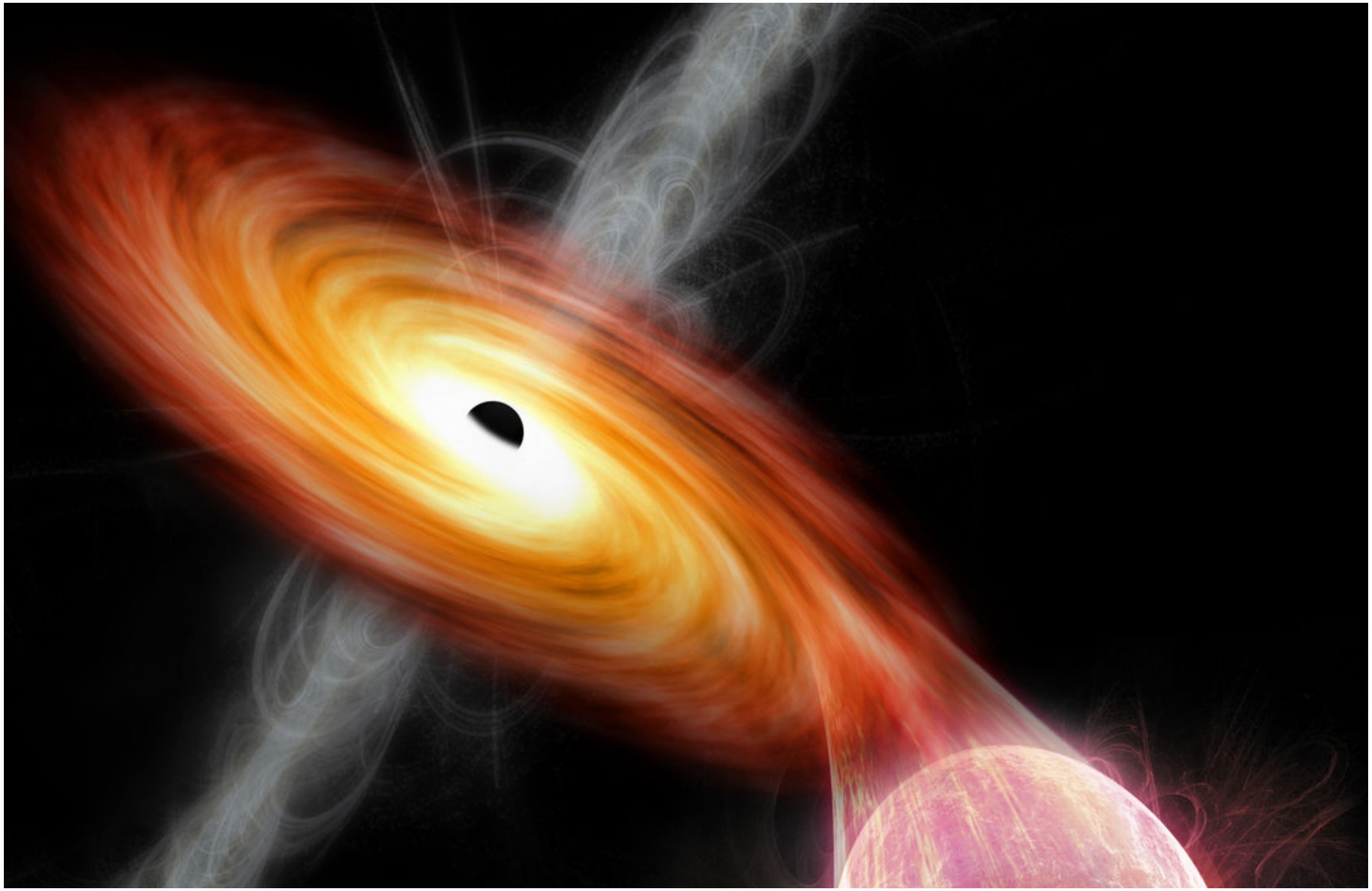
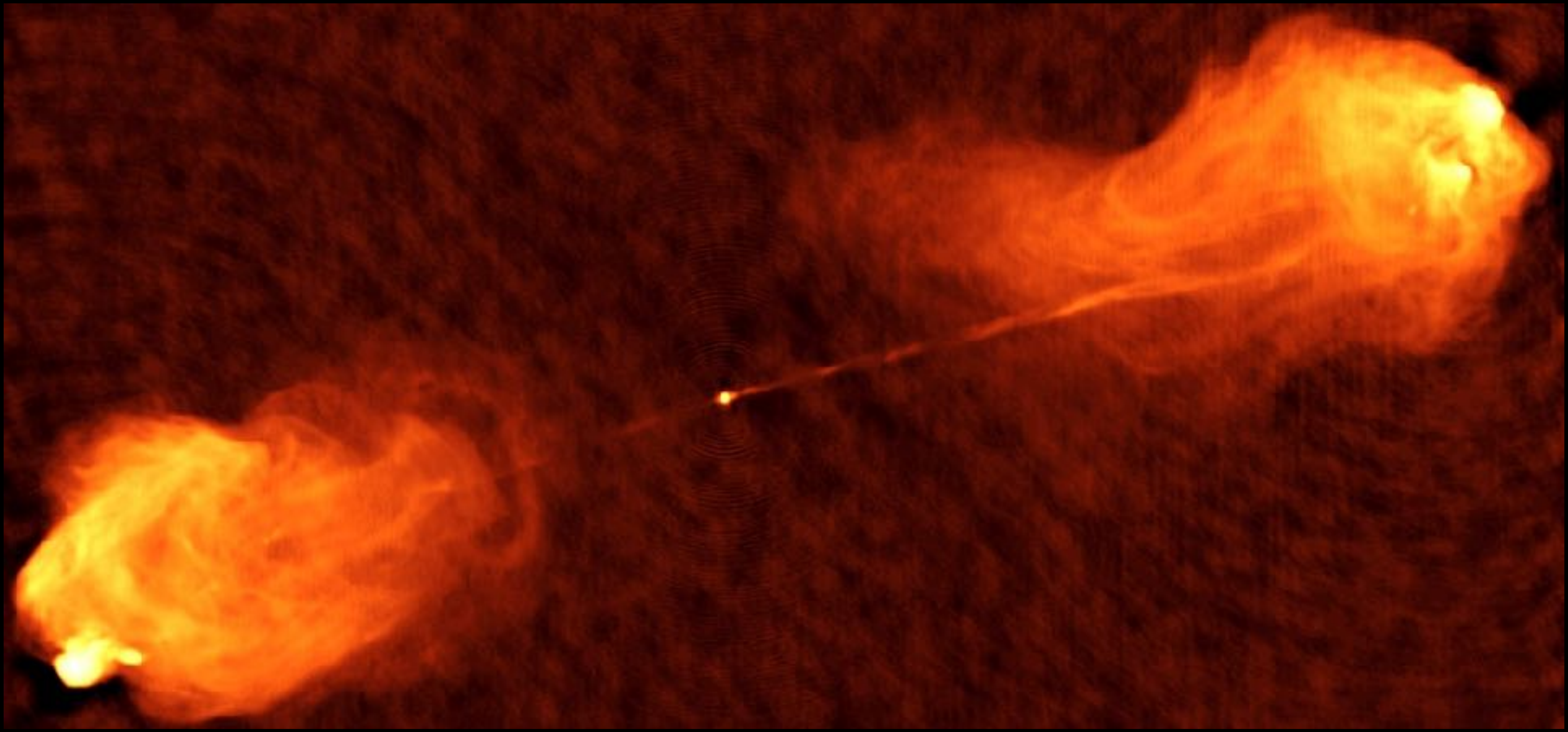
High Energy Astrophysics

