_{\mathrm{p}}}{\sigma_{\mathrm{T}}}$$

from which

$$M = 7 \times 10^{8} M_{\odot}.$$

And from our Eddington accretion rate, using $\epsilon = 0.1$ we have

$$\dot{M}_{\mathrm{Edd}} = \frac{4\pi G M m_{\mathrm{p}}}{\epsilon c \sigma_{\mathrm{T}}} \approx 3 M_{\odot} \mathrm{yr}^{-1}$$

Getting round the Eddington limit

The accretion may not always be Eddington-limited. It is, for example, possible to achieve $\dot{M}$ much greater than would be inferred by using the Eddington luminosity with ϵ = 0.1, by making the disc physically thick, and very low density, so that it is optically thin and matter doesn't have time to radiate away so much energy before it falls over the horizon. This has the advantages of allowing black holes to grow at a very high rate in the early Universe, and also of providing "funnels" which could be a mechanism for collimating outflows from accreting objects.

Unfortunately simple analytical models of these discs are unstable, but the advantages of thick discs are so great that much effort is put into modelling them numerically... including the effects of strong magnetic fields.

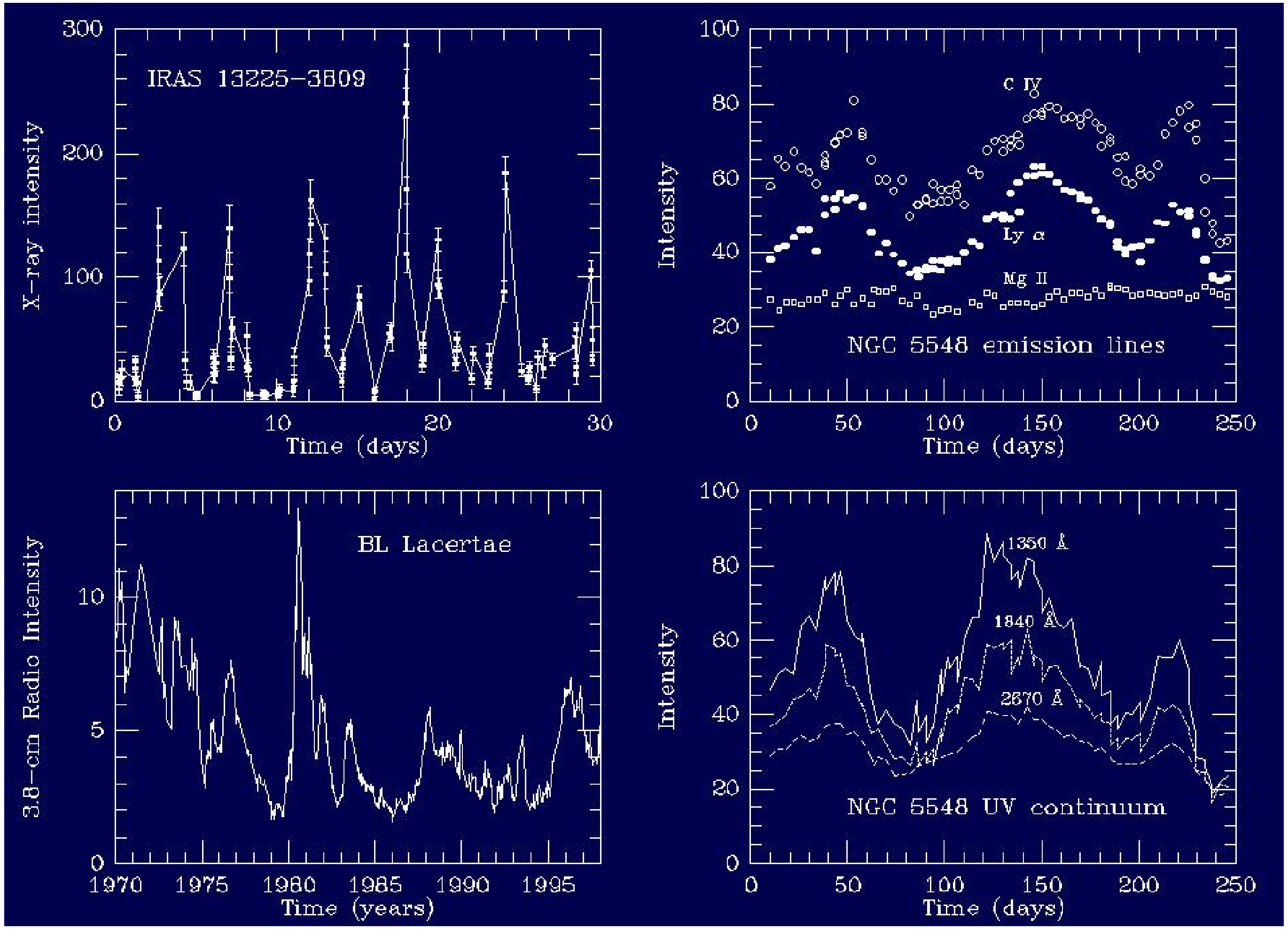
It is also possible for an object to have a luminosity significantly greater than the Eddington luminosity:

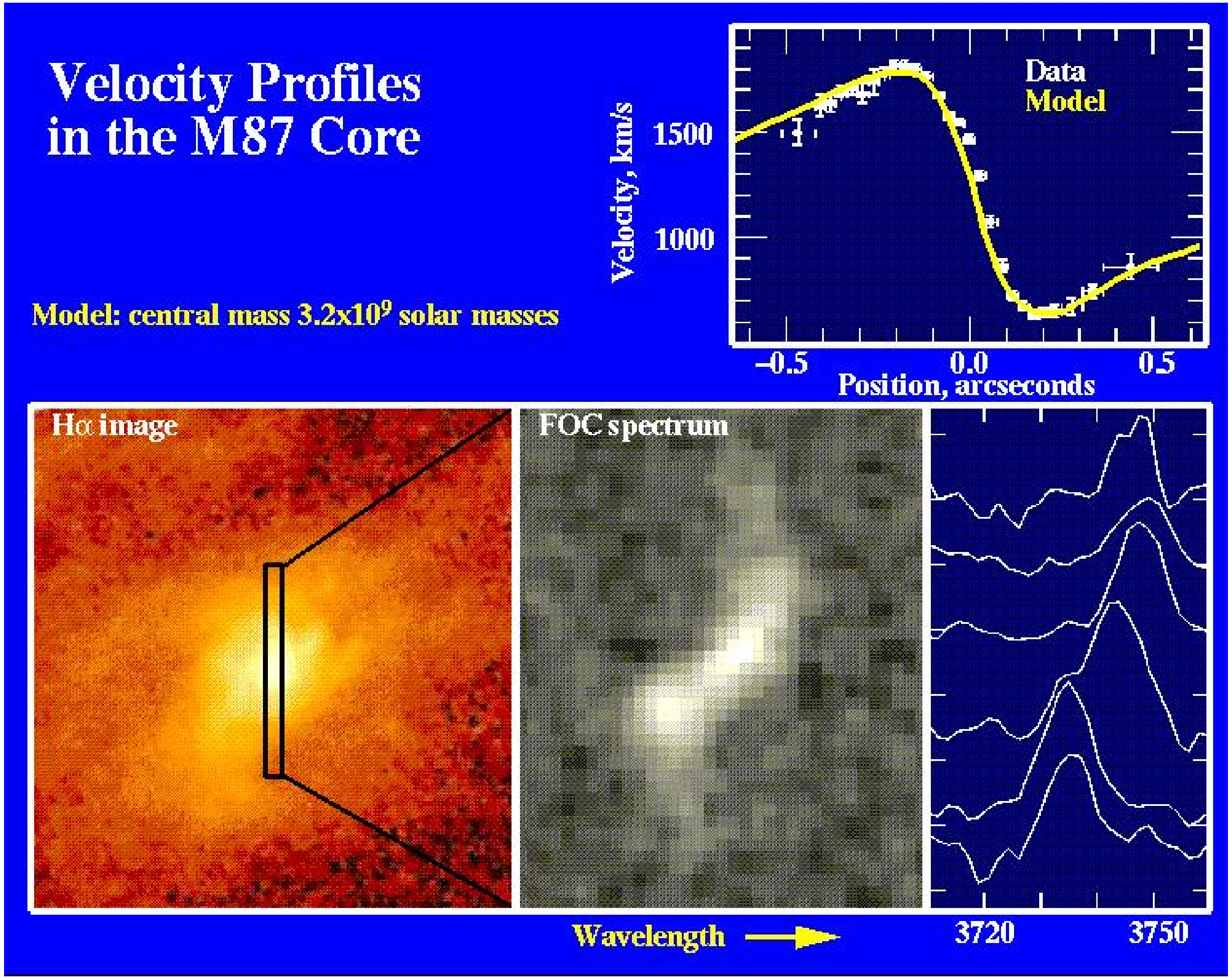
- In supernovae (somewhat trivially!)
- Where spherical symmetry is broken, with extremely collimated radiation in a direction different to the accretion direction
- Where accretion is not steady, e.g. bursts or radiation emitted when discrete clouds of matter fall onto a neutron star or white dwarf.

