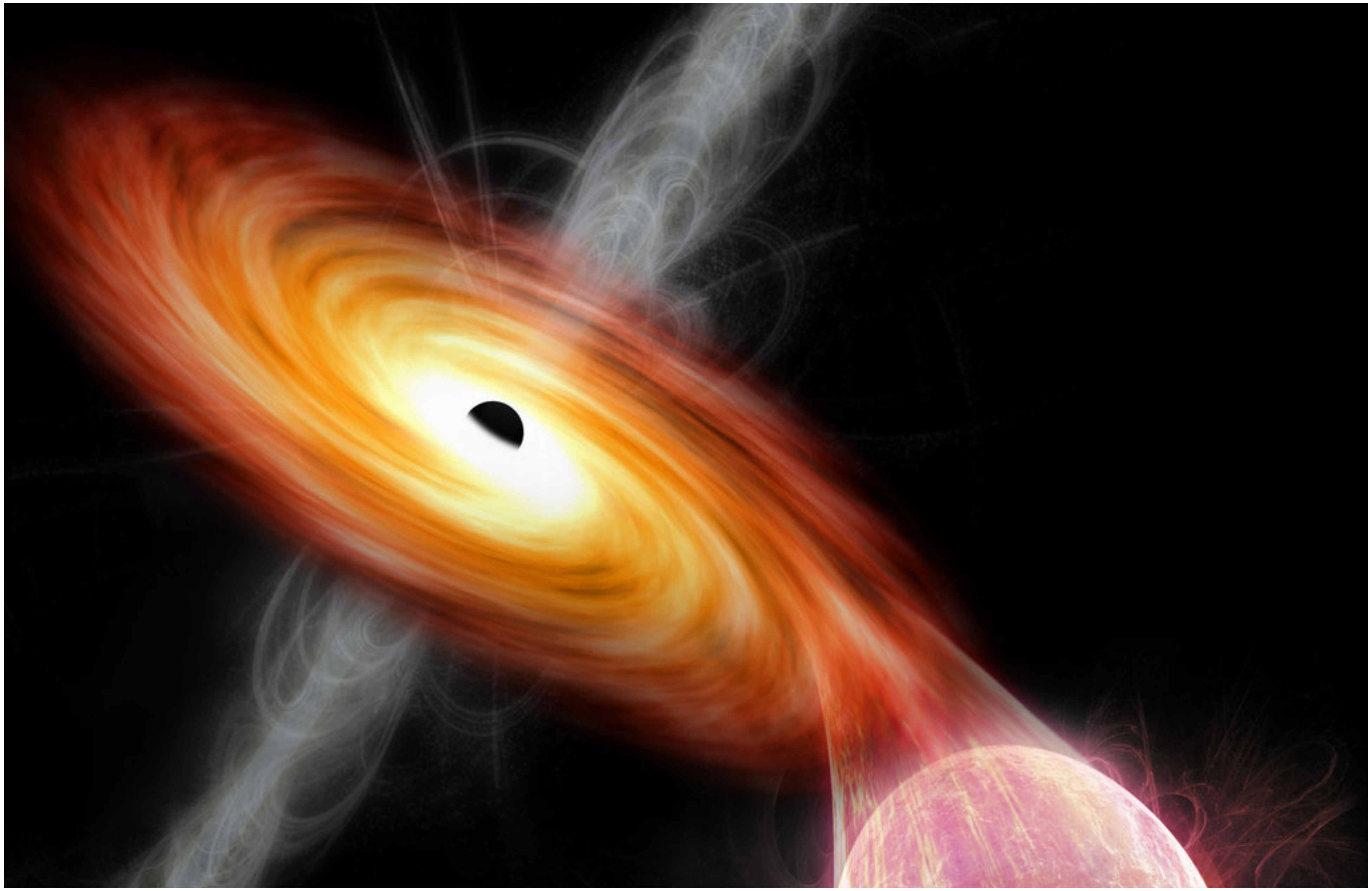
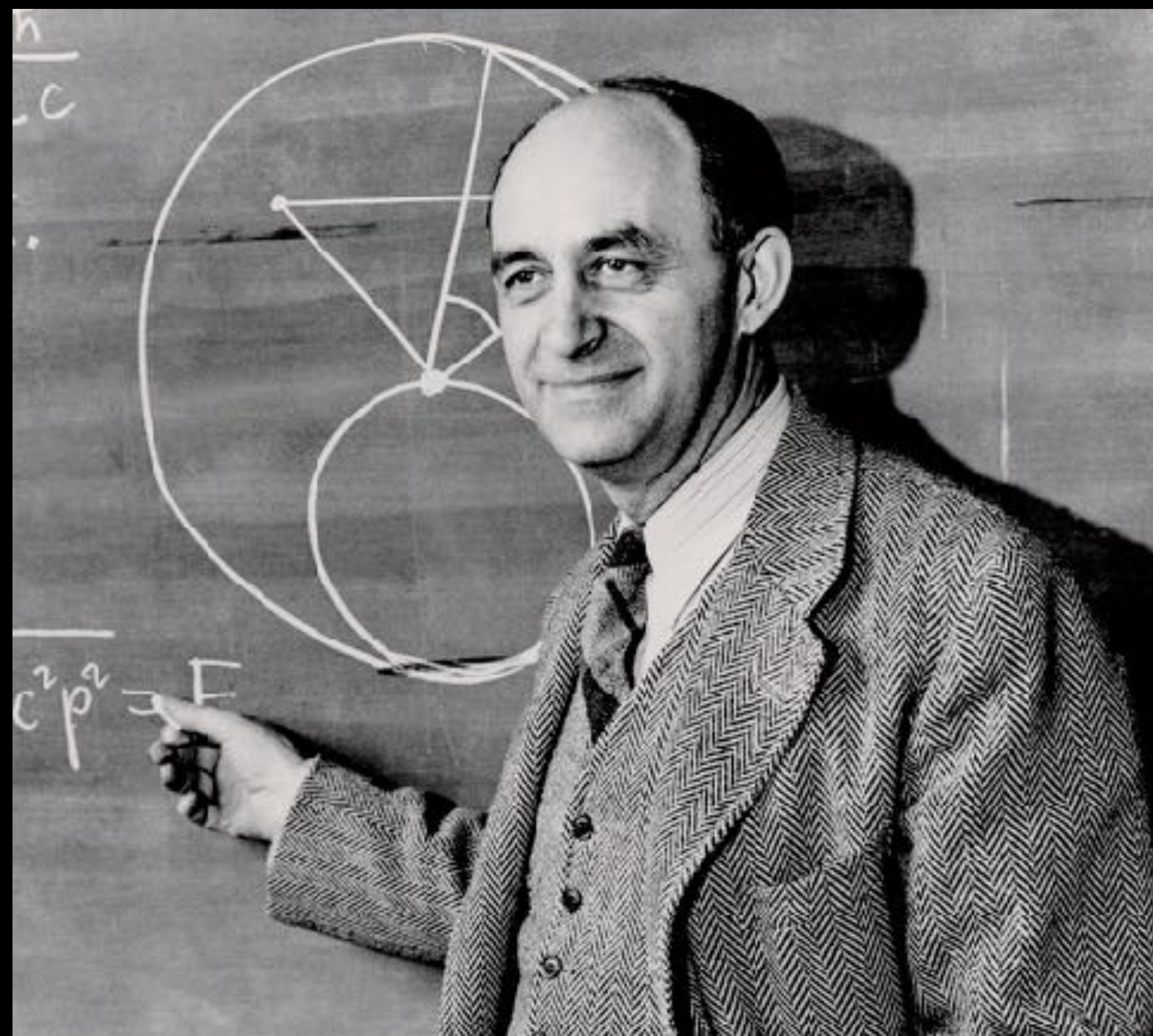
High Energy Astrophysics

Dr. Adam Ingram



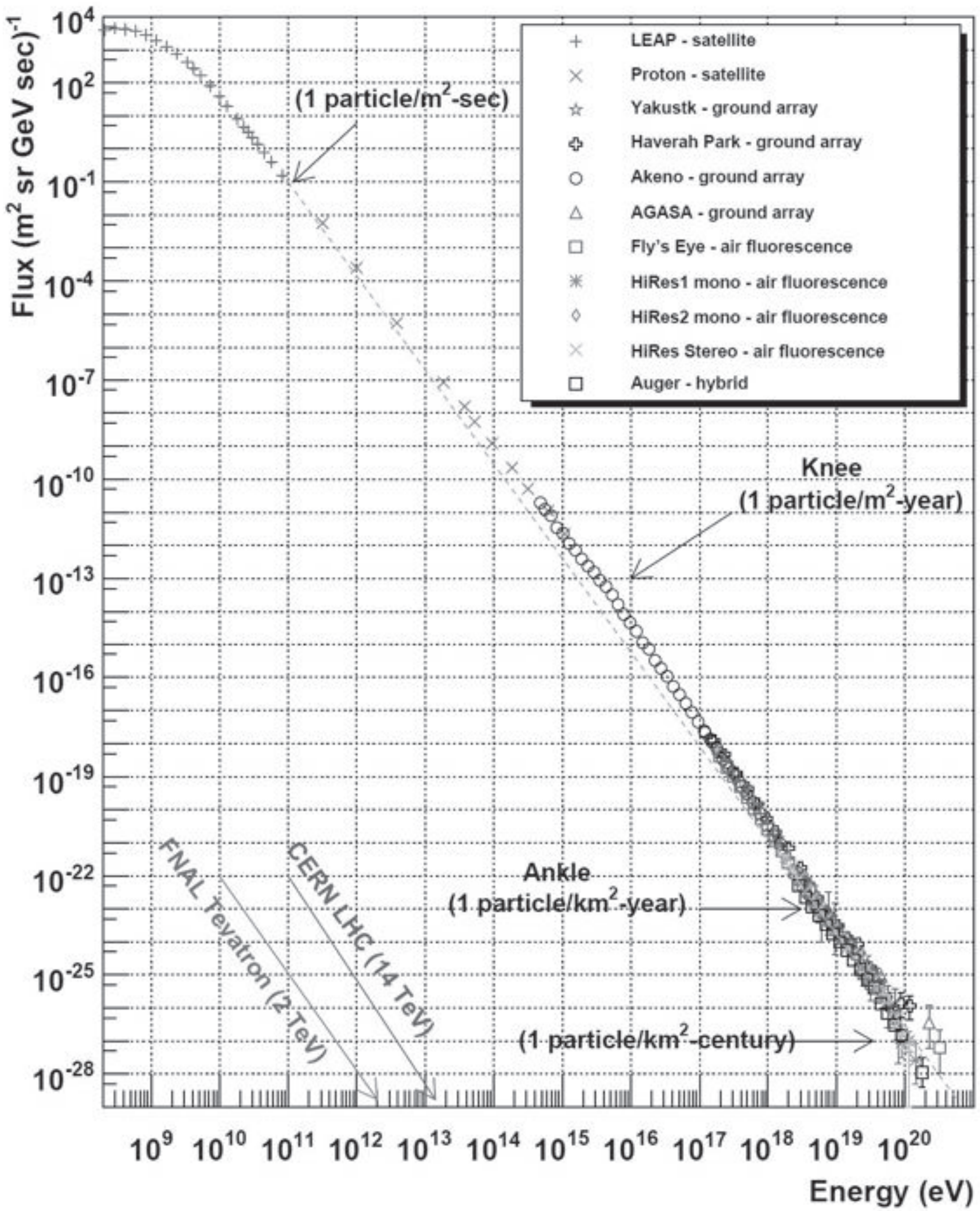
Lecture 2
Shock Acceleration



Enrico Fermi

Cosmic Rays

Earth is hit by highly energetic elementary particles



- Discovered in 1912 by Hess on a balloon flight (1936 Nobel prize)
- ~85% protons, ~12% He nuclei, ~1% heavier nuclei, ~2% electrons.
- ~power-law across many decades in energy:

$$\frac{dN}{dE} \propto E^{-k}, \quad k \sim 2.5$$

- "Ankle and "knee"
- Highest energies ~10^{20} eV!!

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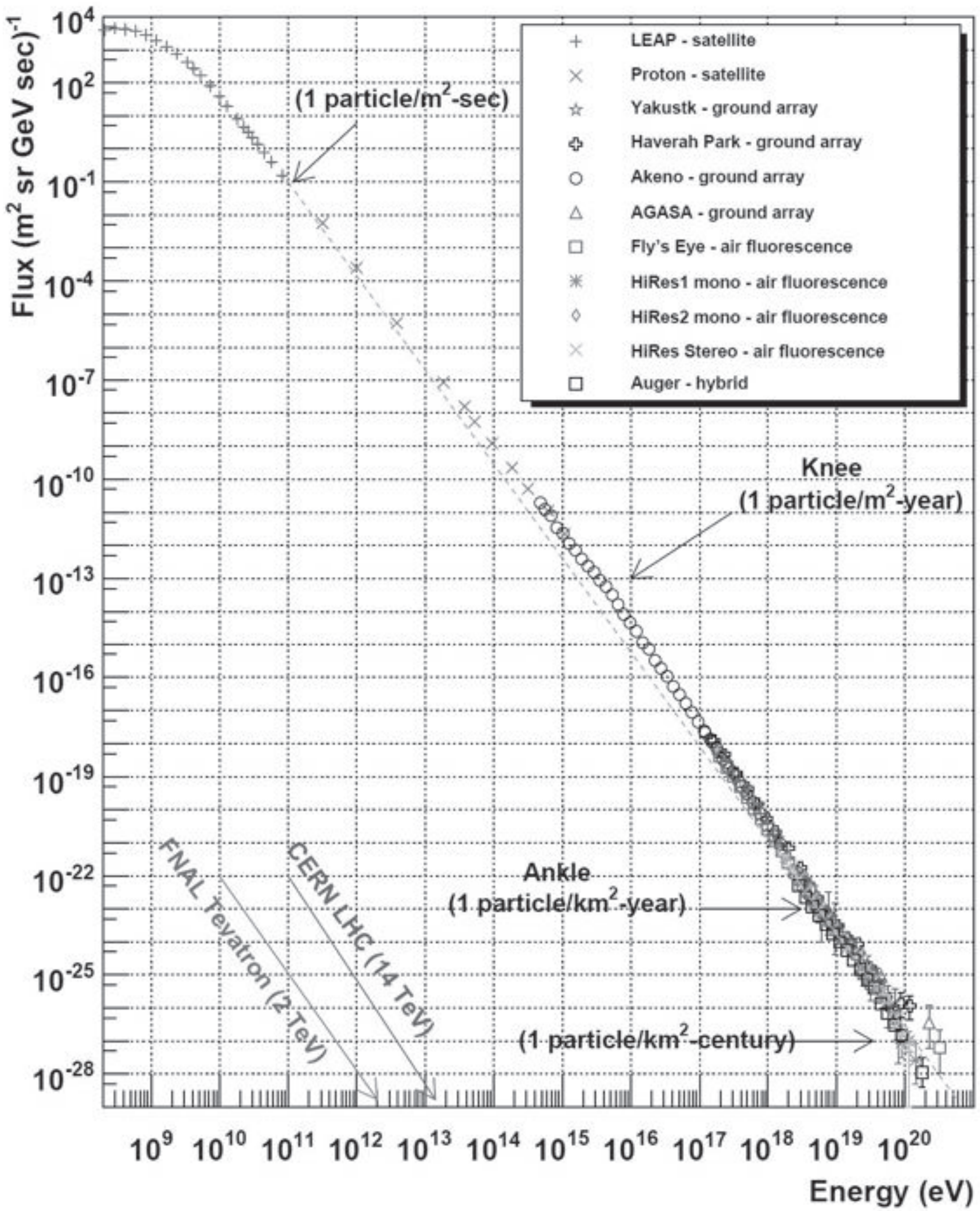