Dr. Adam Ingram



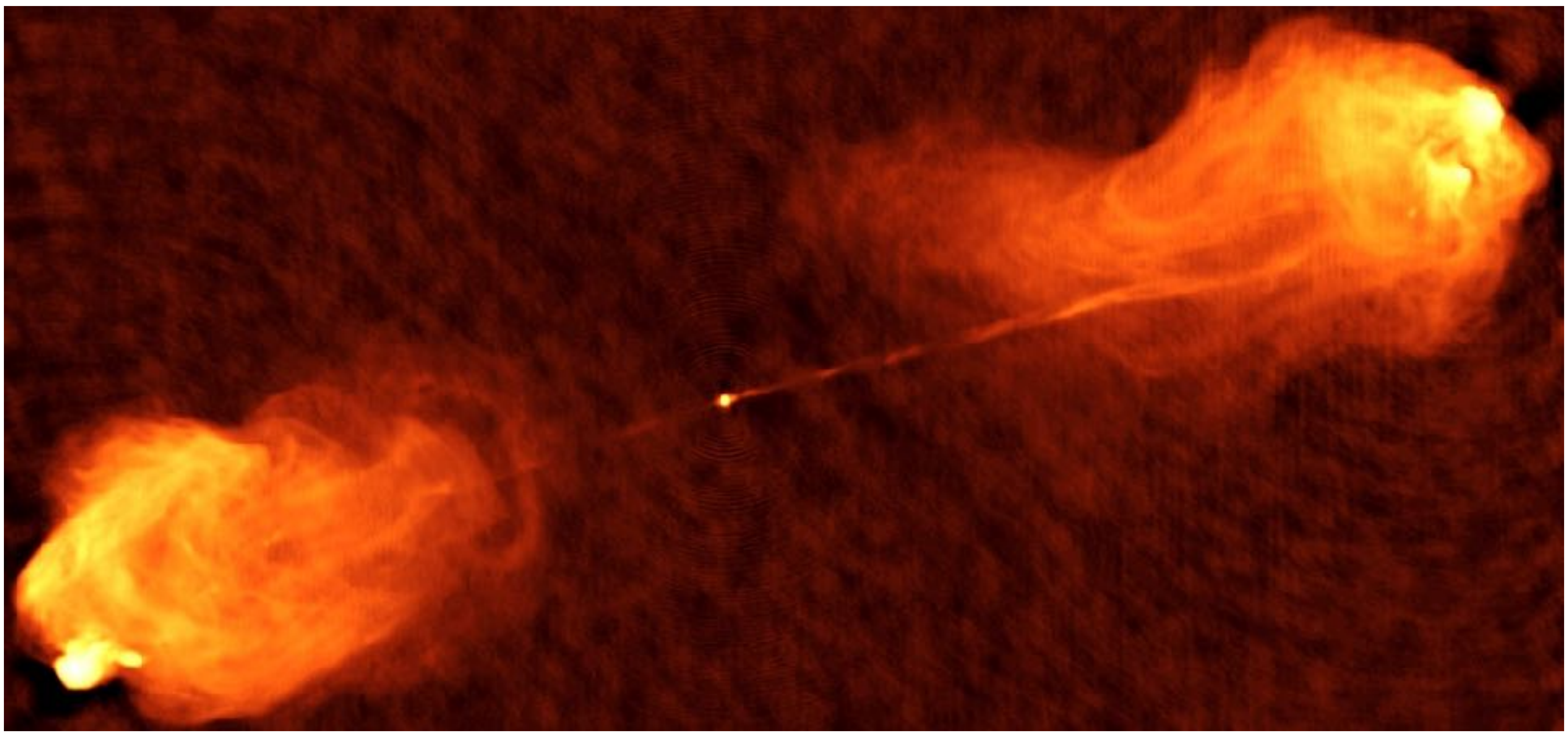
Lecture 3

Synchrotron Radiation



Introduction

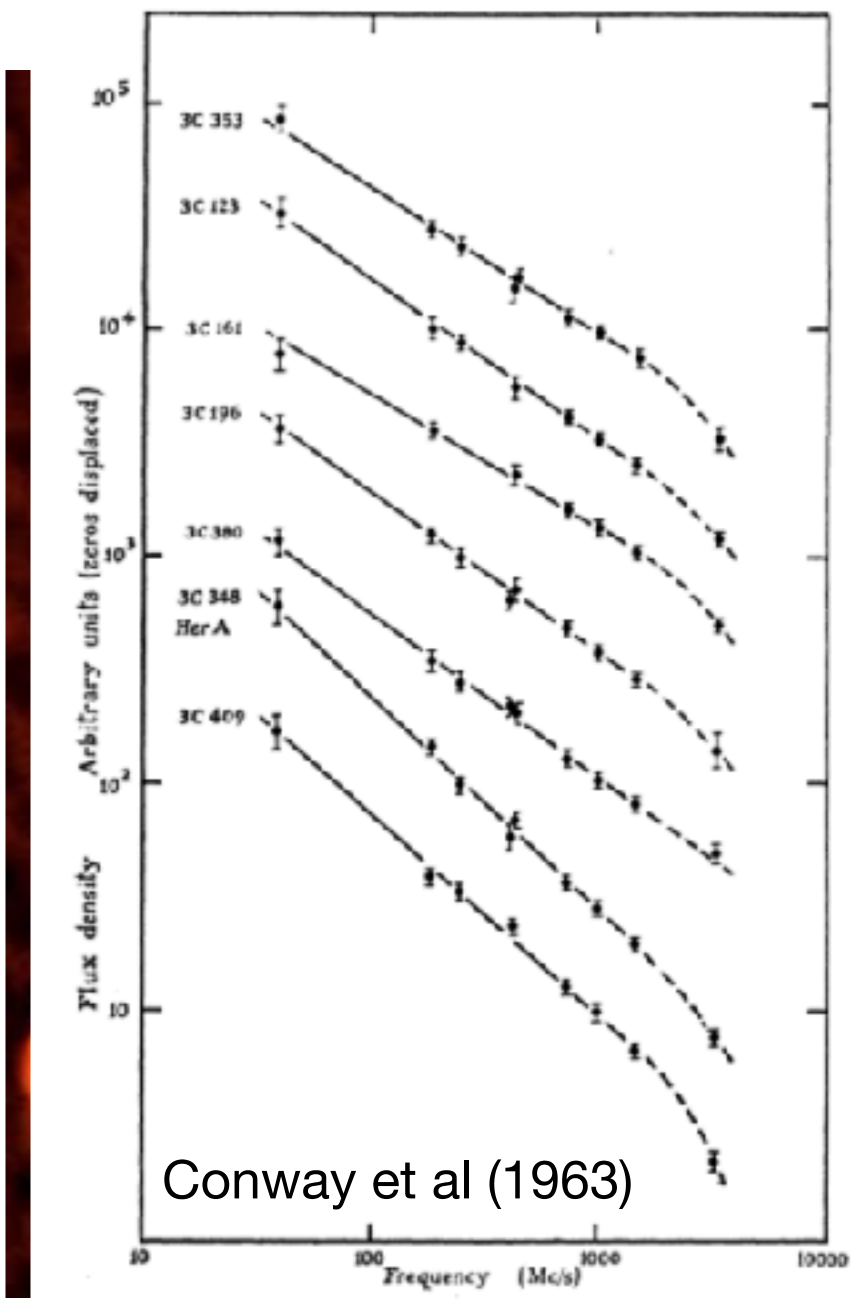
- Electrons accelerated to ultra-relativistic energies at shock fronts (e.g. jet lobes, supernova remnants) will spiral around B-field, resulting in *synchrotron radiation*.
- Observe strong radio emission from such regions.
- e.g. AGN/XRB jet lobes, SN remnants, galaxies (sum of radio emission from SN remnants & XRBs)