Evidence for black holes in AGN

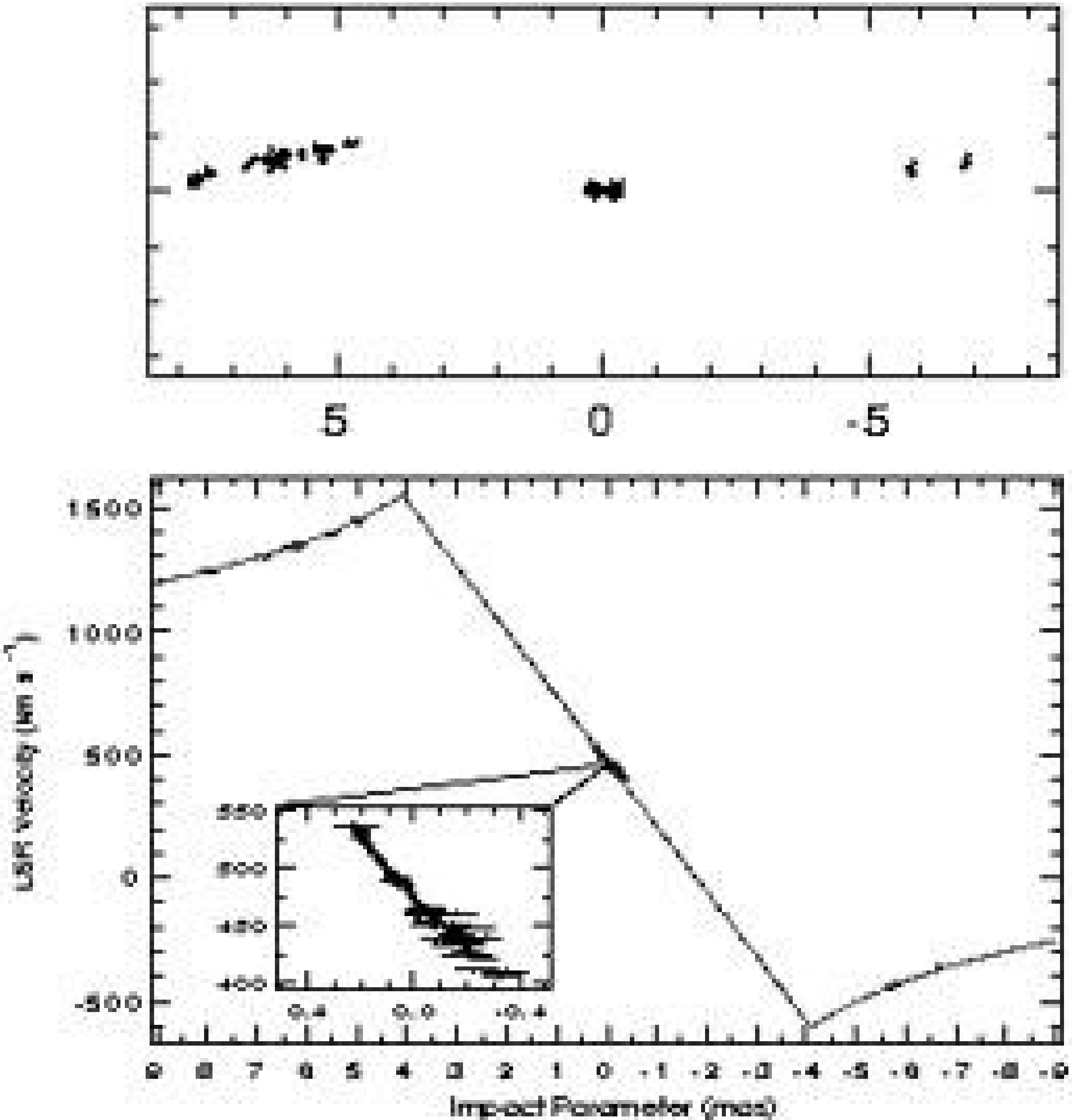
There are several canonical pieces of evidence that supermassive black holes really are there at the heart of AGN. Among these are:

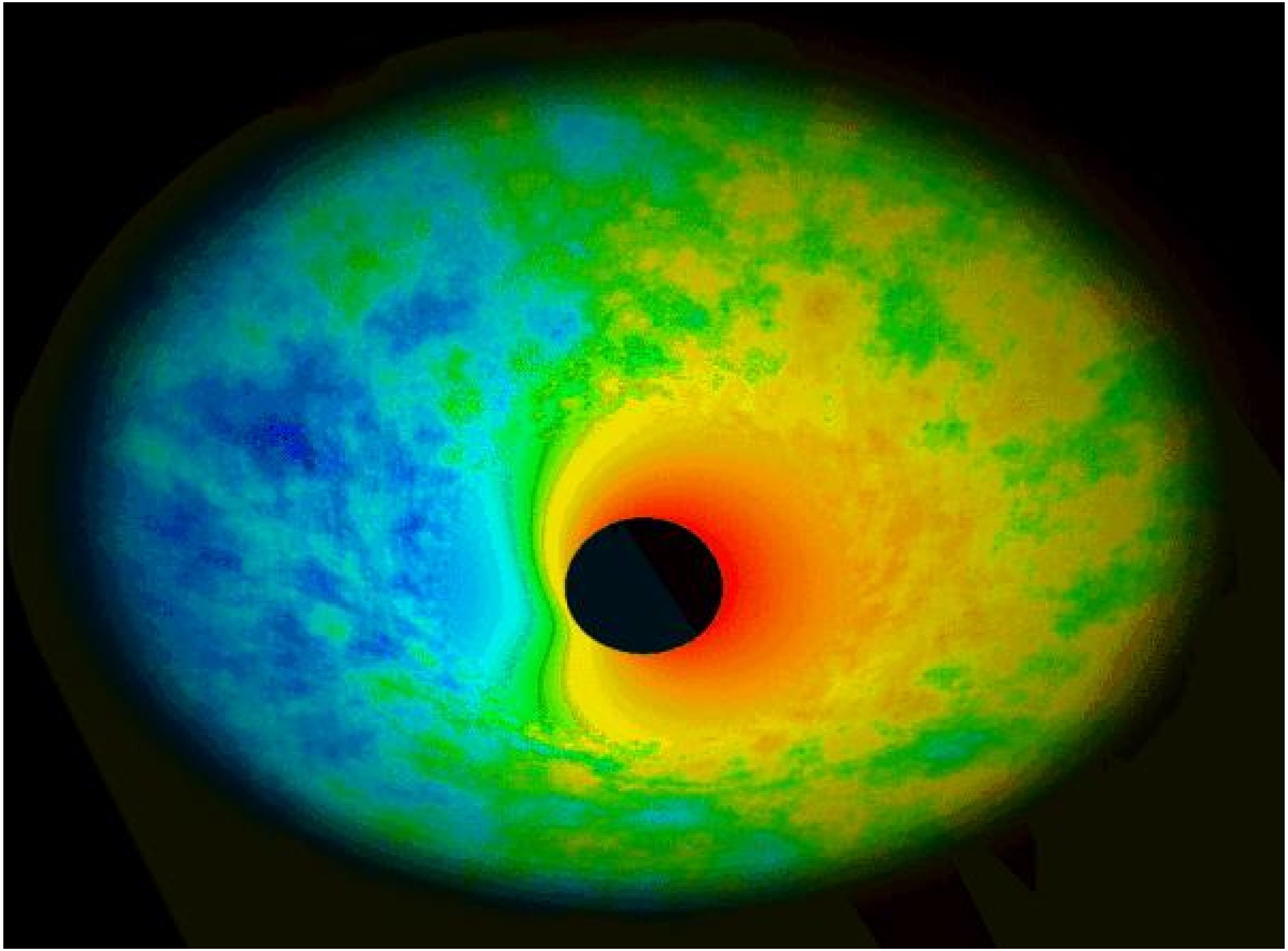
- Variability (in combination with Eddington luminosity).
- Stellar velocity dispersions.
- Rotation speeds inferred from emission lines.
- The controversial history of X-ray line profiles.