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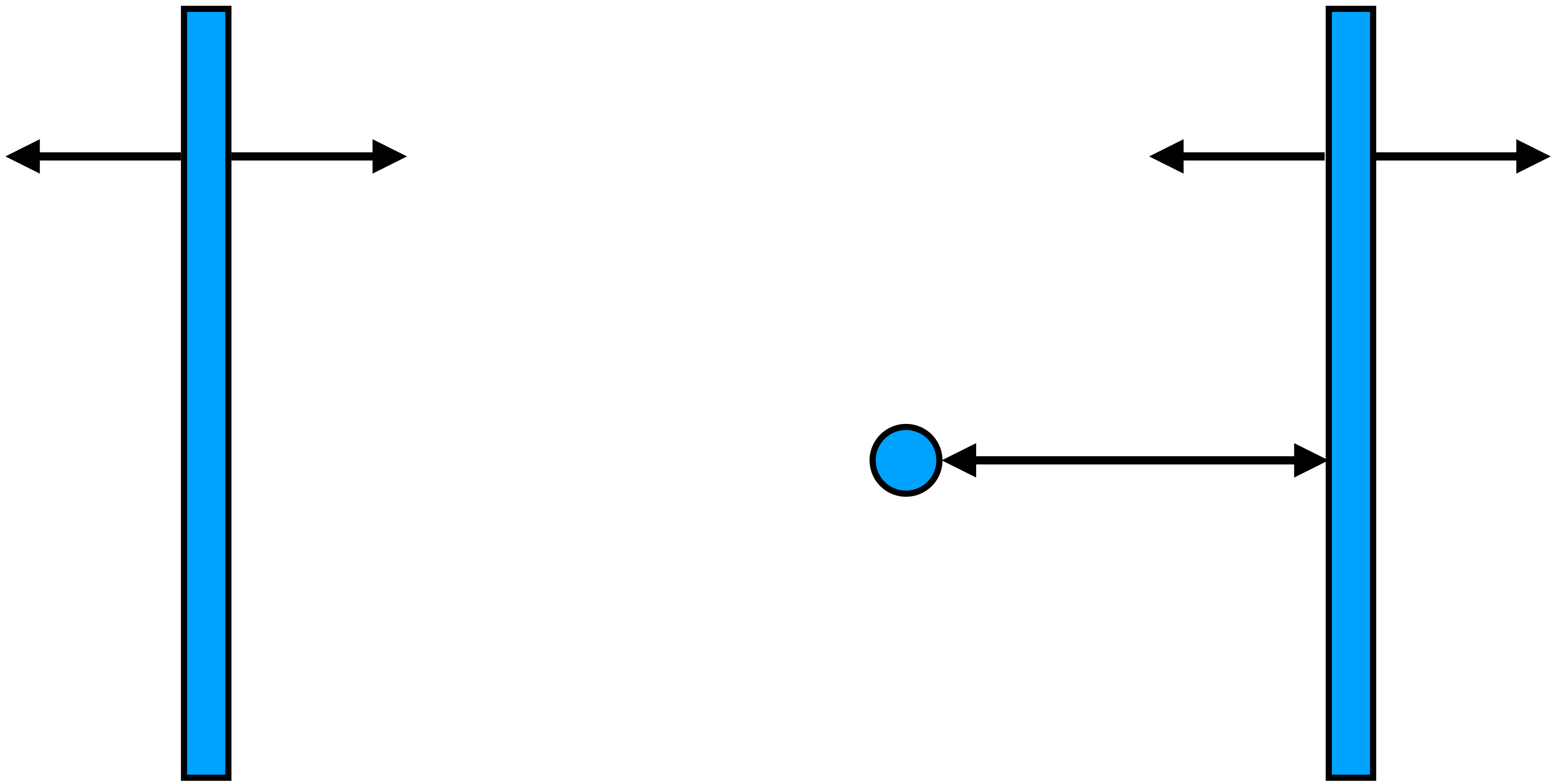
How to accelerate particles to such high energies?

Fermi acceleration

Fermi (1949) considered mildly relativistic particles reflecting off randomly moving “magnetic mirrors” in the Galaxy.

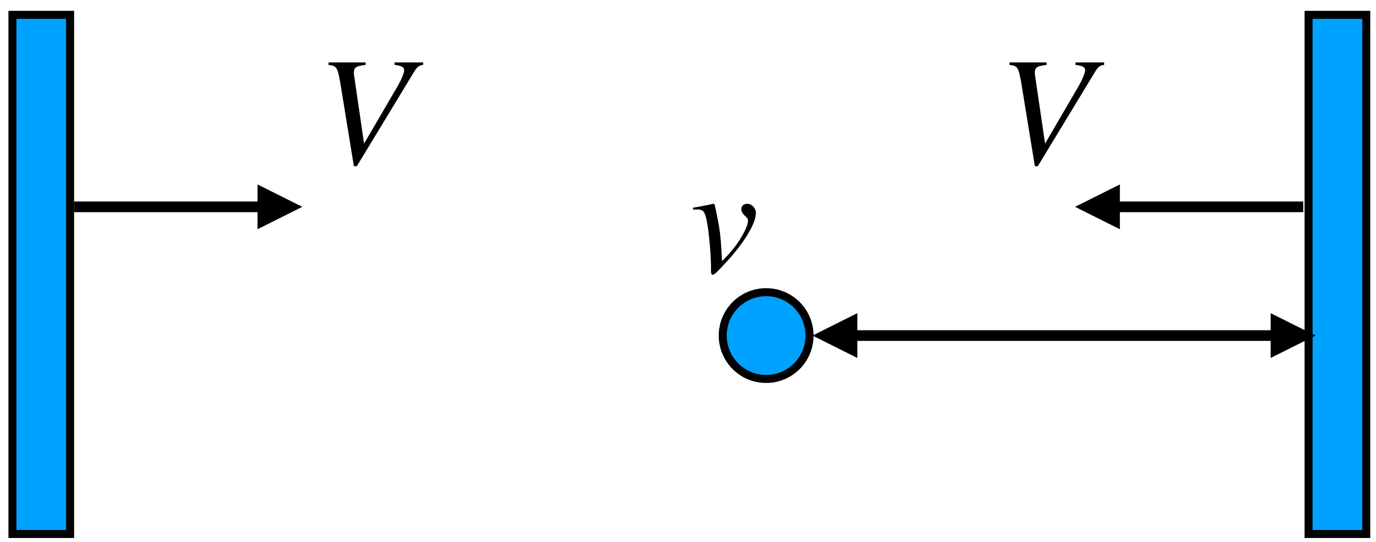


He showed that particles can, on average gain energy from bouncing off these mirrors many times.

Fermi acceleration

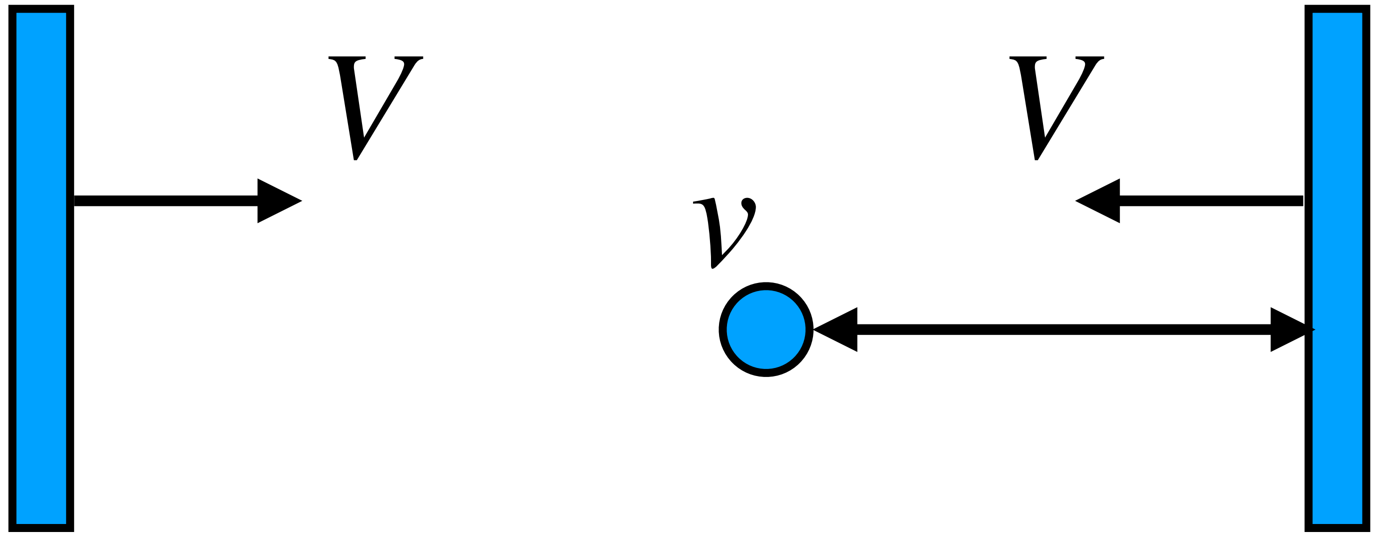
We can understand the concept by thinking of the mirrors moving together like the 'trash compactor' scene in Star Wars.

Fermi acceleration



Assume: $V \ll c$ $v \sim c$ No recoil of mirrors

Fermi acceleration



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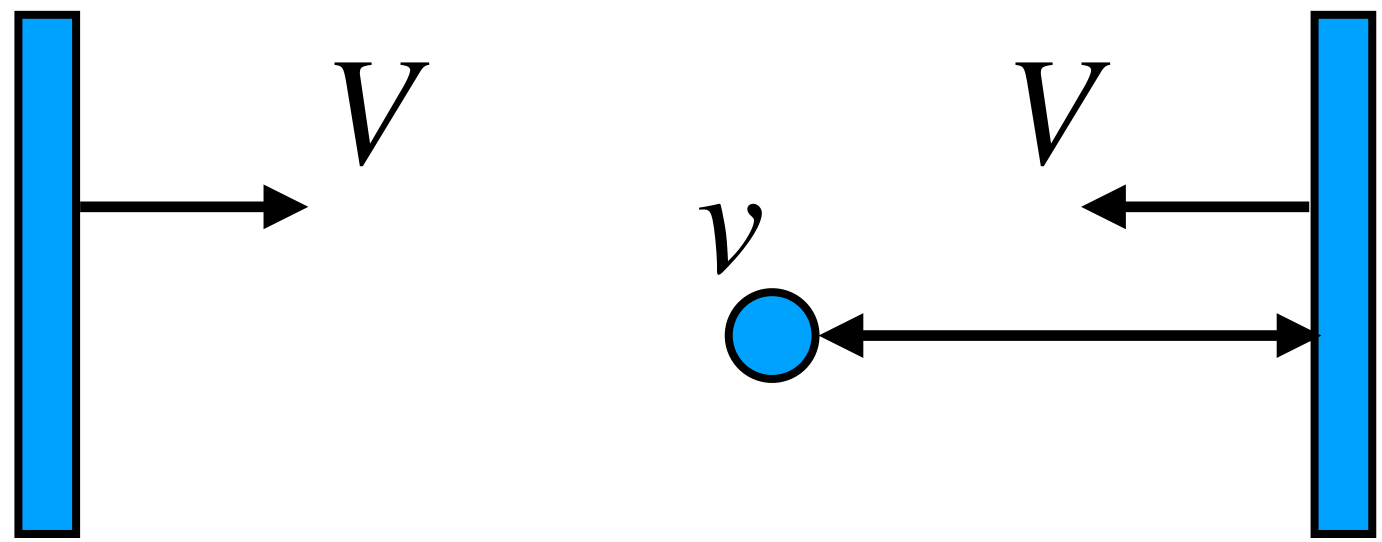
Momentum increase per collision $= \gamma m V$

$$\gamma = \frac{1}{\sqrt{1-(V/c)^2}}$$

Fermi acceleration



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$$E^2 = (pc)^2 + (m_o c^2)^2$$

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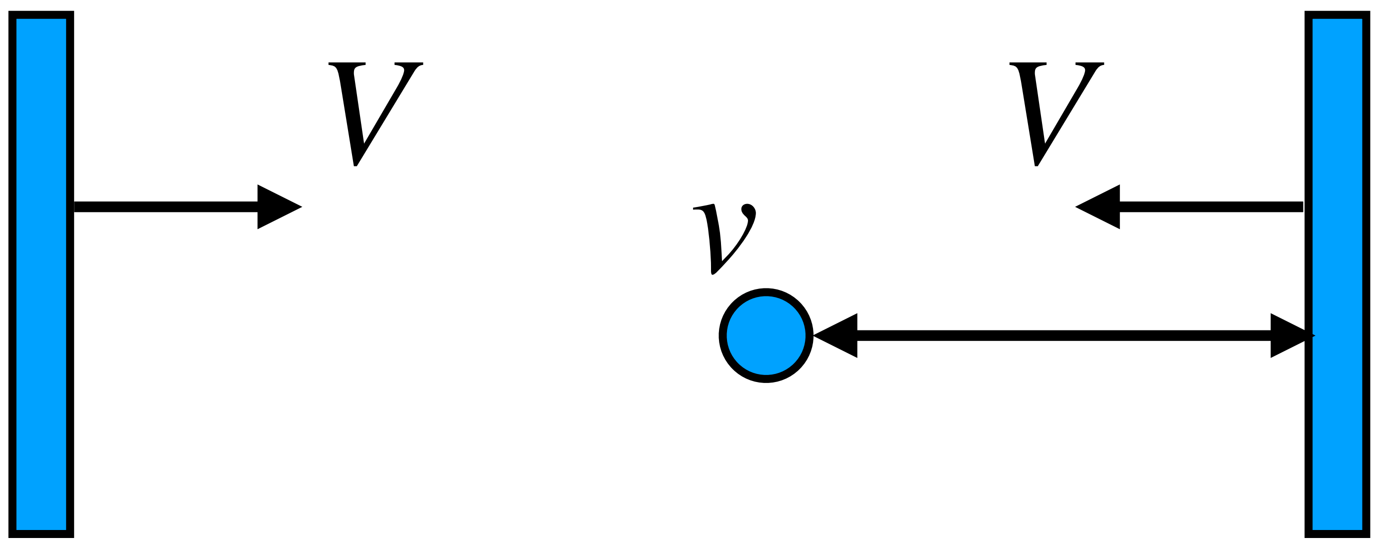