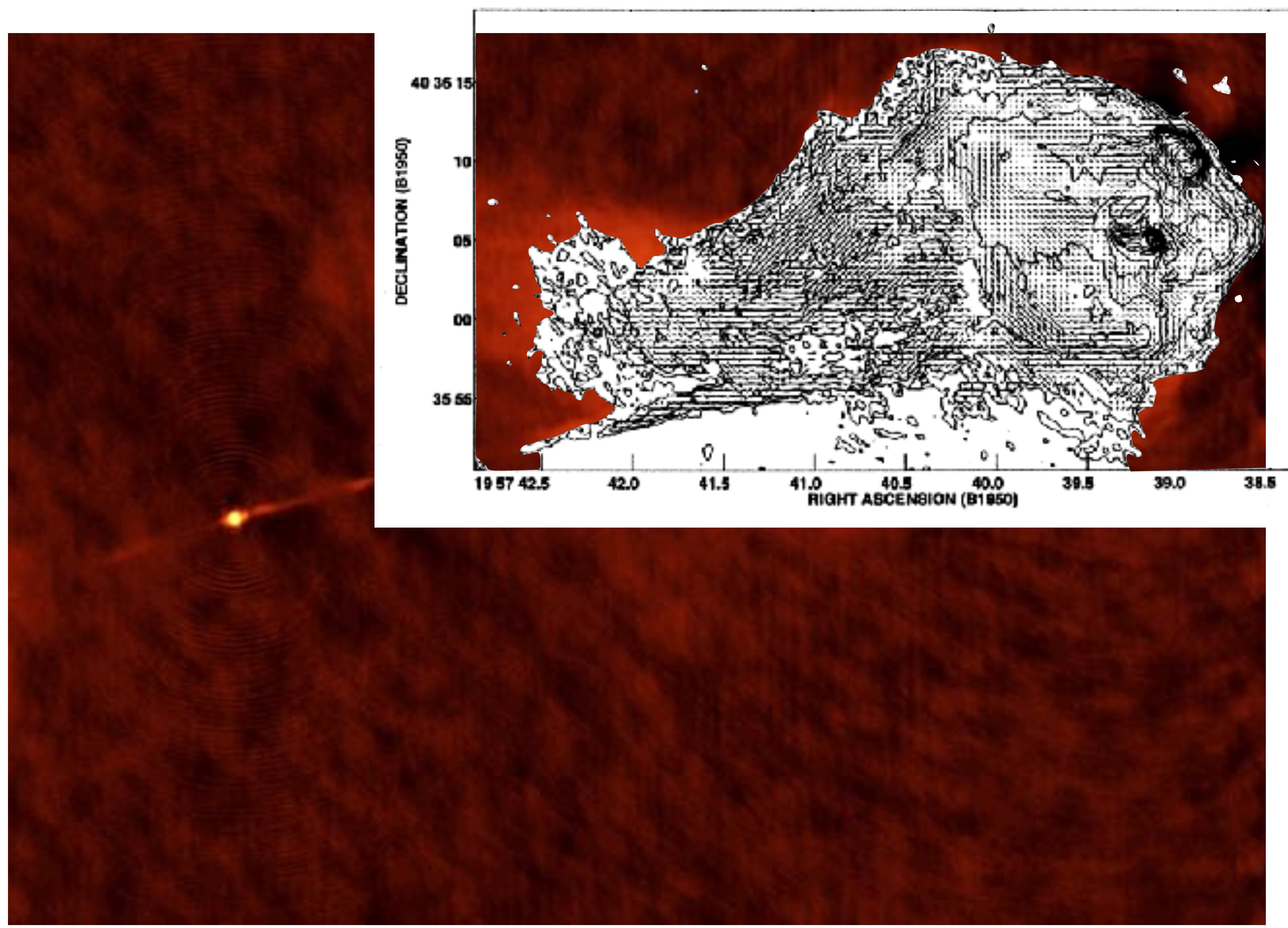
Radio galaxy Cygnus A at 5 GHz (VLA: Carilli and Barthel 1996, A&A Reviews)

Evidence for synchrotron

- Smooth, featureless broadband spectrum over many orders of magnitude in frequency;
- Power-law spectrum (will address turn-over next time);
- High degree of linear polarisation.

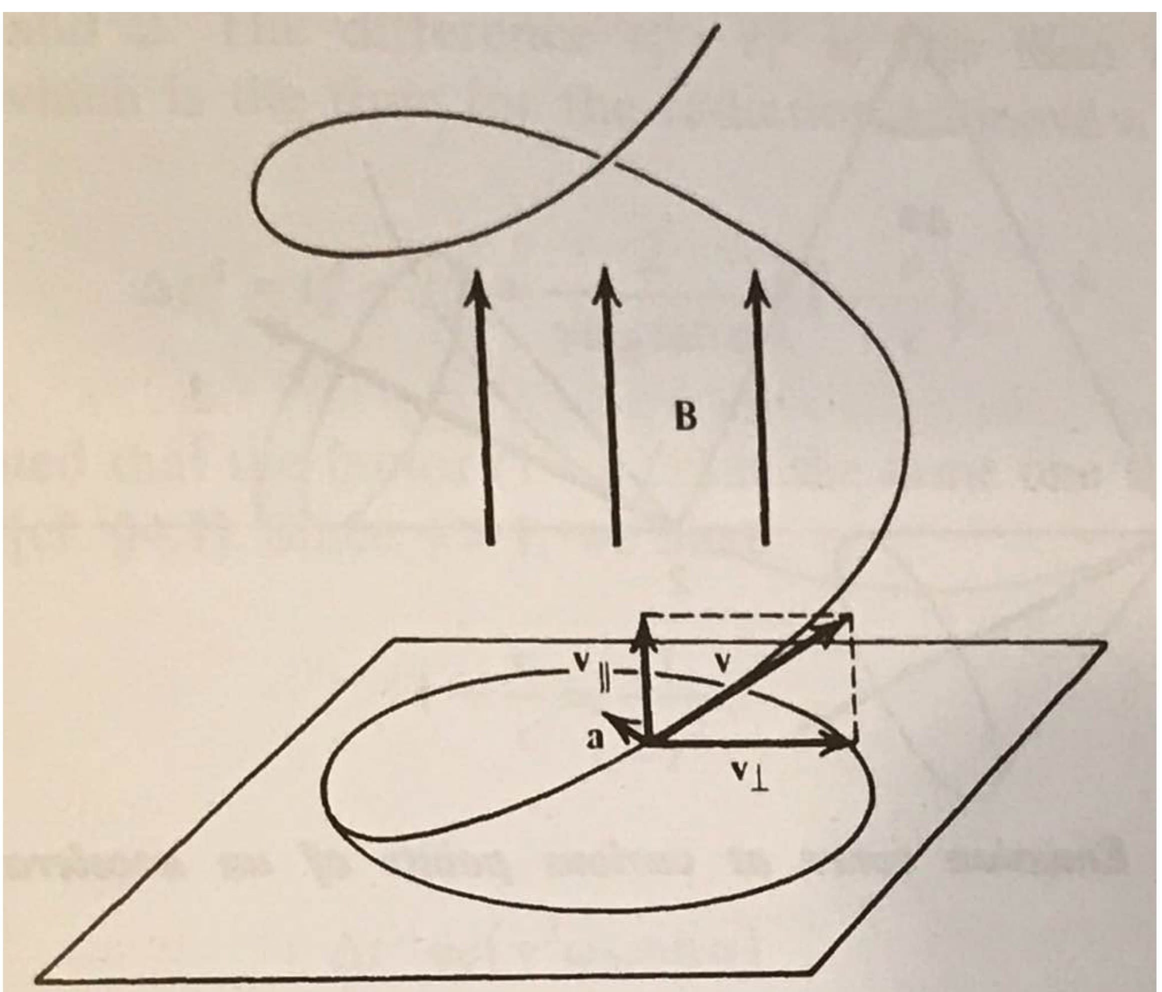


Conway et al (1963)



Synchrotron radiation

- Radiation released by relativistic electrons spiralling around magnetic field lines.