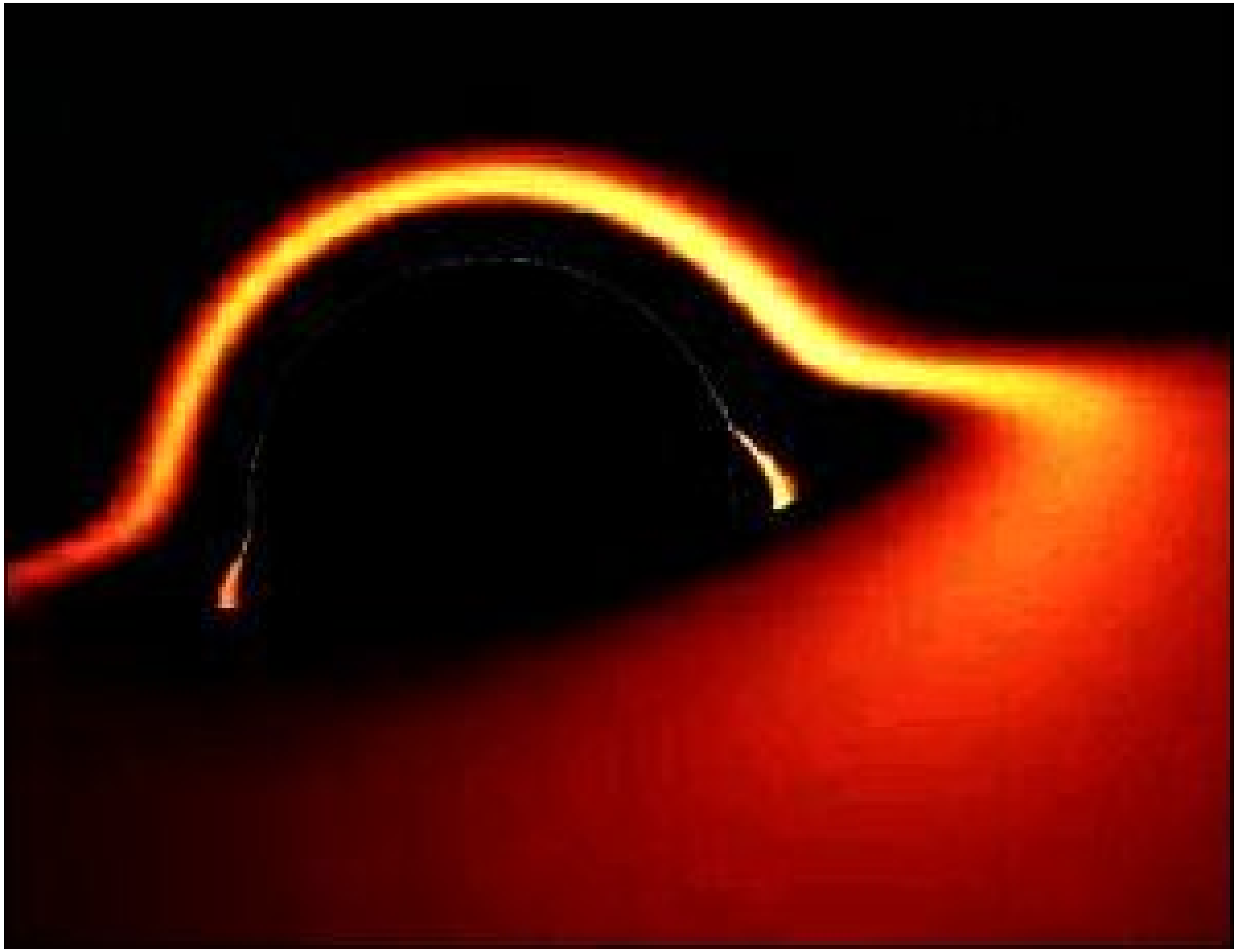

IRAS 13225-3809
X-ray intensity
Time (days)
C IV
Ly α
Mg II
NGC 5548 emission lines
Intensity
Time (days)
BL Lacertae
3.8-cm Radio Intensity
Time (years)
1350 Å
1840 Å
2670 Å
NGC 5548 UV continuum
Intensity
Time (days)

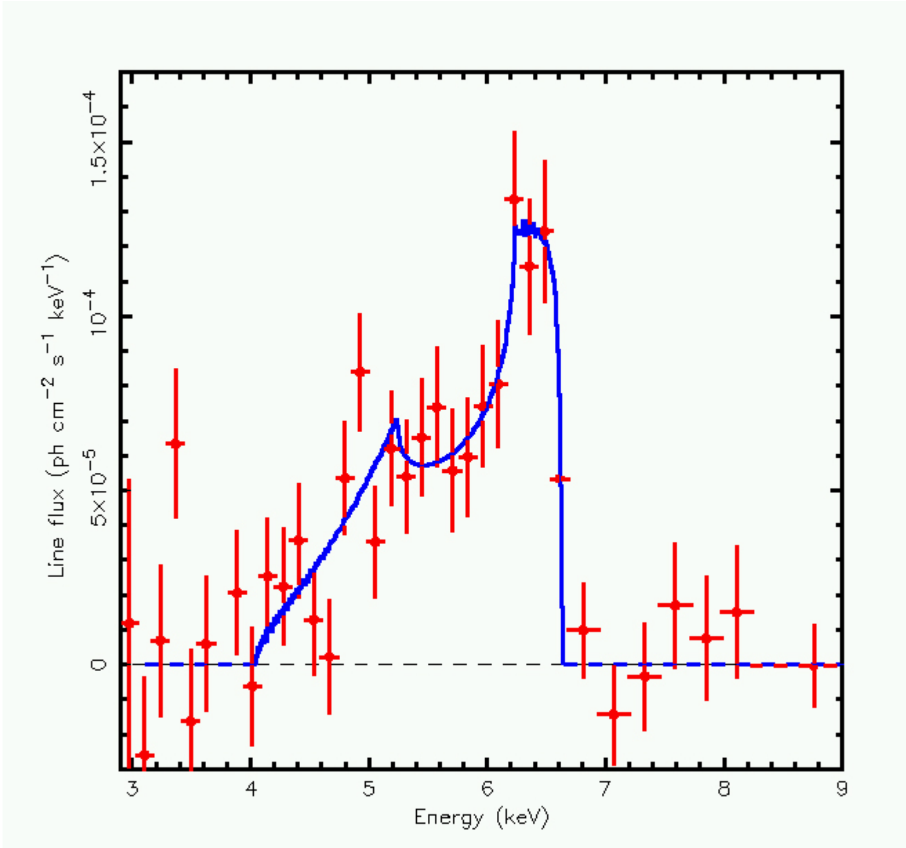


Velocity Profiles
in the M87 Core
Model: central mass 3.2x10^9 solar masses
Data
Model
Velocity, km/s
1500
1000
–0.5
0.0
0.5
Position, arcseconds
Hα image
FOC spectrum
Wavelength
3720
3750


5
0
-5
1500
1000
500
0
-500
LSR Velocity (km s^{-1})
550
500
450
400
0.4
0.0
-0.4
9 8 7 6 5 4 3 2 1 0 -1 -2 -3 -4 -5 -6 -7 -8 -9
Impact Parameter (mas)







Line flux (ph cm^{-2} s^{-1} keV^{-1})
1.5×10^{-4}
10^{-4}
5×10^{-5}
0
3
4
5
6
7
8
9
Energy (keV)