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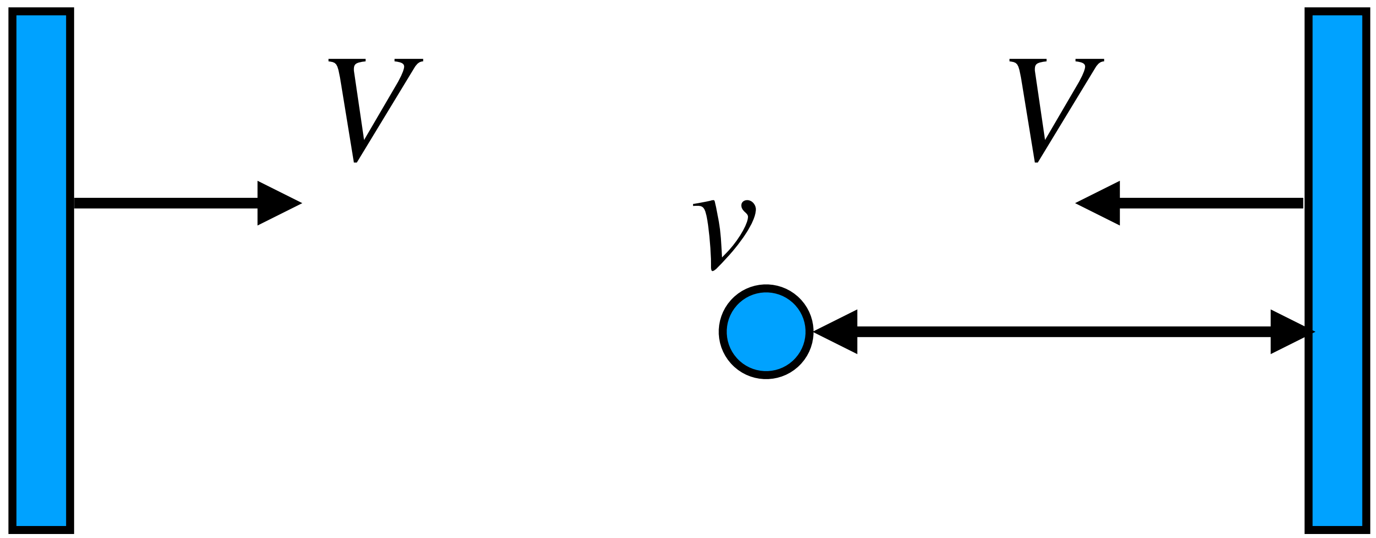

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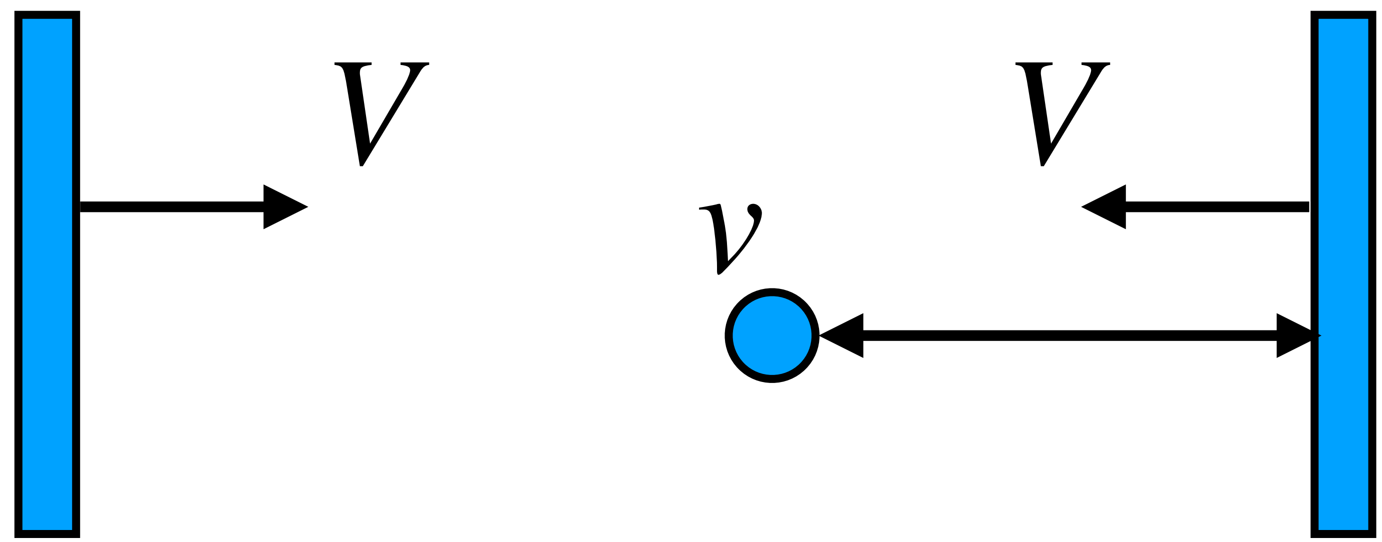

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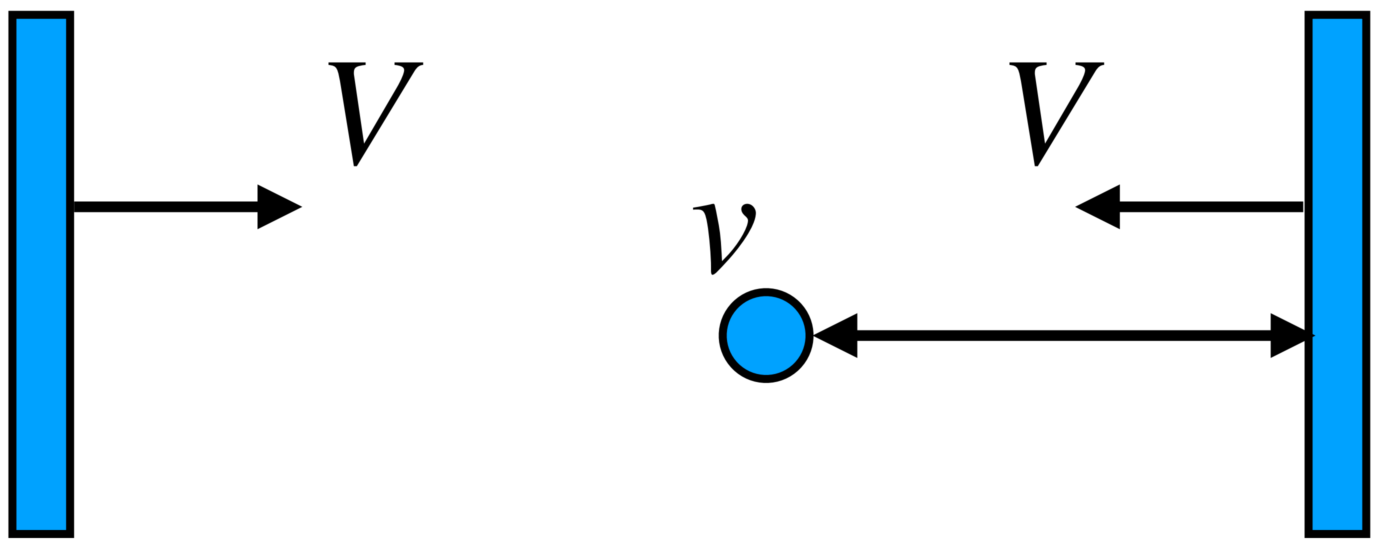

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$$\text{Energy increase per collision} \approx \gamma m V c = \gamma m c^2 \frac{V}{c} = \frac{V}{c} E$$

$$\therefore\ \beta \equiv \frac{\text{Energy after collision}}{\text{Energy before collision}} = 1 + \frac{\Delta E}{E} \approx 1 + \frac{V}{c}$$

Fermi acceleration

- Same fractional energy gain per crossing
- Energy after n crossings: $E = E_0\beta^n$

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$$\frac{\ln(N/N_0)}{\ln(E/E_0)} = \frac{n \ln(P)}{n \ln(\beta)}$$

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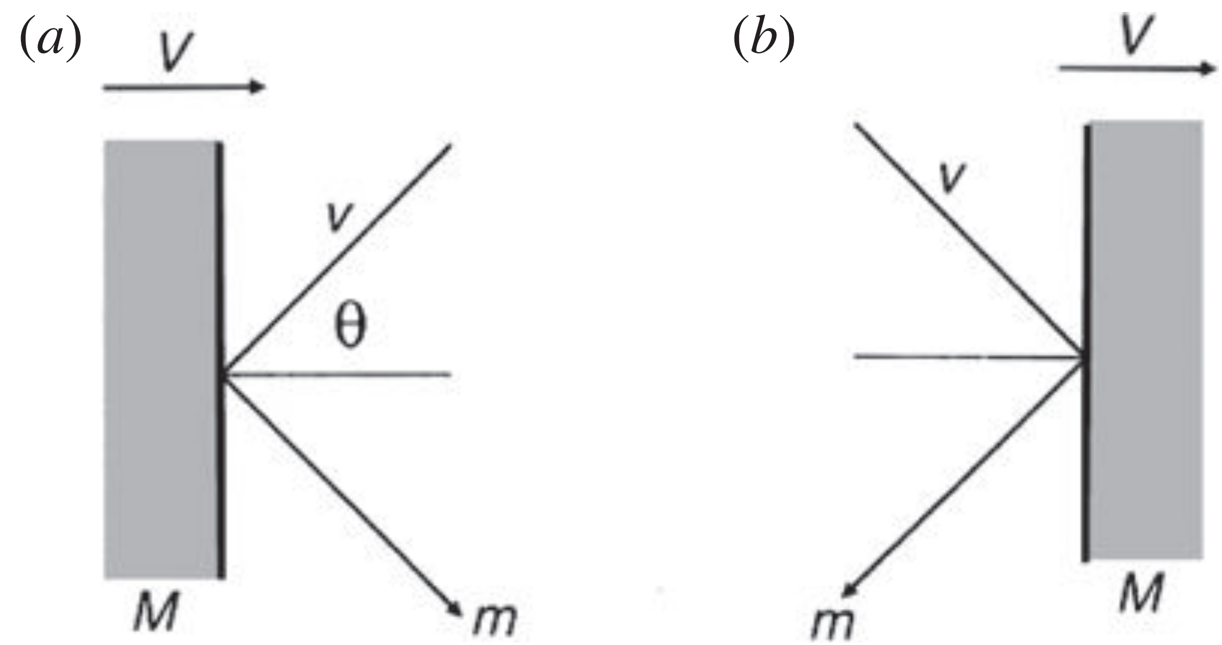
$$\therefore \frac{dN}{dE} \propto E^{\ln(P)/\ln(\beta)-1}$$

…get a power-law spectrum!

Fermi acceleration



From Longair (1994)

Illustrating the collision between a particle of mass m and a cloud of mass M: (a) a head-on collision; (b) a following collision. The probabilities of head-on and following collisions are proportional to the relative velocities of approach of the particle and the cloud, namely, $v + V\cos\theta$ for (a) and $v - V\cos\theta$ for (b). Since $v \approx c$, the probabilities are proportional to $1 + (V/c)\cos\theta$ where $0 < \theta < \pi$.

Fermi imagined randomly moving magnetic mirrors, and particles moving at all angles to the mirrors. Collisions with approaching mirrors more likely than those with receding mirrors, so net energy gain. Net gain per collision is $\Delta E = (8/3)(V/c)^2 E$ (see e.g. Longair 1994 for derivation). For this reason, Fermi's original model is called <u>*second order Fermi acceleration*</u>.

Fermi acceleration

Problems

1. How to get particles travelling at mildly relativistic v in the first place?
2. Random velocities of Galactic clouds is $V/c \sim 10^{-4}$, so fractional energy gain per collision is $\sim \Delta E/E \sim 10^{-8}$. Mean free path between clouds is ~0.1 pc, so very slow energy gain!
3. Get a power-law spectrum, but why specifically $\sim E^{-2.5}$?