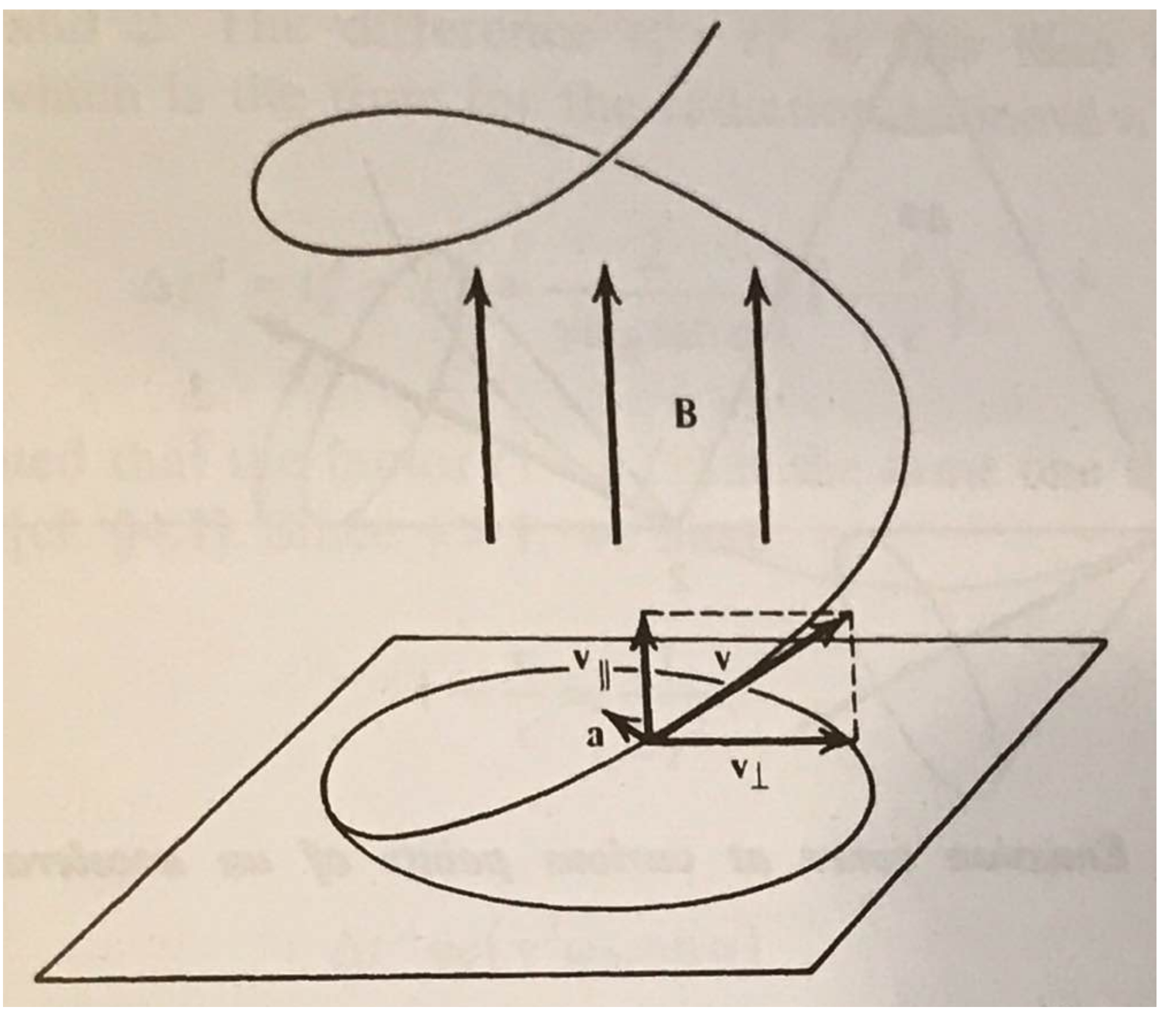


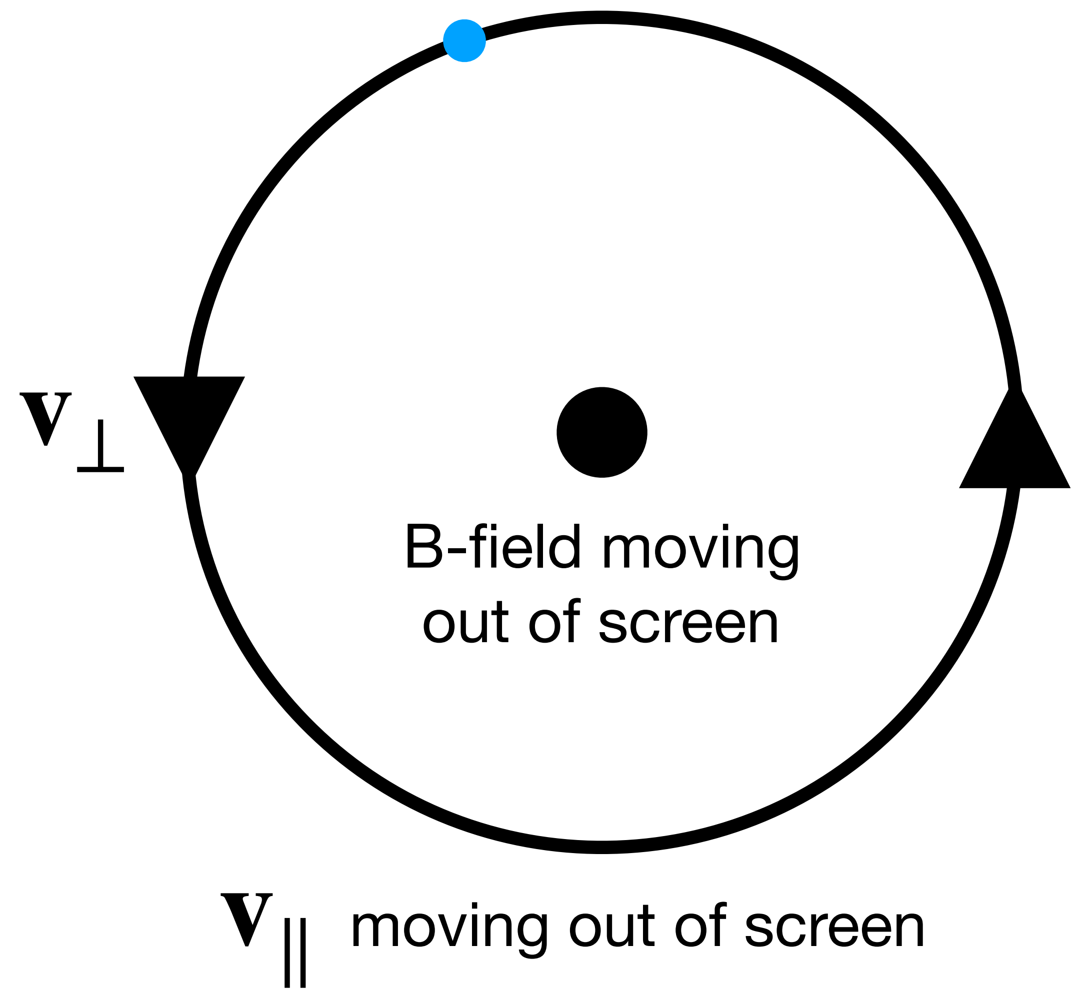
Synchrotron radiation

- Radiation released by relativistic electrons spiralling around magnetic field lines.
- Motion is <u>*helical*</u>:

$\mathbf{v}_{\|} = \text{constant}$ ...velocity parallel to B-field

$\mathbf{v}_{\perp} = \text{circular}$ ...velocity perpendicular to B-field





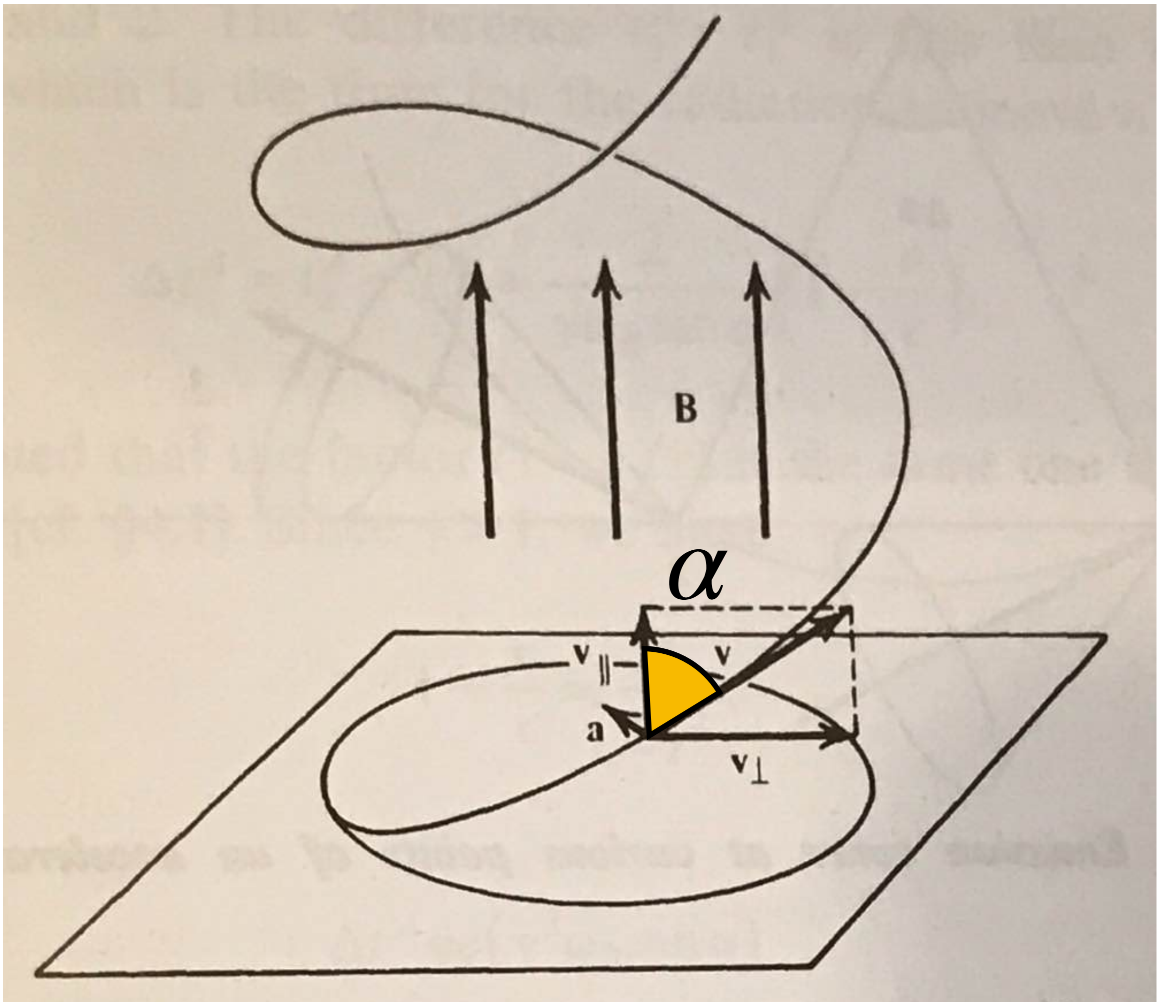
Synchrotron radiation

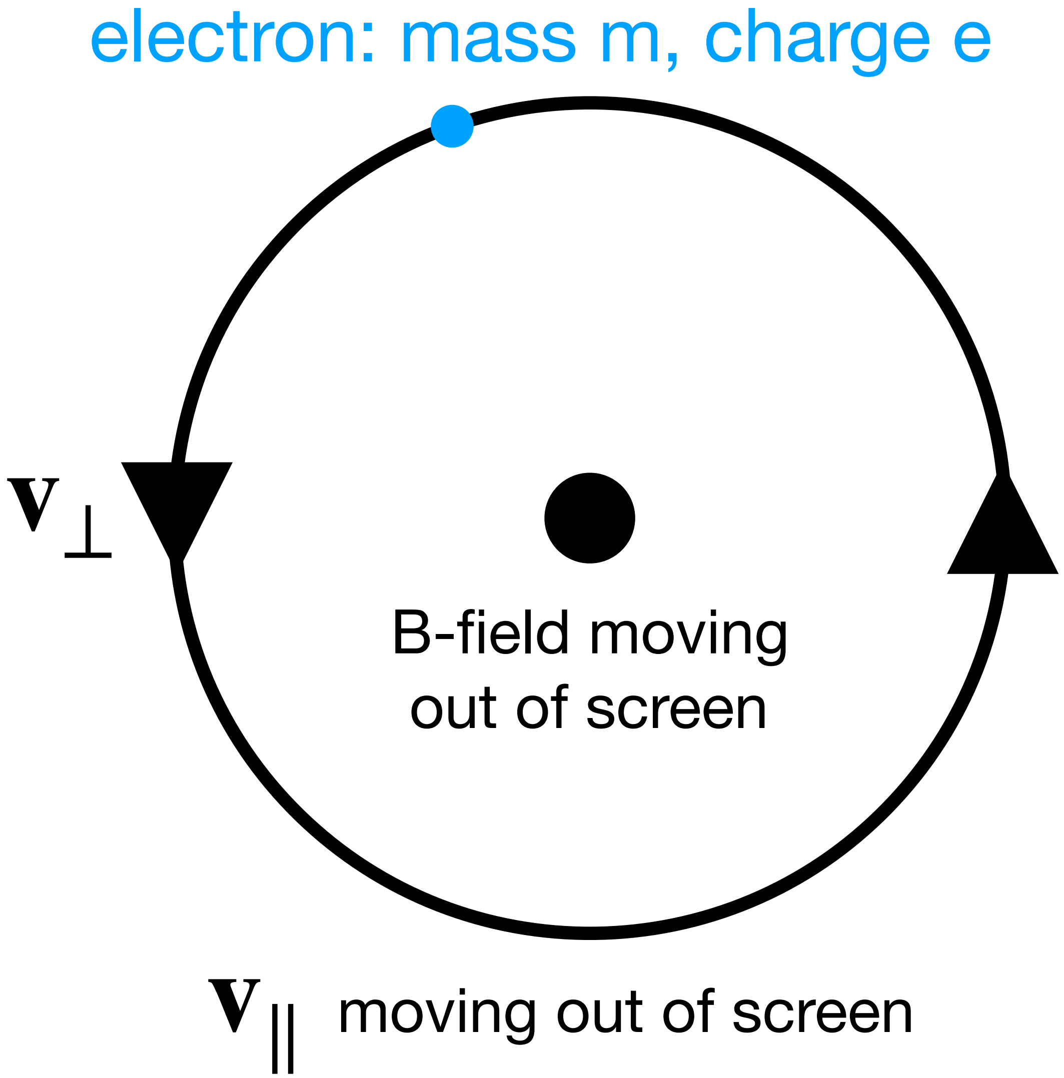
- Radiation released by relativistic electrons spiralling around magnetic field lines.
- Motion is *helical*:

$$\mathbf{v}_{||} = \text{constant}$$ ...velocity parallel to B-field

$$\mathbf{v}_{\perp} = \text{circular}$$ ...velocity perpendicular to B-field

Pitch angle = angle between $\mathbf{v}$ and $\mathbf{B}$ = α





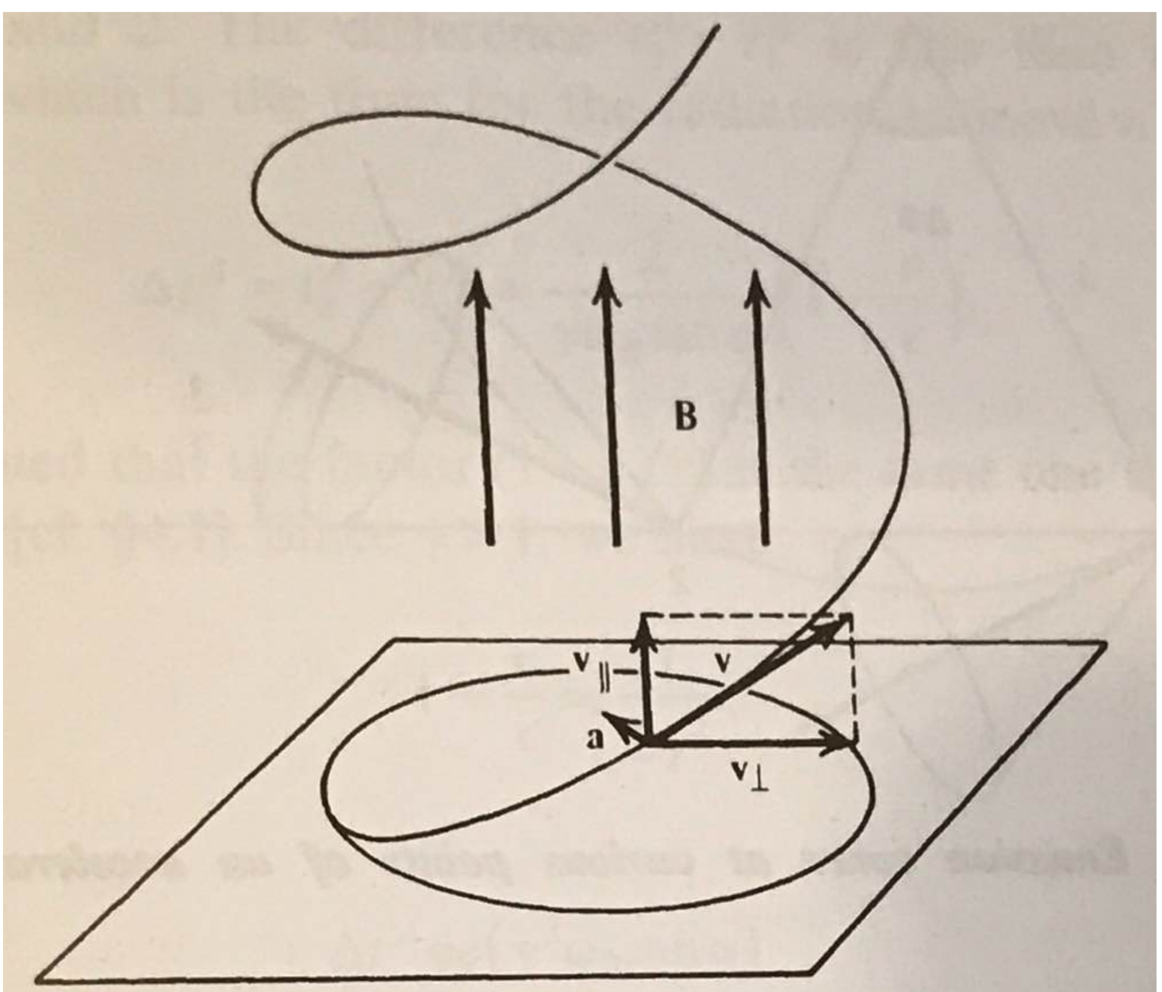
Acceleration of electron

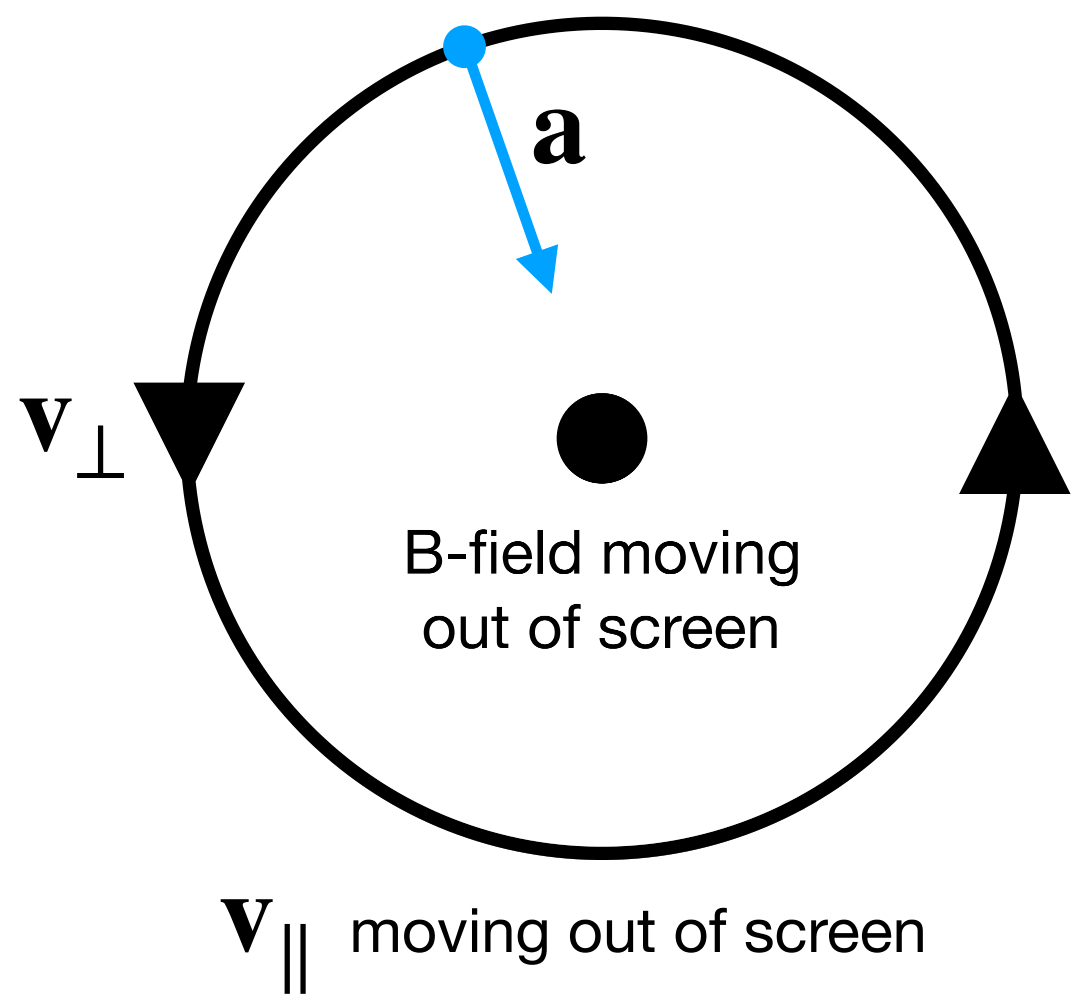
- Lorentz force: $$\mathbf{F} = q(\mathscr{E} + \mathbf{v} \times \mathbf{B}) = -e(\mathscr{E} + \mathbf{v} \times \mathbf{B})$$

Electron has -ve charge

Electric field vector
(E is reserved for electron energy)