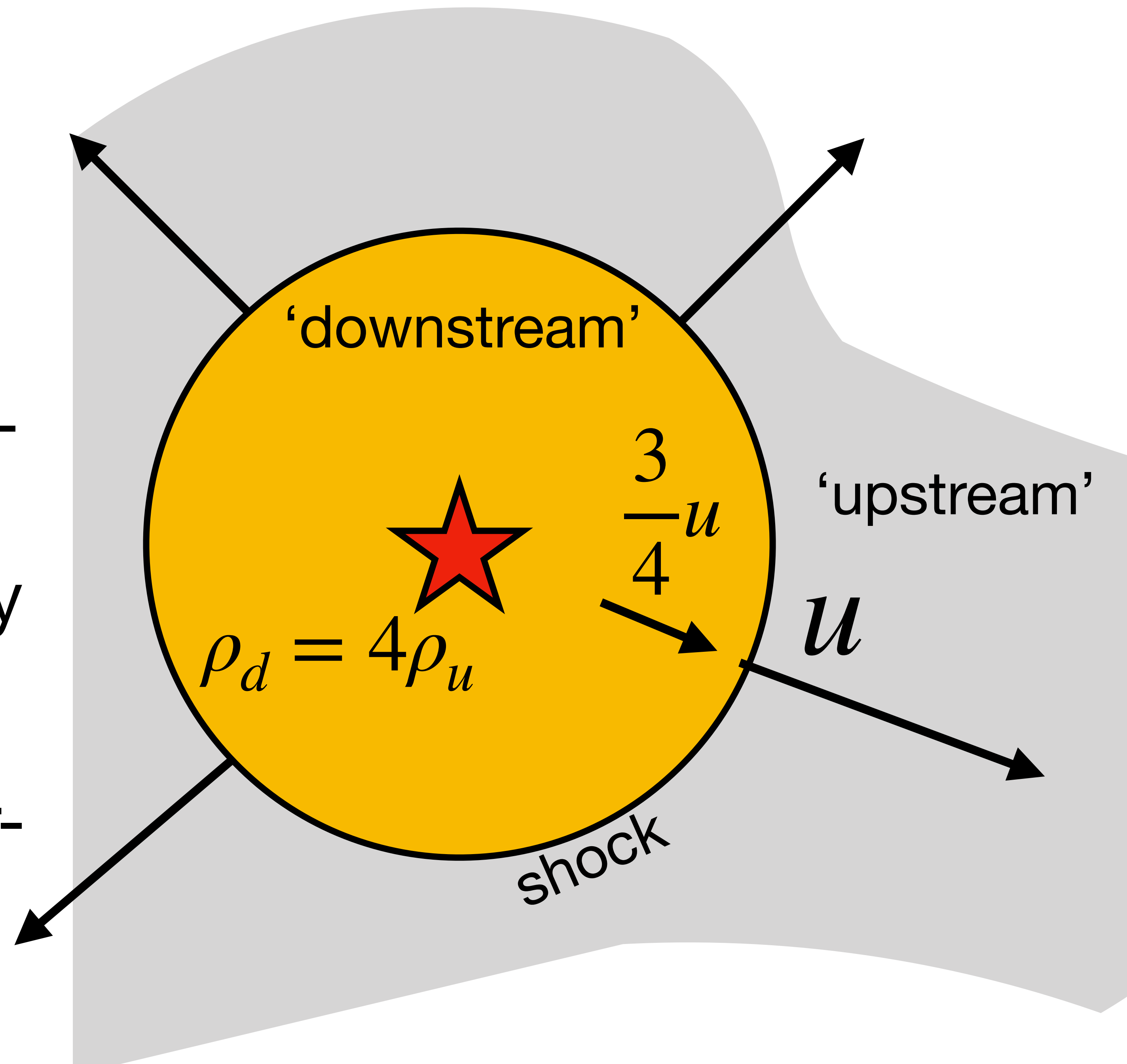
Shock acceleration

Considering particles at a shock front solves all the problems:

1. Already some mildly relativistic particles from the supernova / black hole jet causing the shock wave;

2. Mechanism to only get head-on collisions so ΔE/E~V/c, therefore easier to get to very high energies;

3. Can predict a specific power-law index.



Bell (1978)

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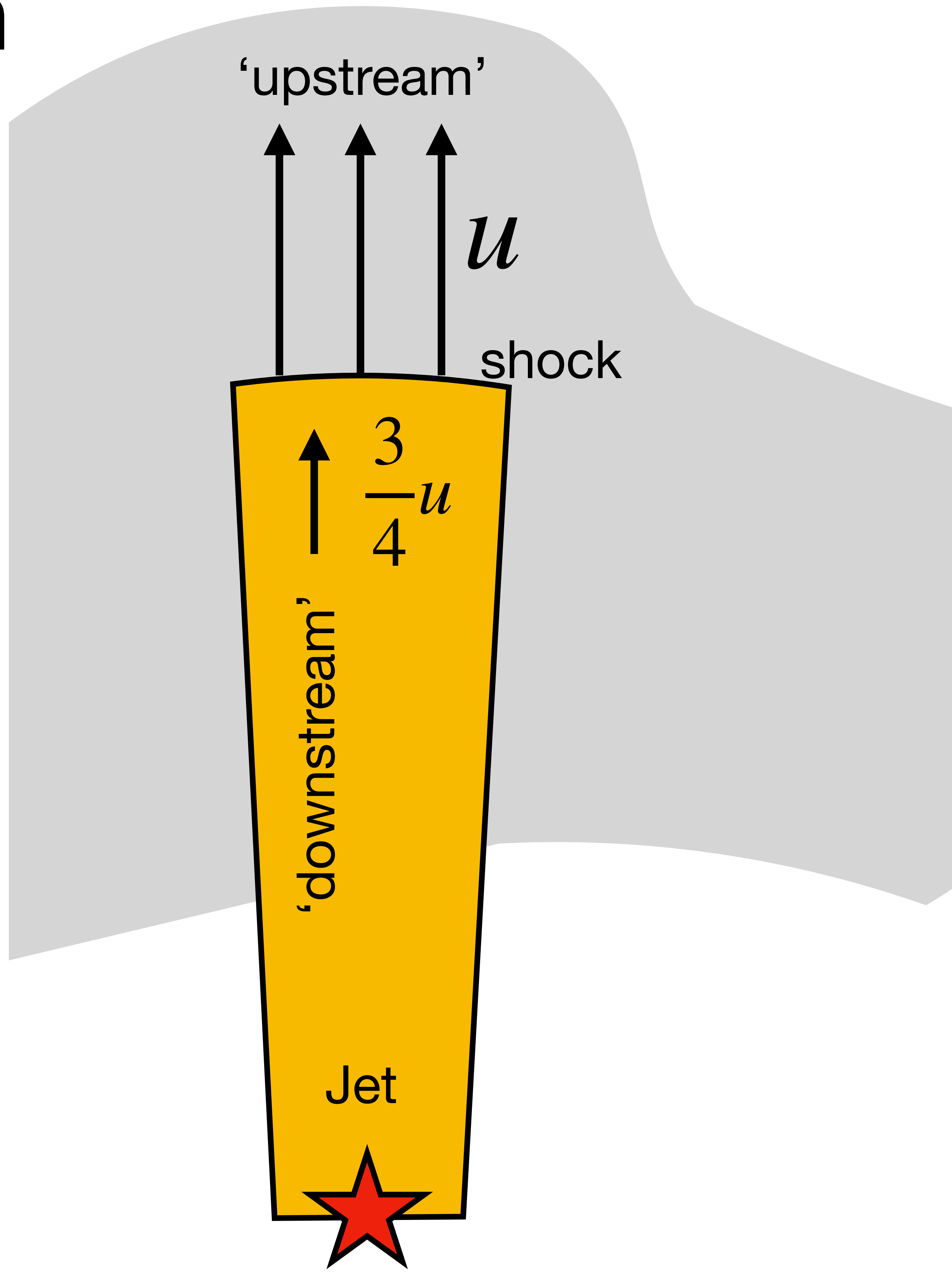

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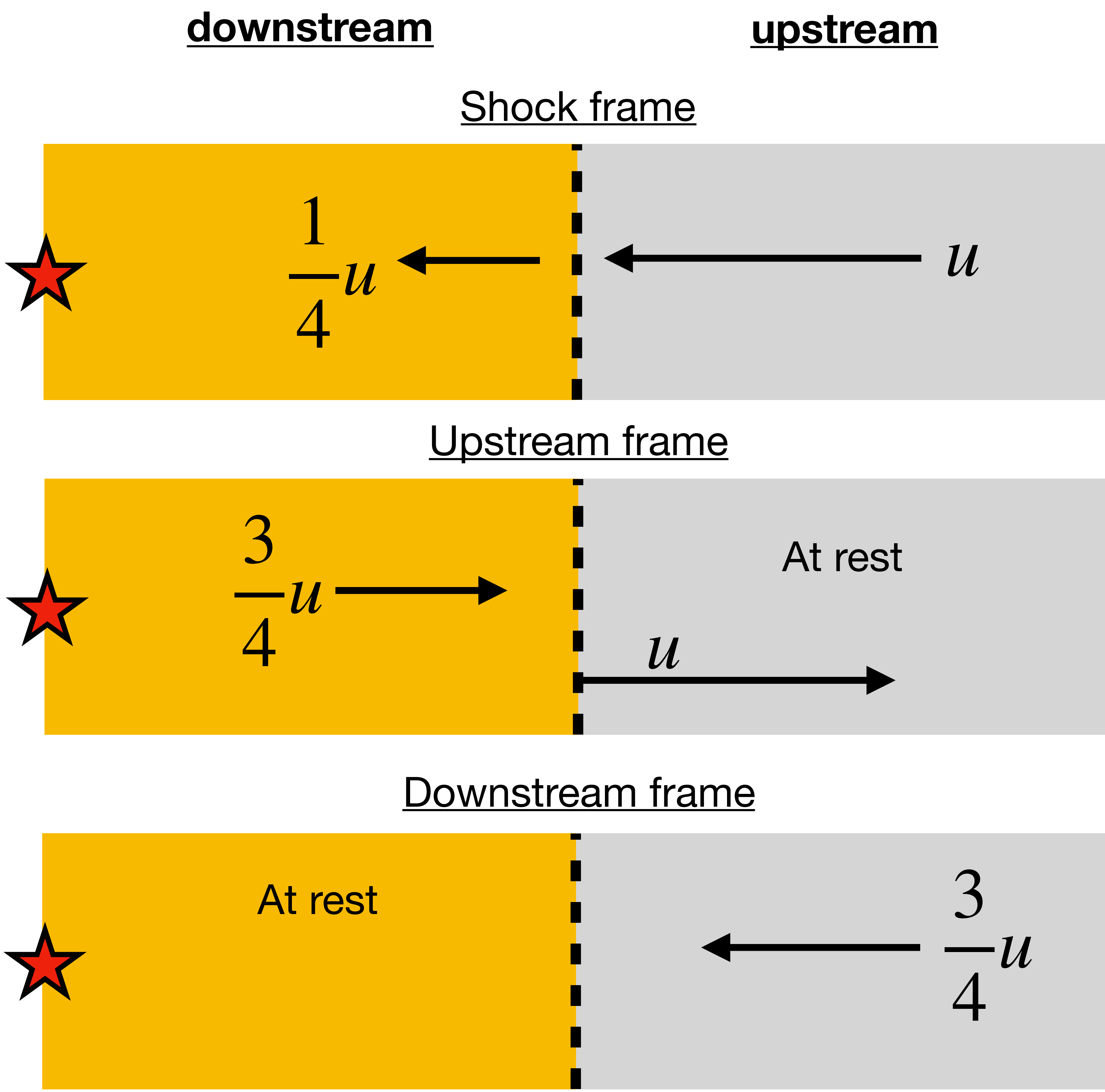

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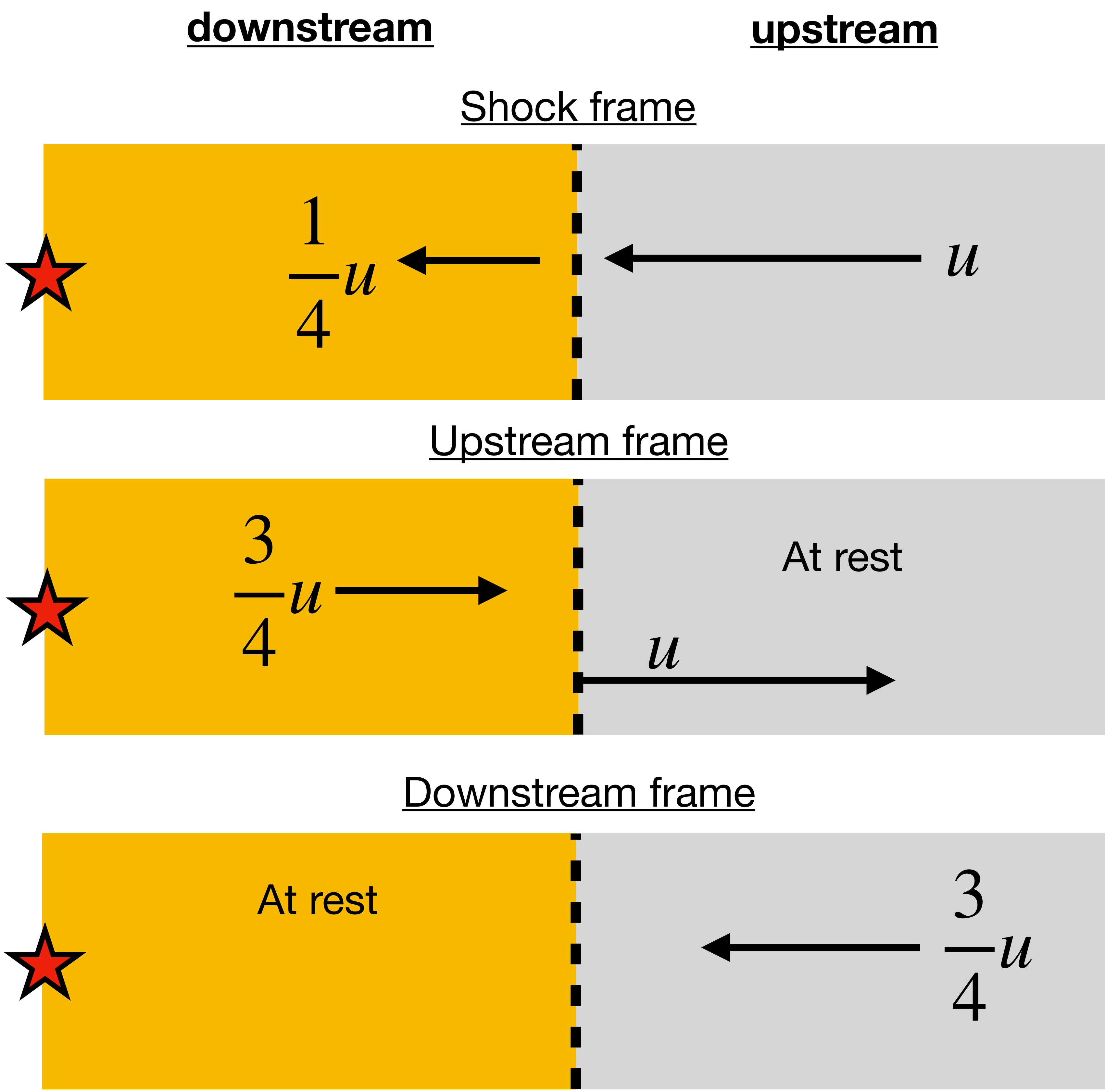
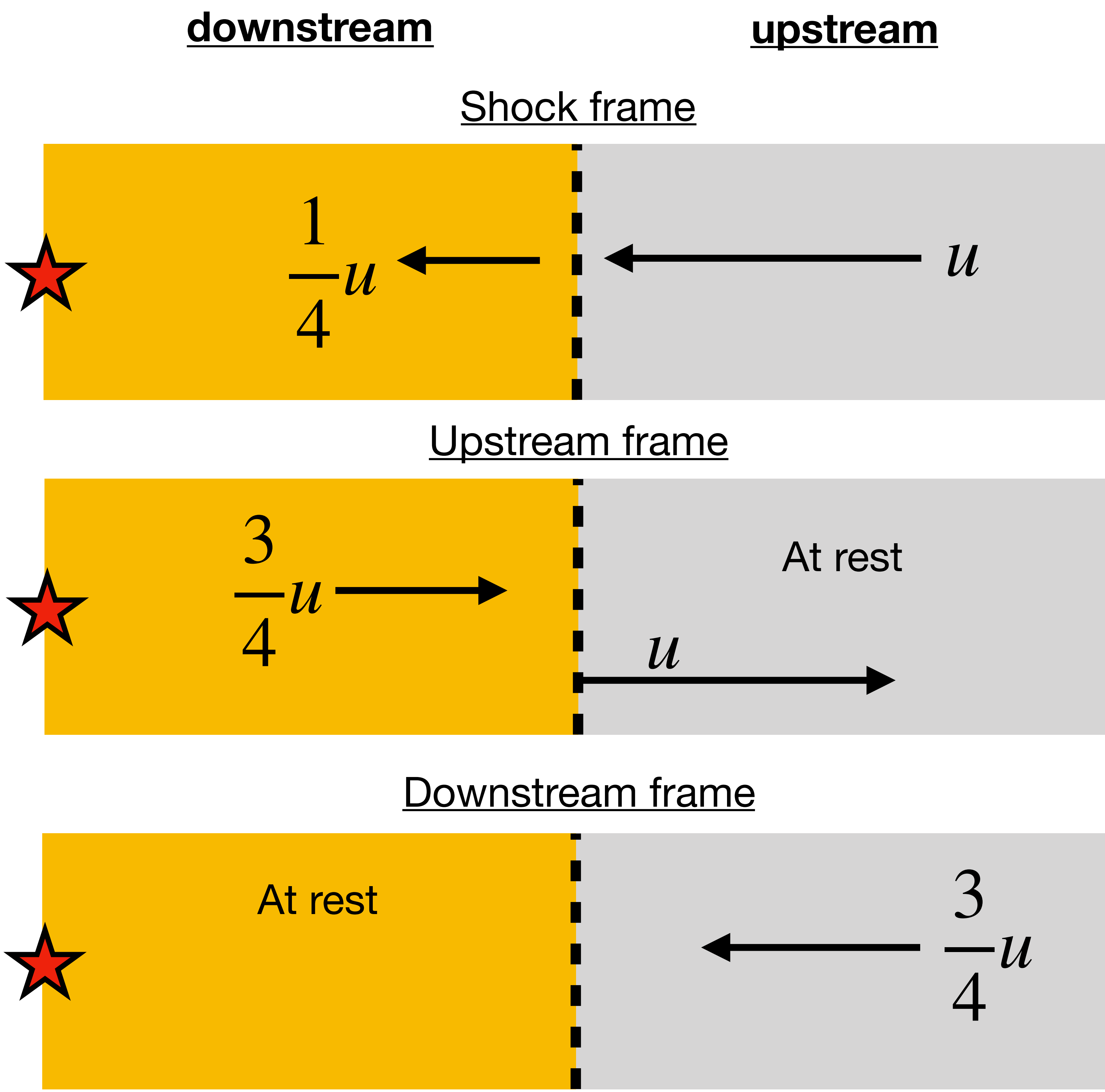


Upstream particles see downstream particles moving towards them at V=(3/4)u

Shock acceleration



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Downstream particles see upstream particles moving towards them at V=(3/4)u

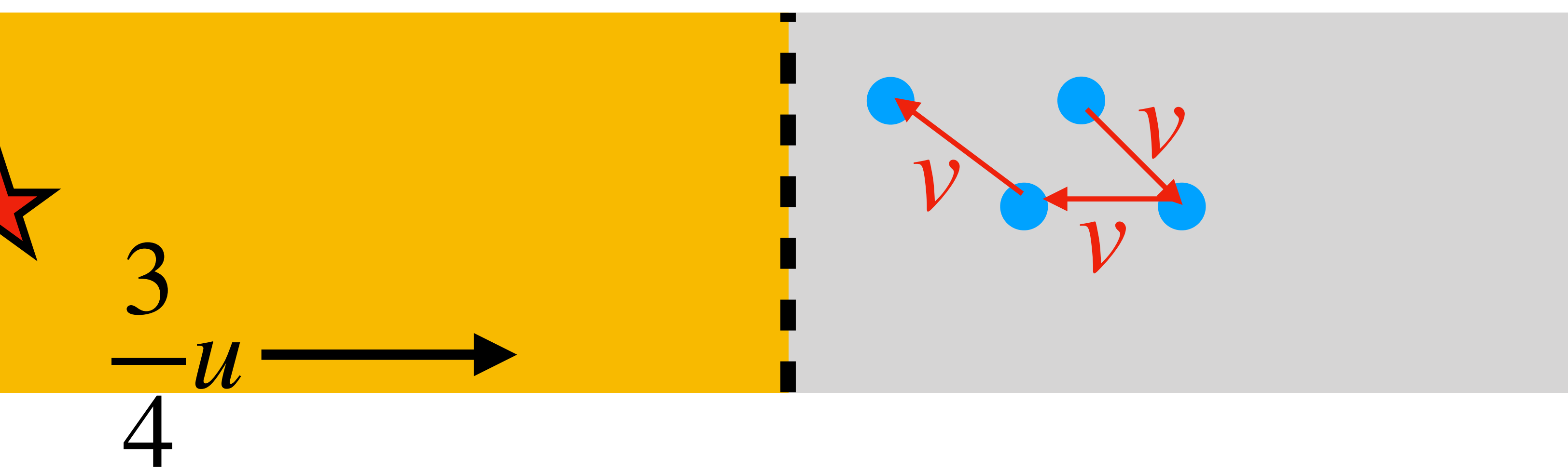
Shock acceleration

downstream **upstream**

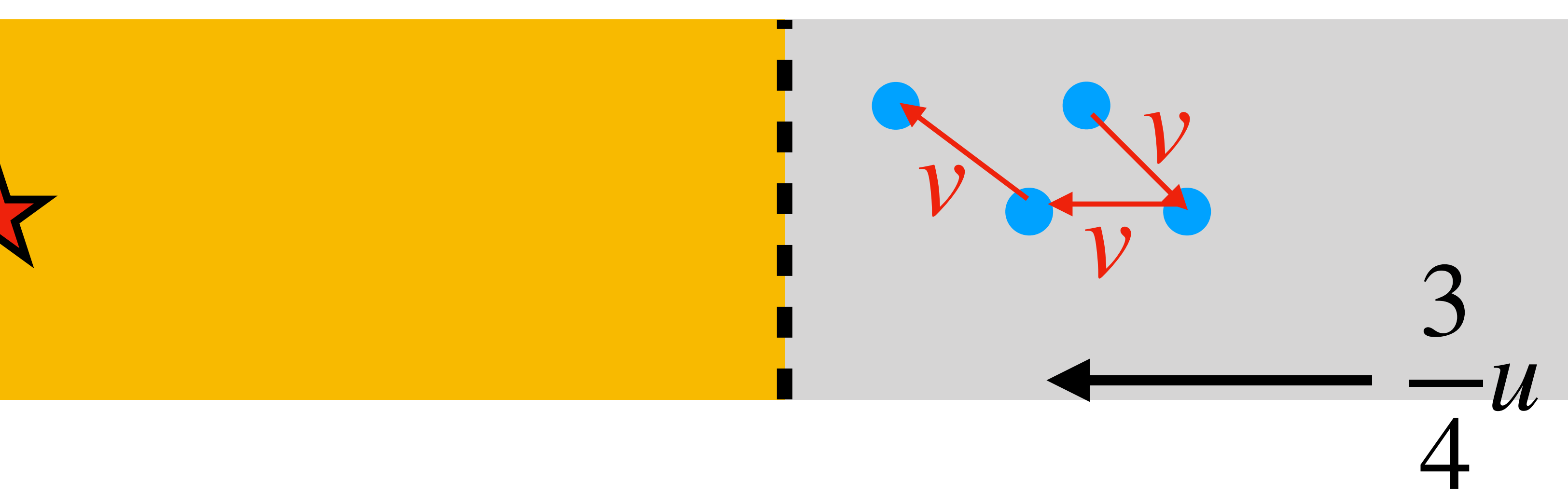
$$u << c$$

$$v \sim c$$

Upstream frame



Downstream frame



- Particles scattering on upstream side with isotropic velocity distribution in upstream rest frame.

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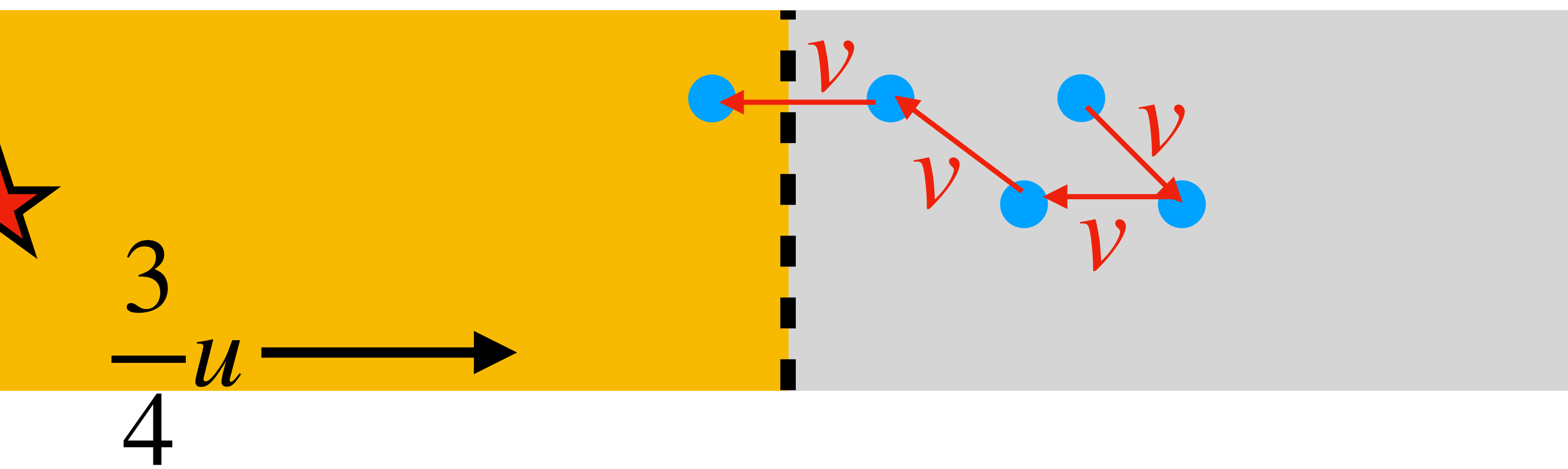

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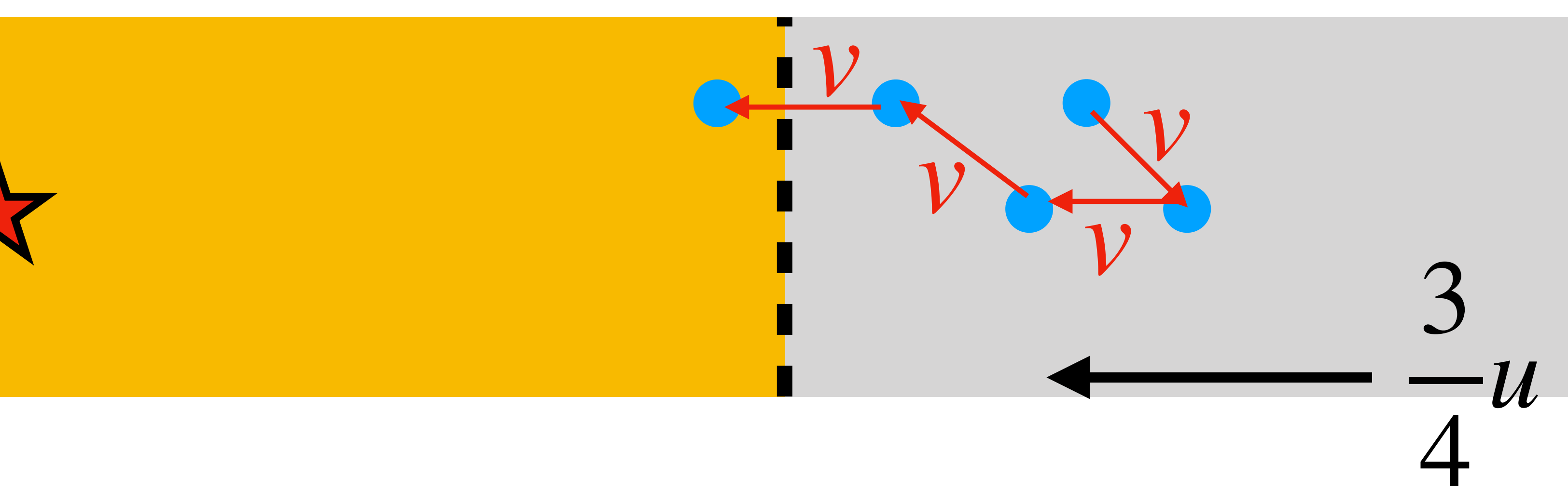