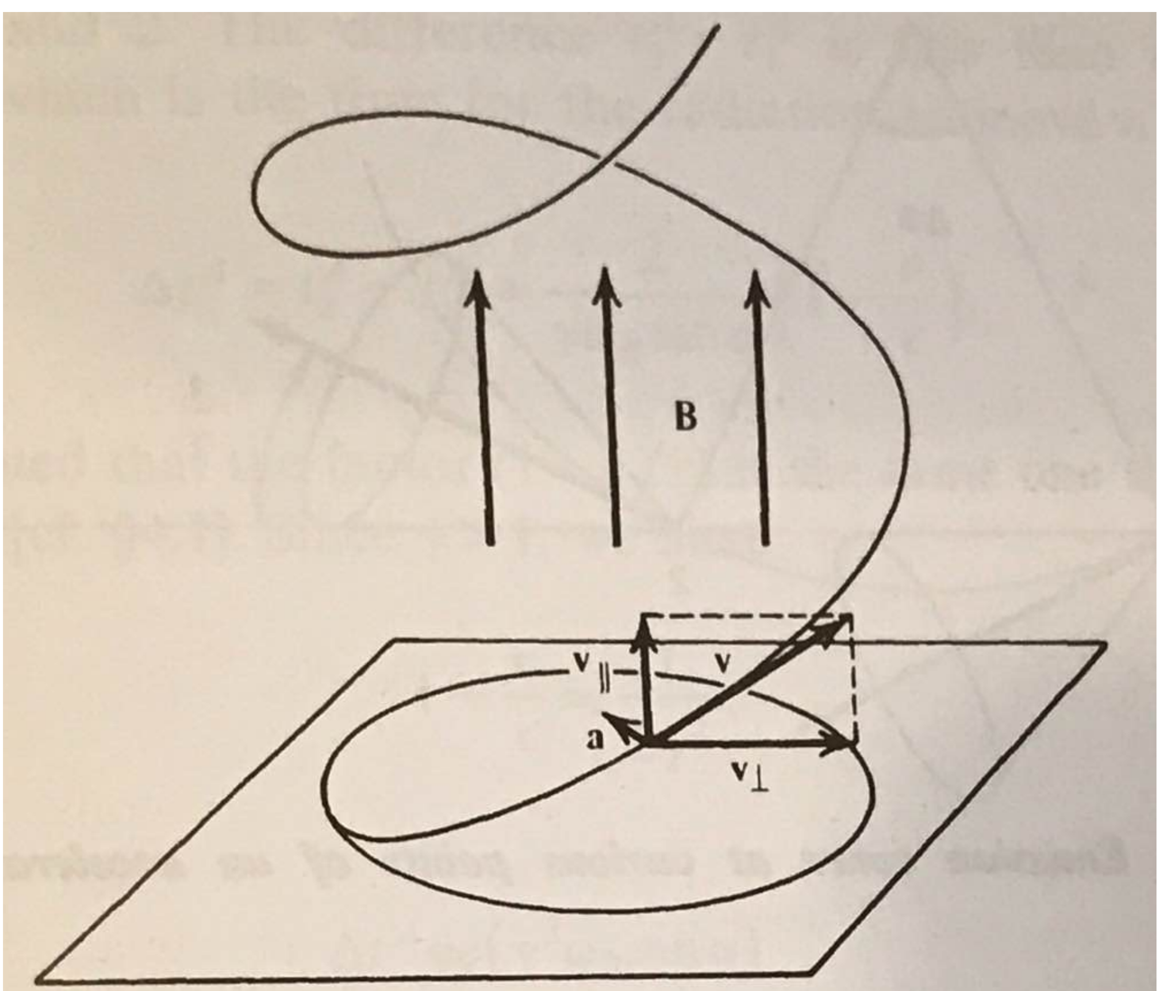
Electric charge

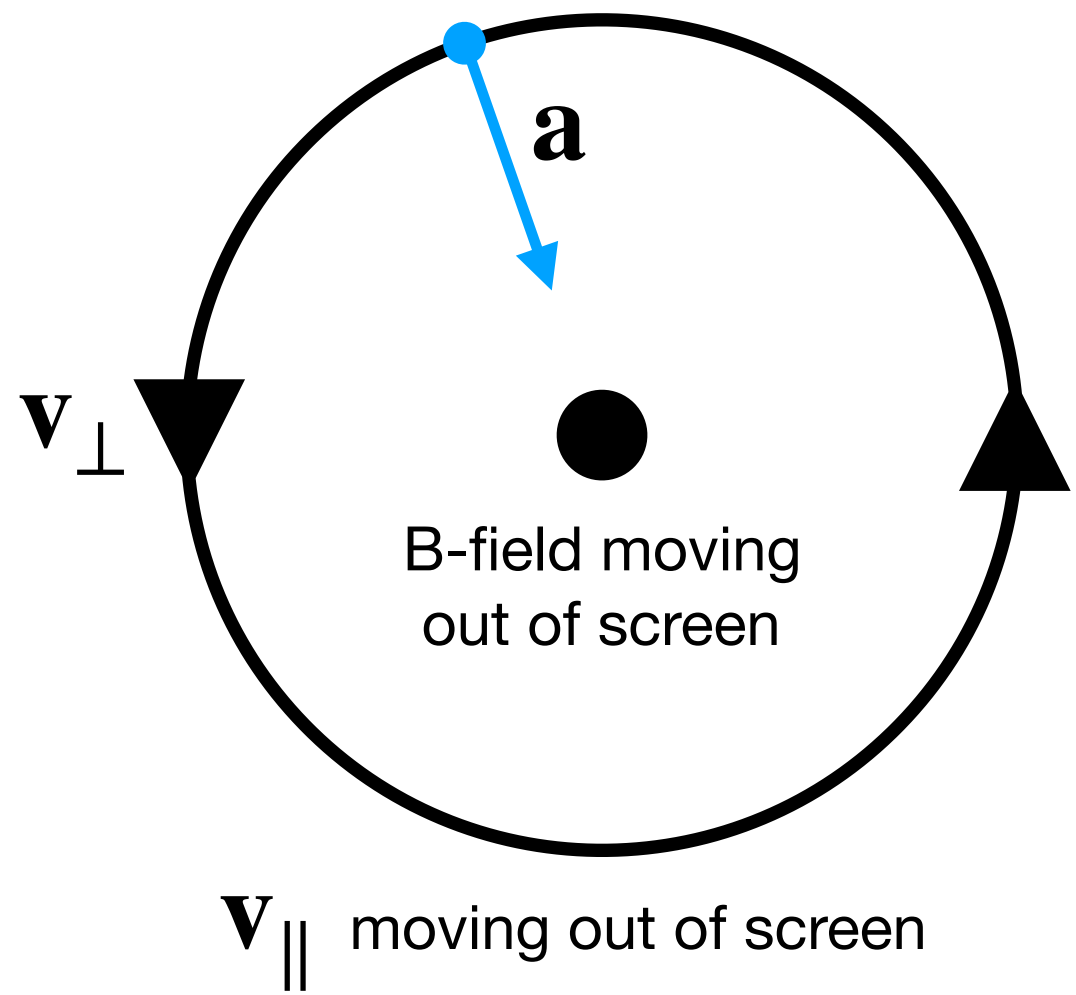




Acceleration of electron

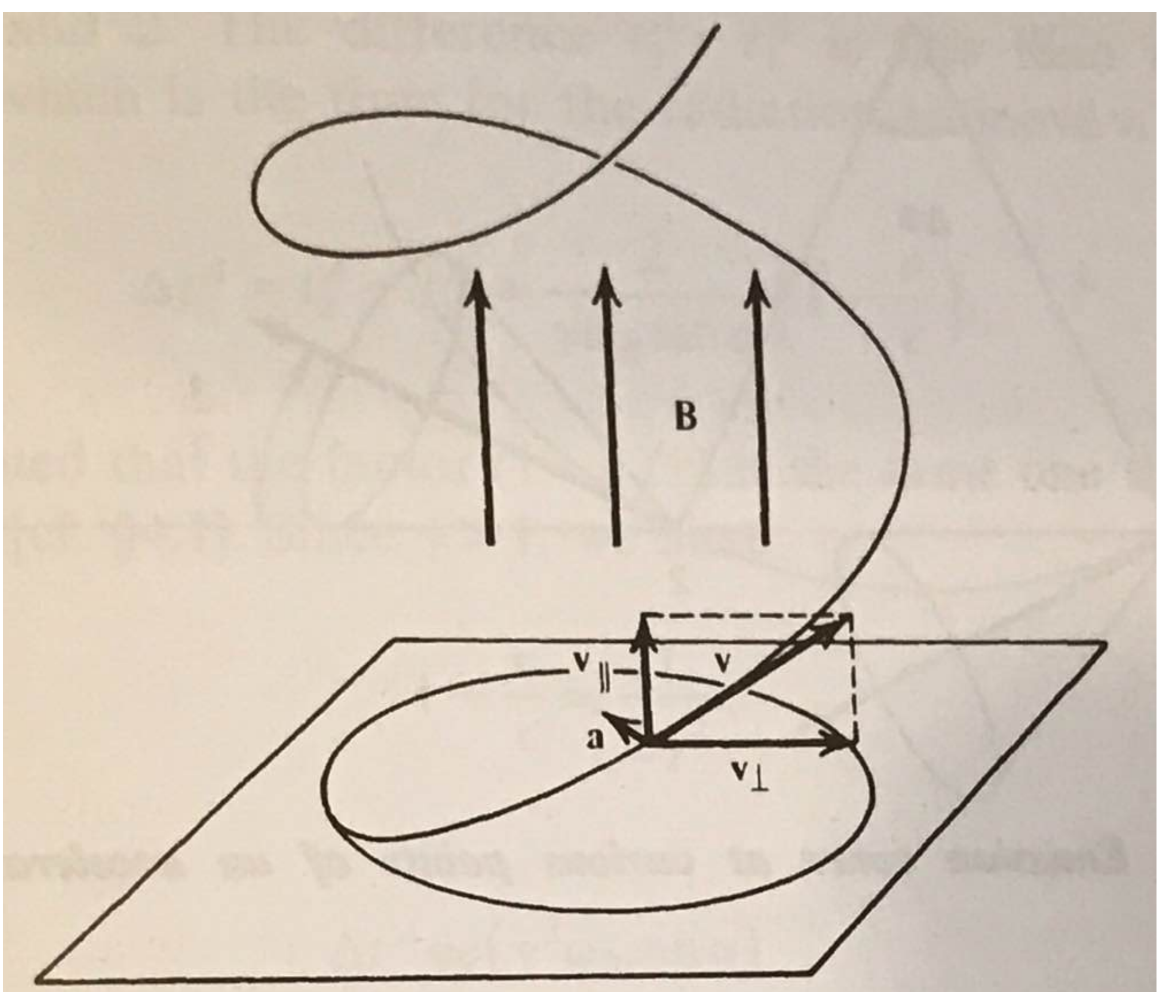
- Lorentz force: $\mathbf{F} = q(\mathscr{E} + \mathbf{v} \times \mathbf{B}) = -e(\mathscr{E} + \mathbf{v} \times \mathbf{B})$
- Static B-field $\implies \mathbf{F} = -e\mathbf{v} \times \mathbf{B}$