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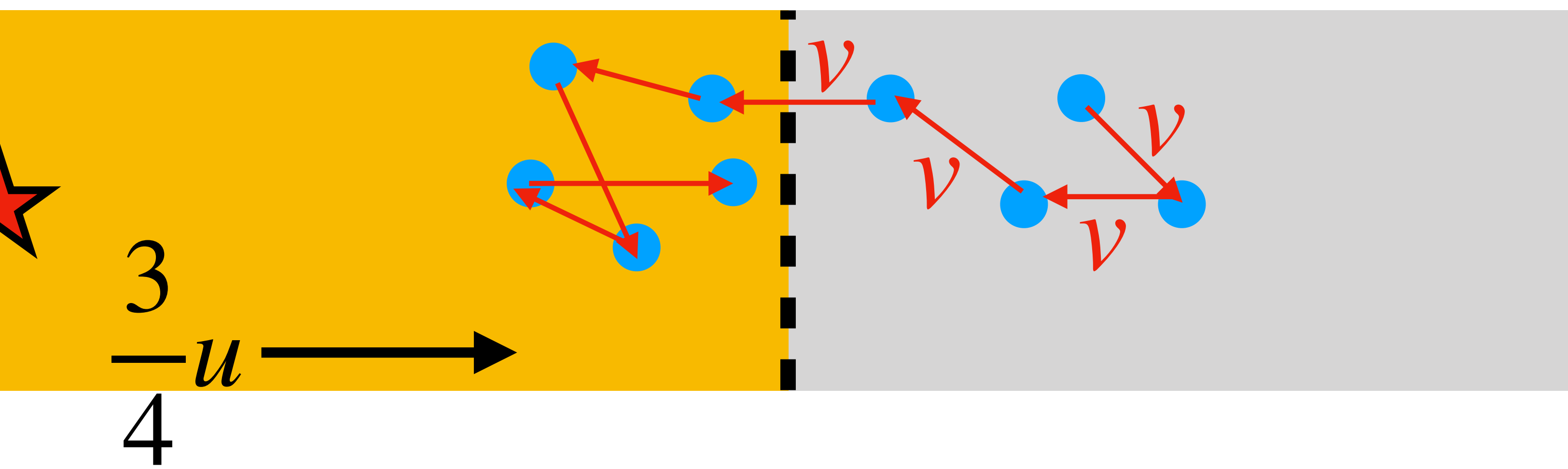

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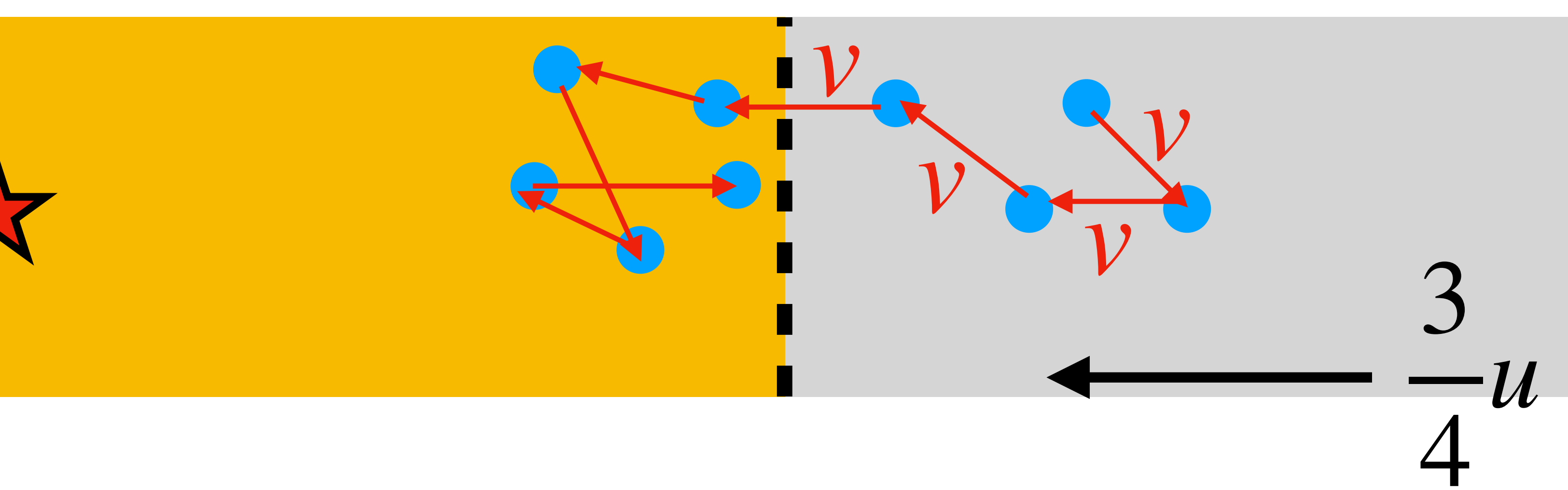

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Shock acceleration

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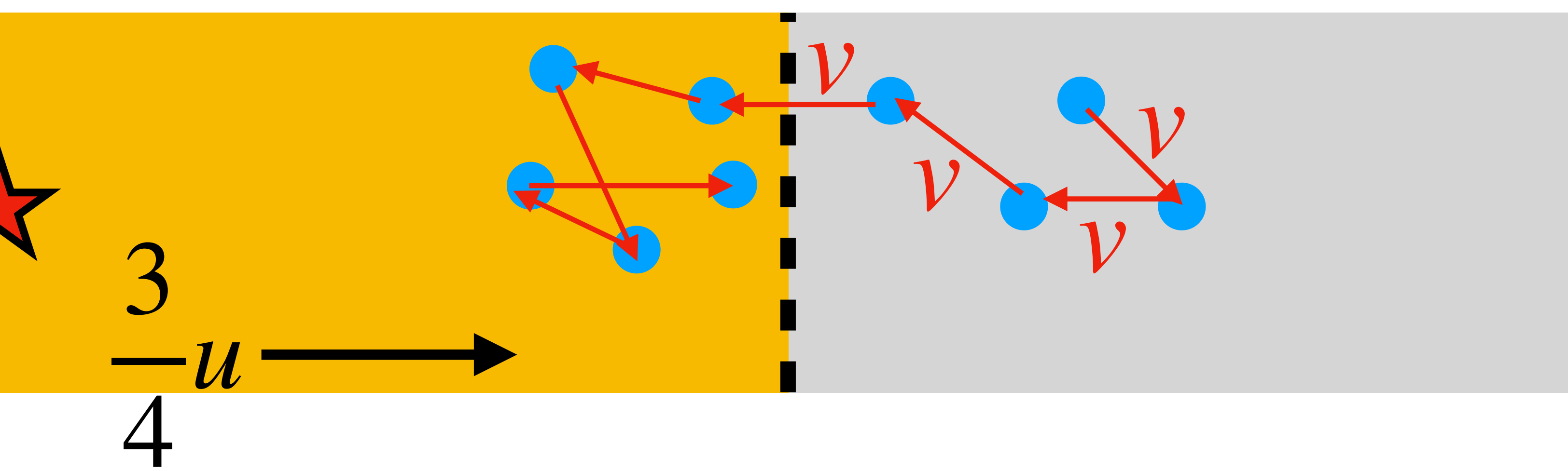

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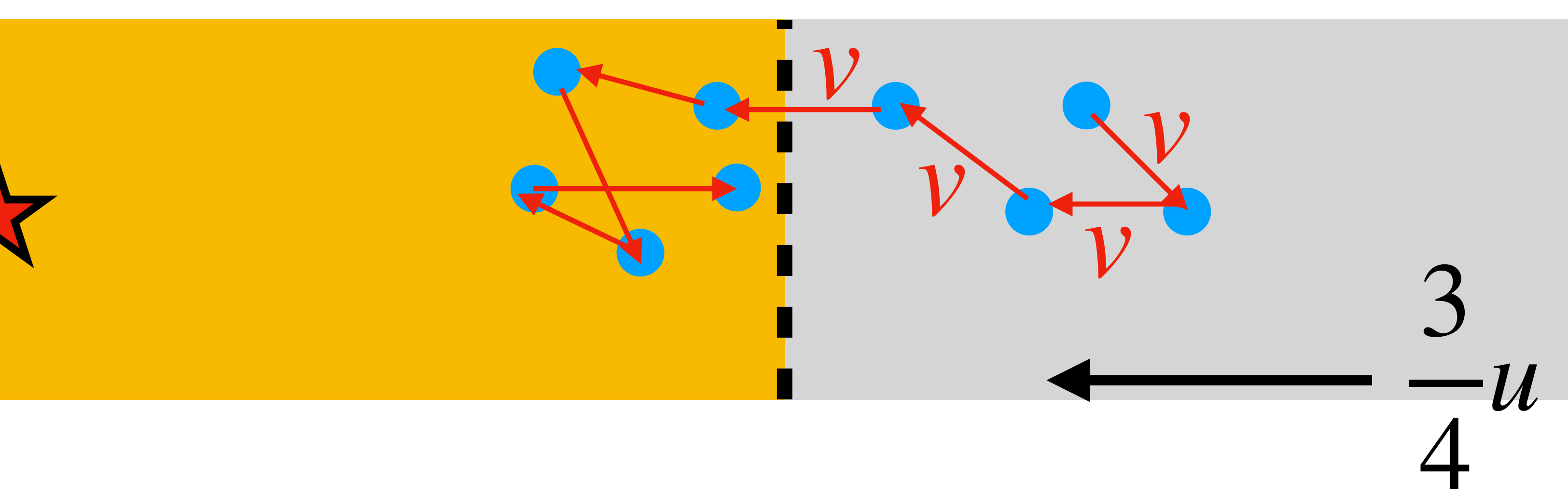

- Particles scattering on upstream side with isotropic velocity distribution in upstream rest frame.
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Shock acceleration

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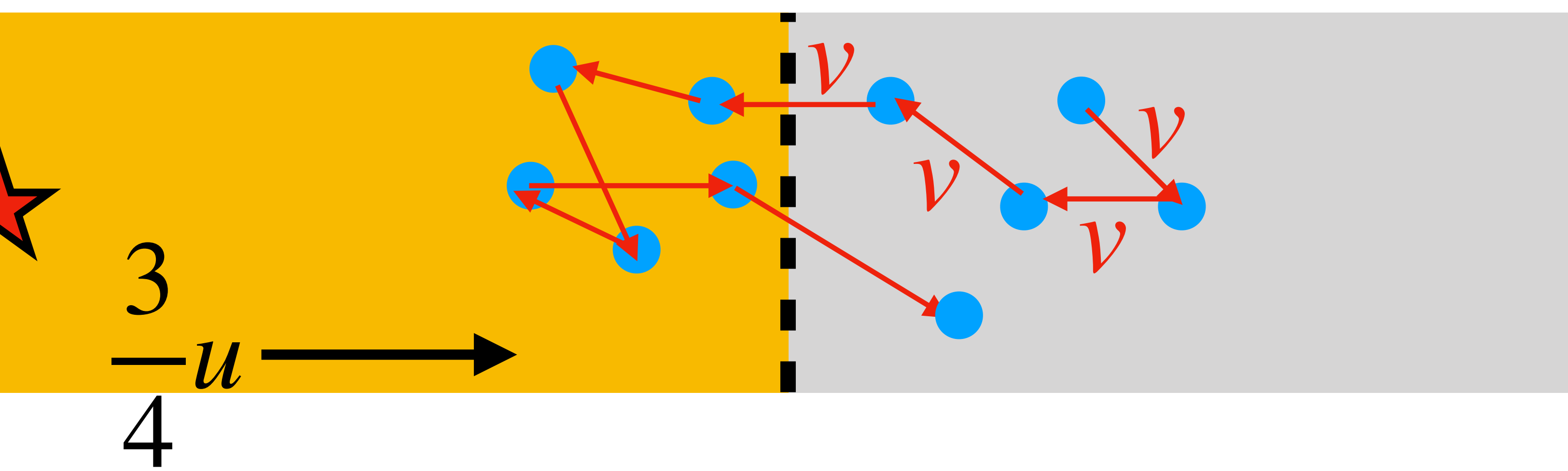

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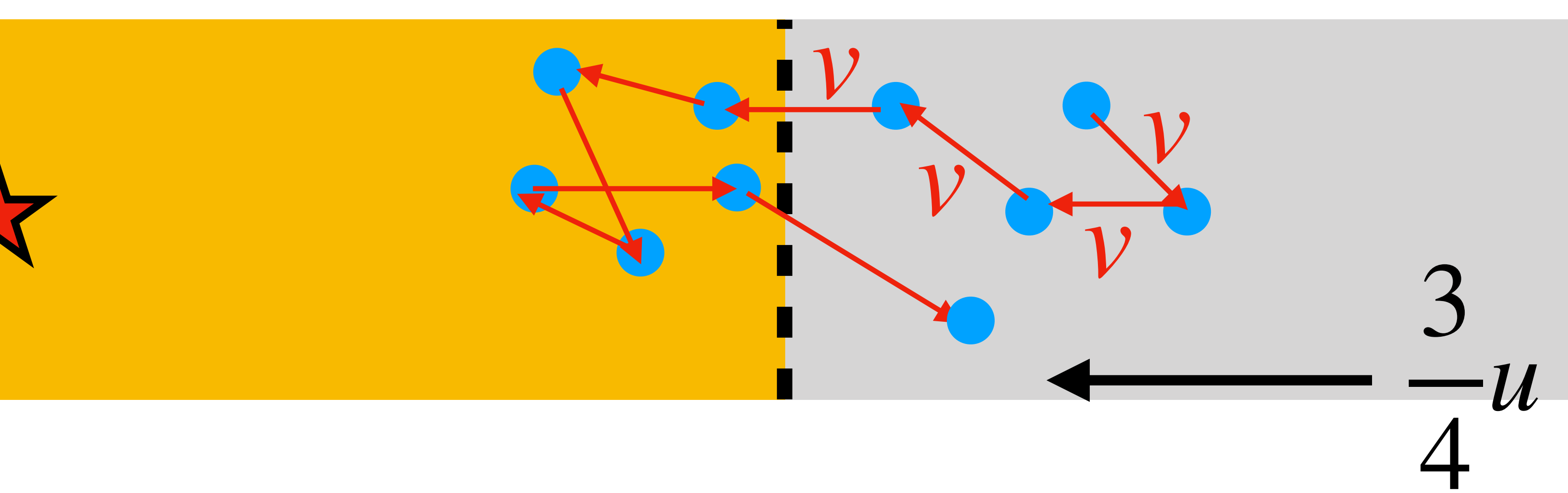

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Shock acceleration

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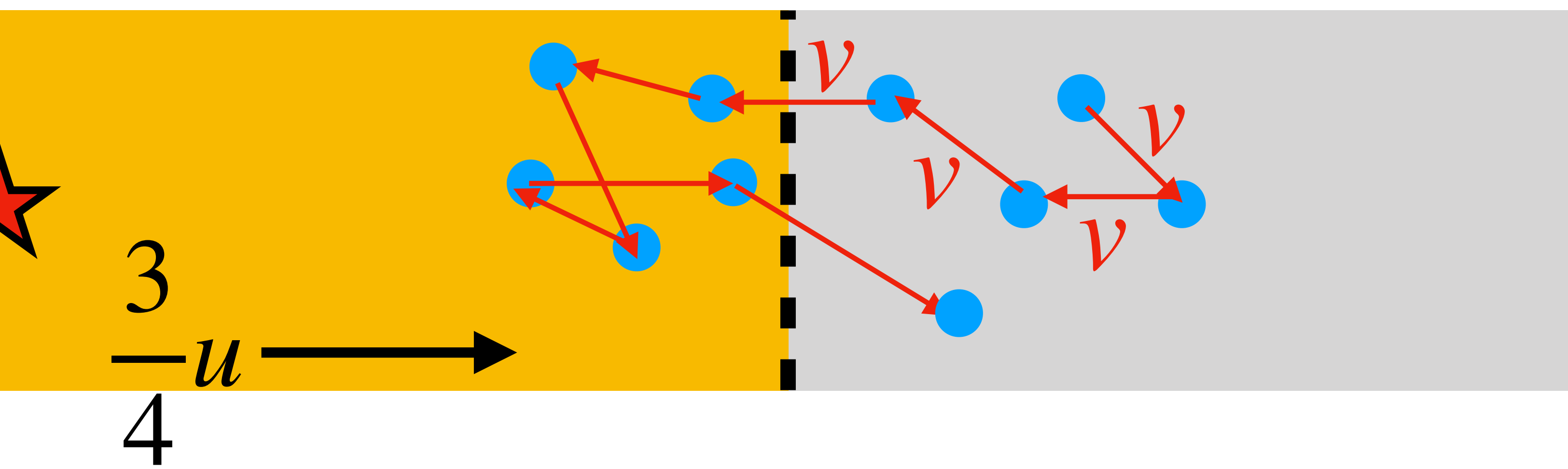

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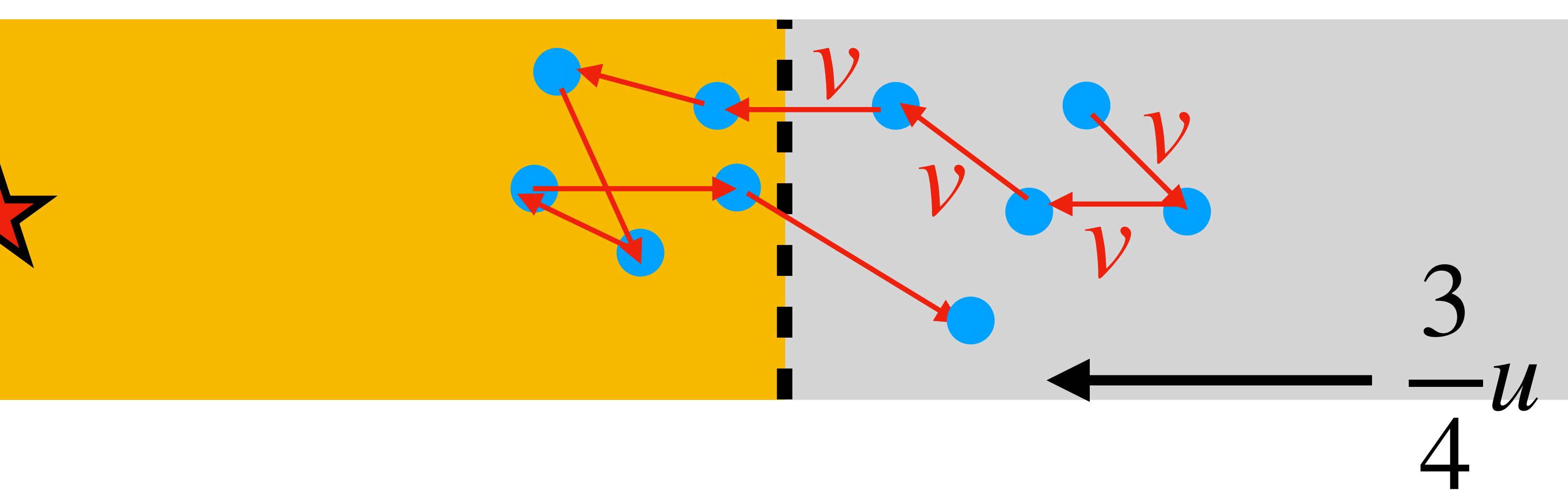

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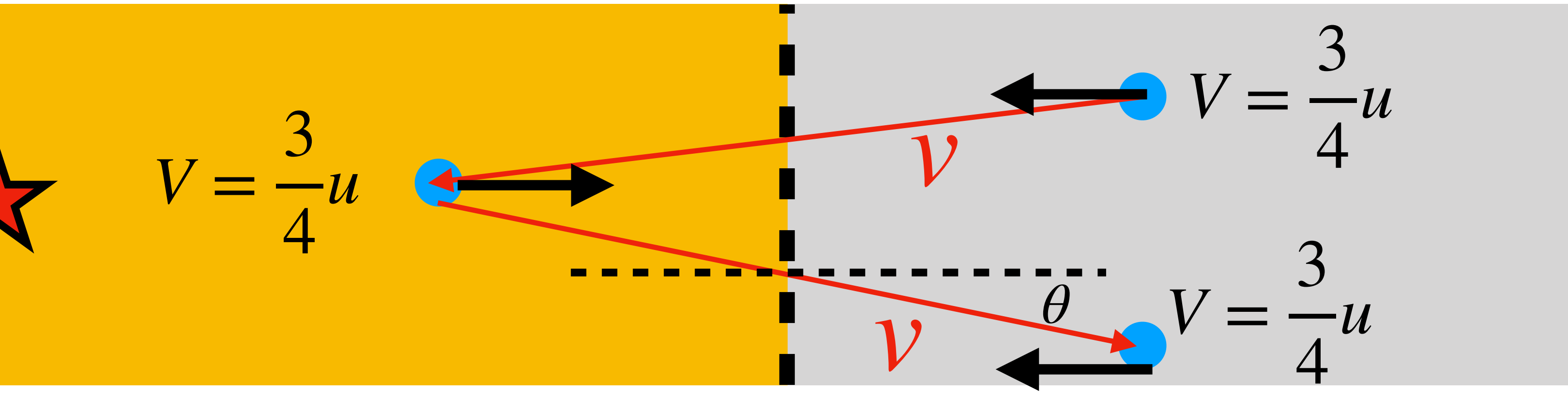
- More scatterings, pick up bulk velocity V=(3/4)u towards explosion.

Shock acceleration



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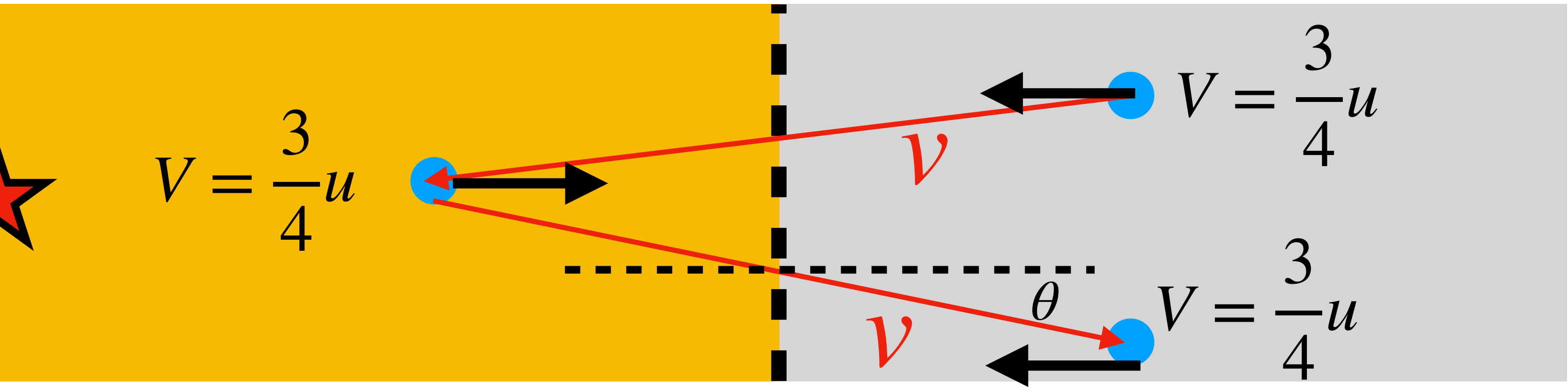
Shock acceleration



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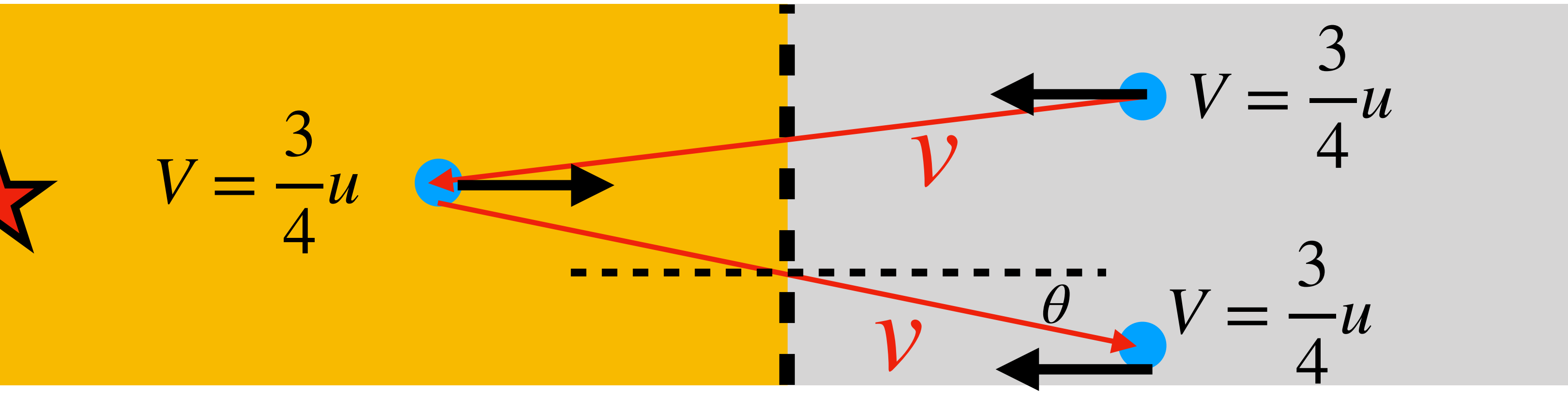

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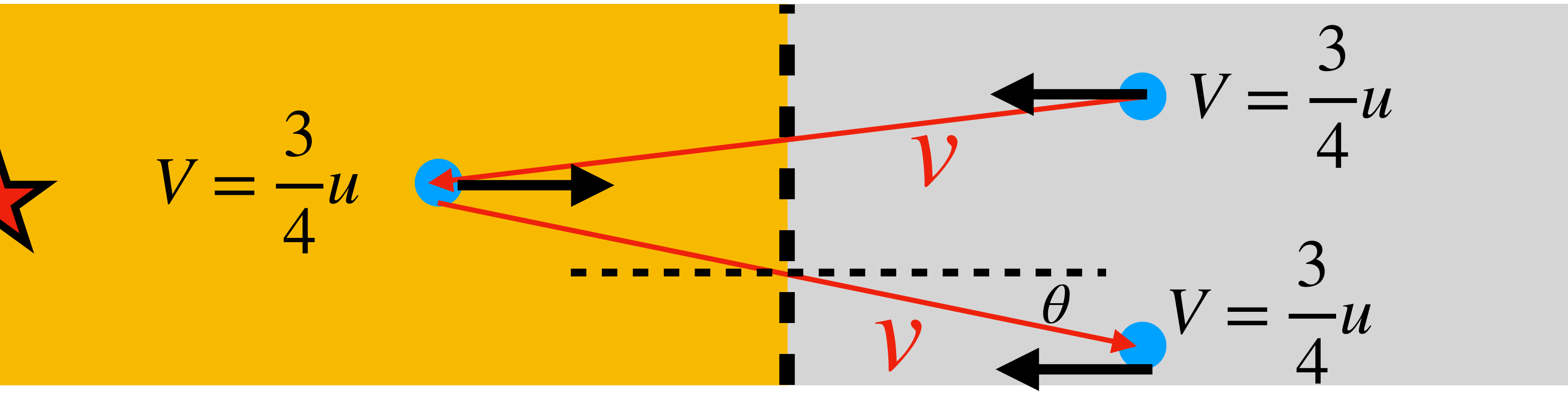

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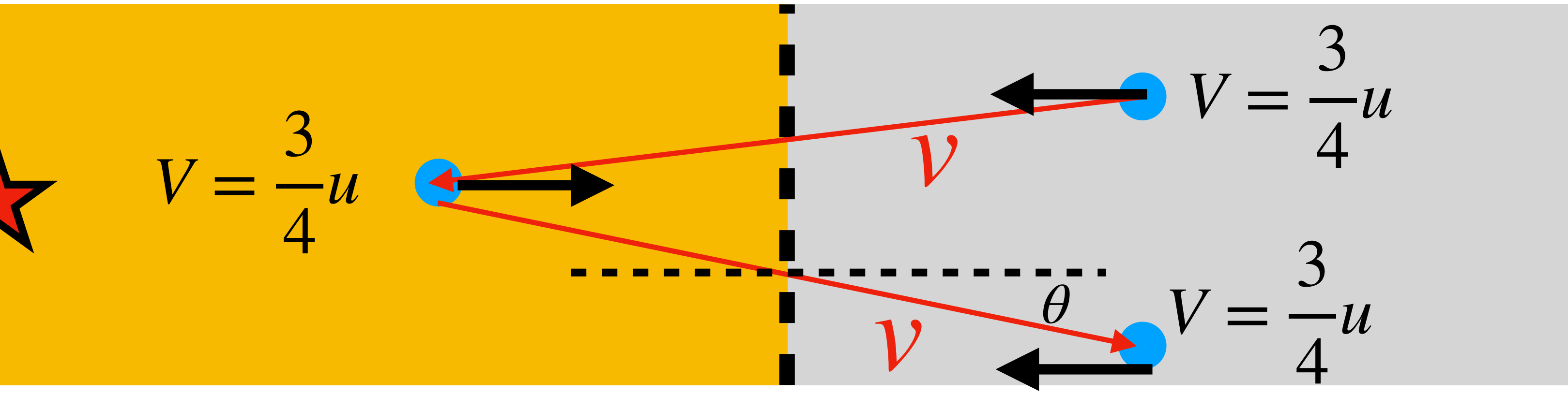

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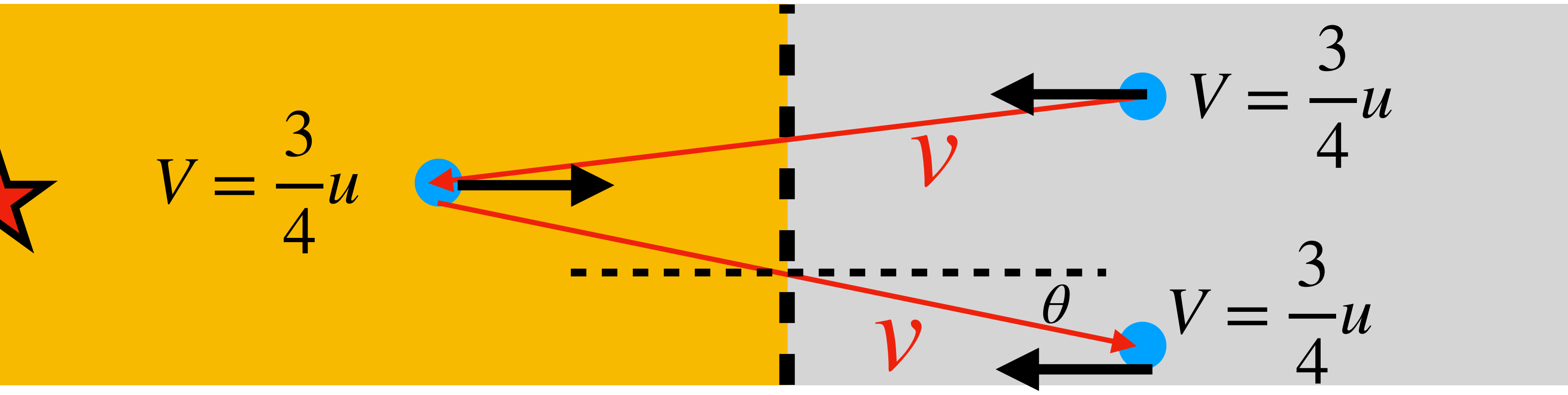
$$\int_0^{\pi/2} p(\theta)d\theta = 1 \quad \Longrightarrow \quad p(\theta)d\theta = 2\cos\theta d\cos\theta$$ …v~c

Shock acceleration



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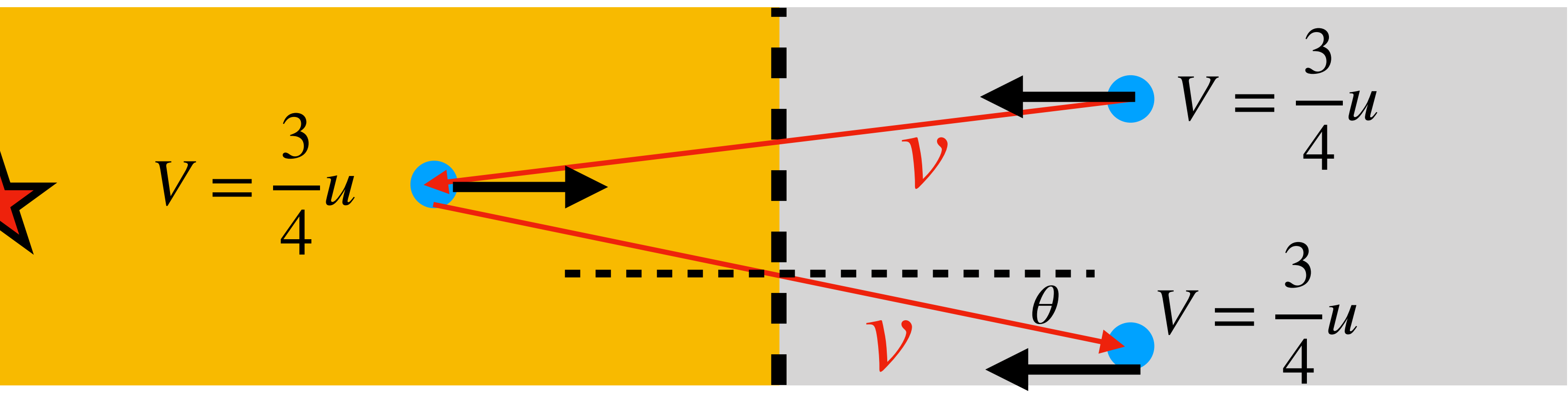
- Average fractional energy gain per crossing:

$$\langle \Delta E/E \rangle = \int_0^{\pi/2} \frac{V}{c}\cos\theta p(\theta)d\theta = 2\frac{V}{c}\int_0^1 \cos^2\theta d\cos\theta = \frac{2}{3}\frac{V}{c}$$

Shock acceleration



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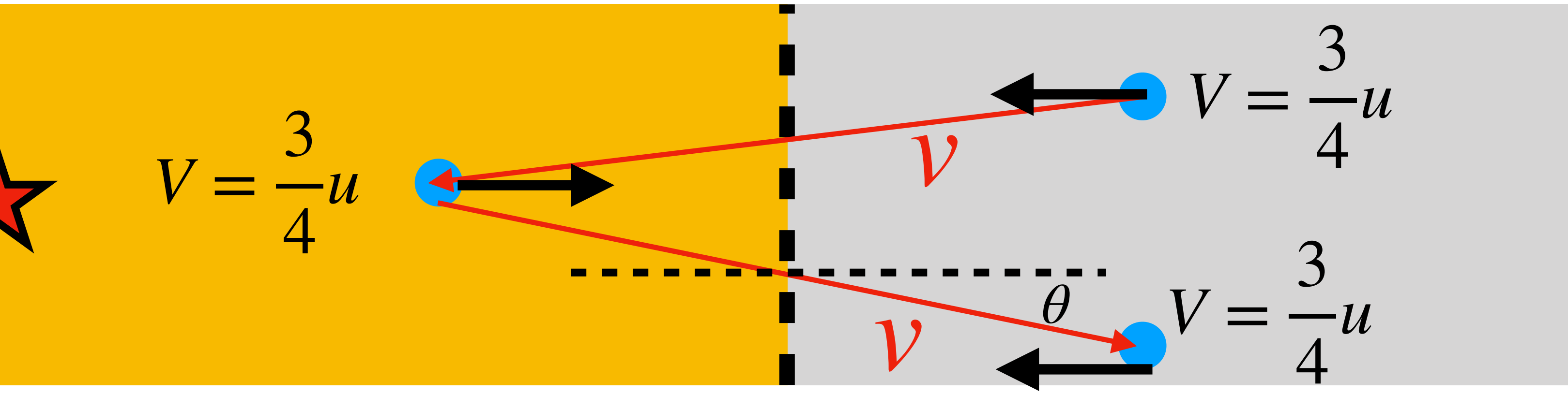
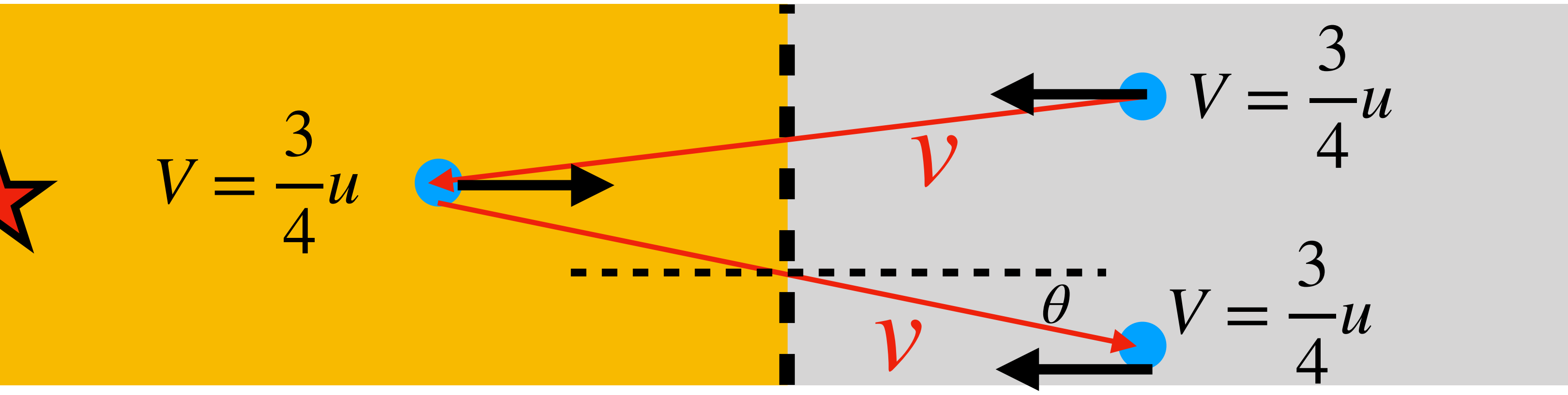
Shock acceleration



- Average fractional energy gain per crossing: $\langle \Delta E/E \rangle = \frac{2}{3}\frac{V}{c}$
- Average fractional energy gain per round trip: $\langle \Delta E/E \rangle = \frac{4}{3}\frac{V}{c} = \frac{u}{c}$
- Therefore: $\beta = 1 + \frac{u}{c}$

Shock acceleration



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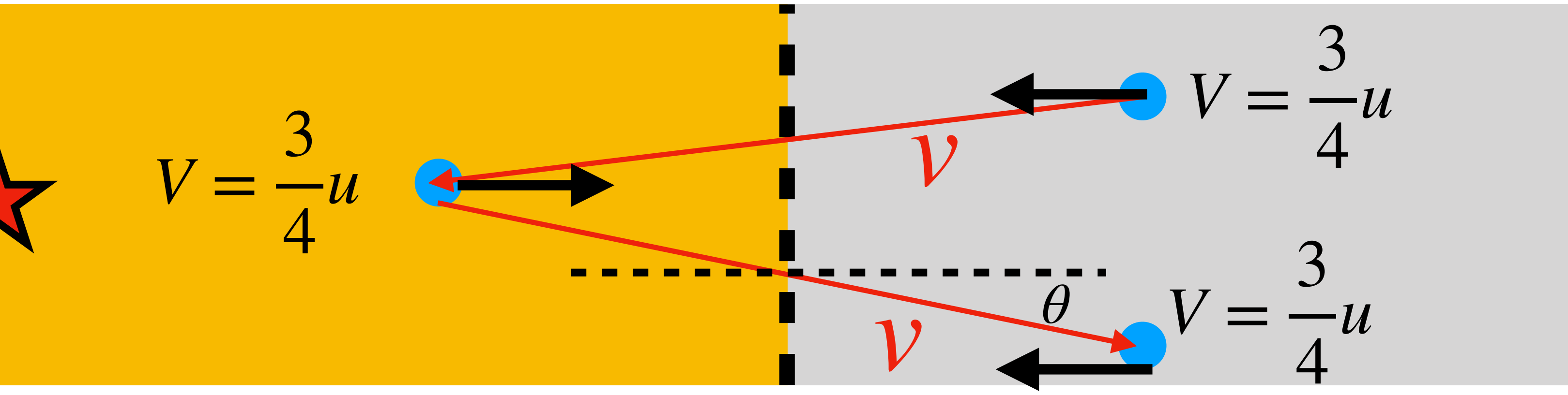
Because fractional energy gain goes as (u/c) instead of $(u/c)^2$, this diffusive shock acceleration process is known as *first order Fermi acceleration*. Now much easier to accelerate particles to the observed E~10^{20} eV!

Shock acceleration



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- Therefore: $\beta = 1 + \frac{u}{c}$

Because fractional energy gain goes as (u/c) instead of $(u/c)^2$, this diffusive shock acceleration process is known as *first order Fermi acceleration*. Now much easier to accelerate particles to the observed E~10^{20} eV!

Fermi did not come up with the shock front idea though, this dates back to the late 1970s, with many of the key insights being made by Tony Bell (Oxford): https://ui.adsabs.harvard.edu/abs/1978MNRAS.182..147B/abstract; https://ui.adsabs.harvard.edu/abs/1978MNRAS.182..443B/abstract

Shock acceleration

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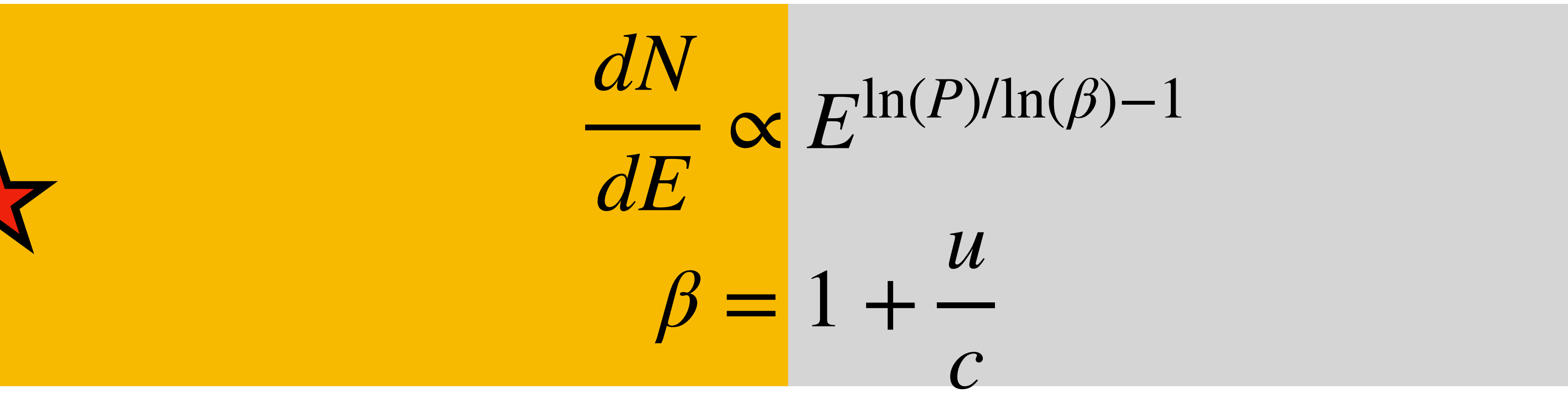
To get power-law index, must calculate probability of particles remaining near the shock region after each round trip, P.

Shock acceleration



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- Close to observed $E^{-2.5}$, but not exactly!
- Still active area of research.
- Mechanism so popular because it explains why you can get the *same* power-law spectrum for particles accelerated in diverse array of astrophysical objects (e.g. supernova remnants, AGN etc). There only needs to be a strong shock!