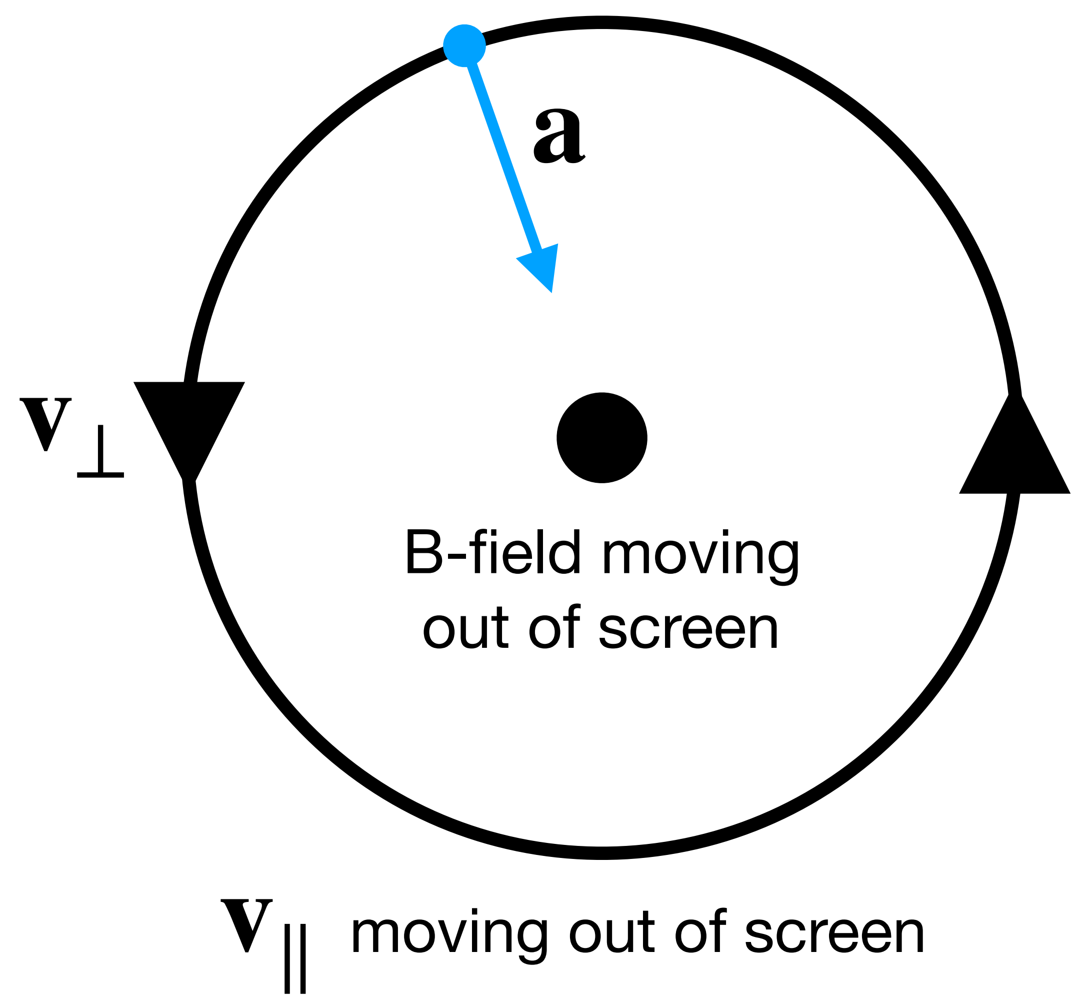




Acceleration of electron

- Lorentz force: $\mathbf{F} = q(\mathscr{E} + \mathbf{v} \times \mathbf{B}) = -e(\mathscr{E} + \mathbf{v} \times \mathbf{B})$
- Static B-field $\implies \mathbf{F} = -e\mathbf{v} \times \mathbf{B}$
- Therefore equation of motion: $\dfrac{d}{dt}(\gamma m \mathbf{v}) = -e\mathbf{v} \times \mathbf{B}$

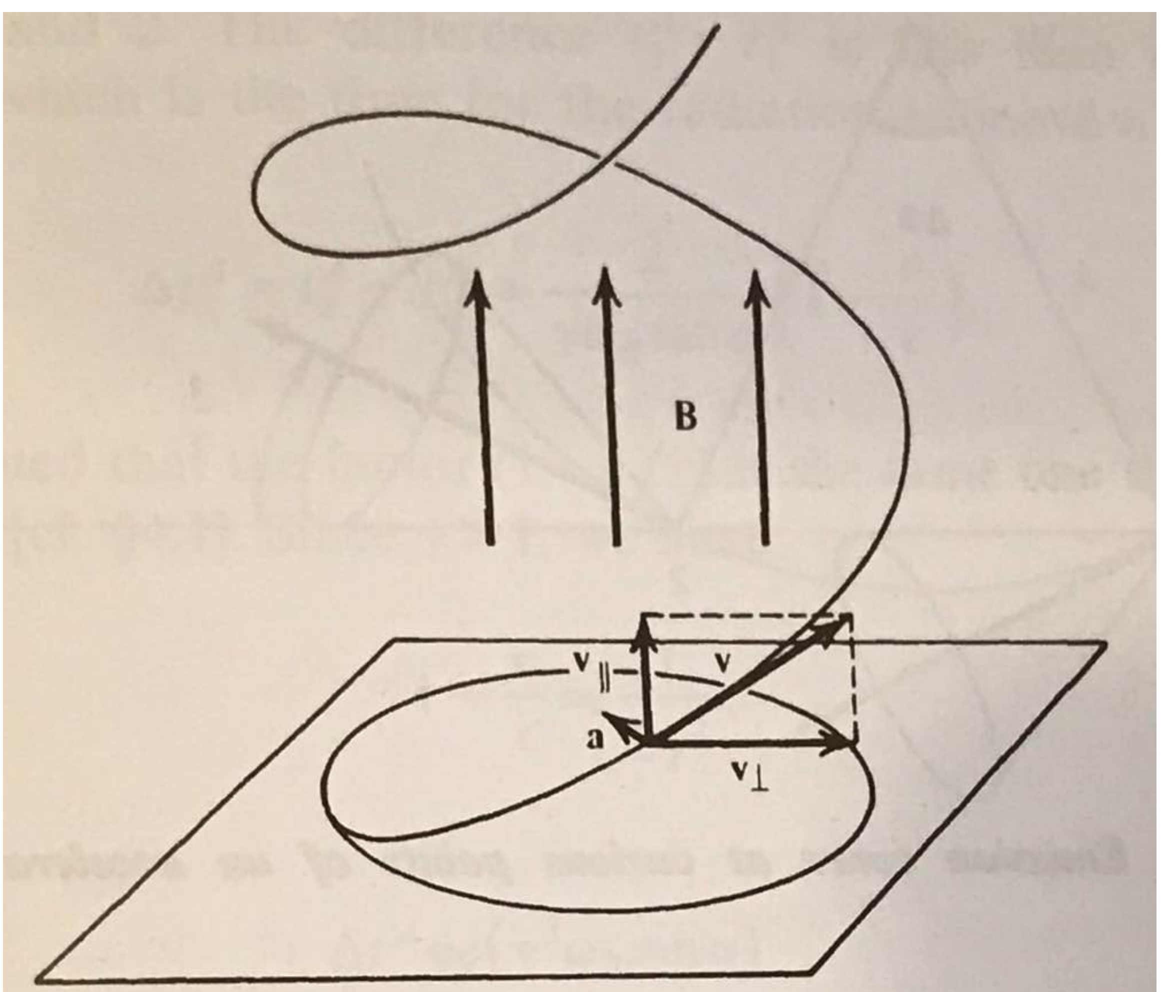


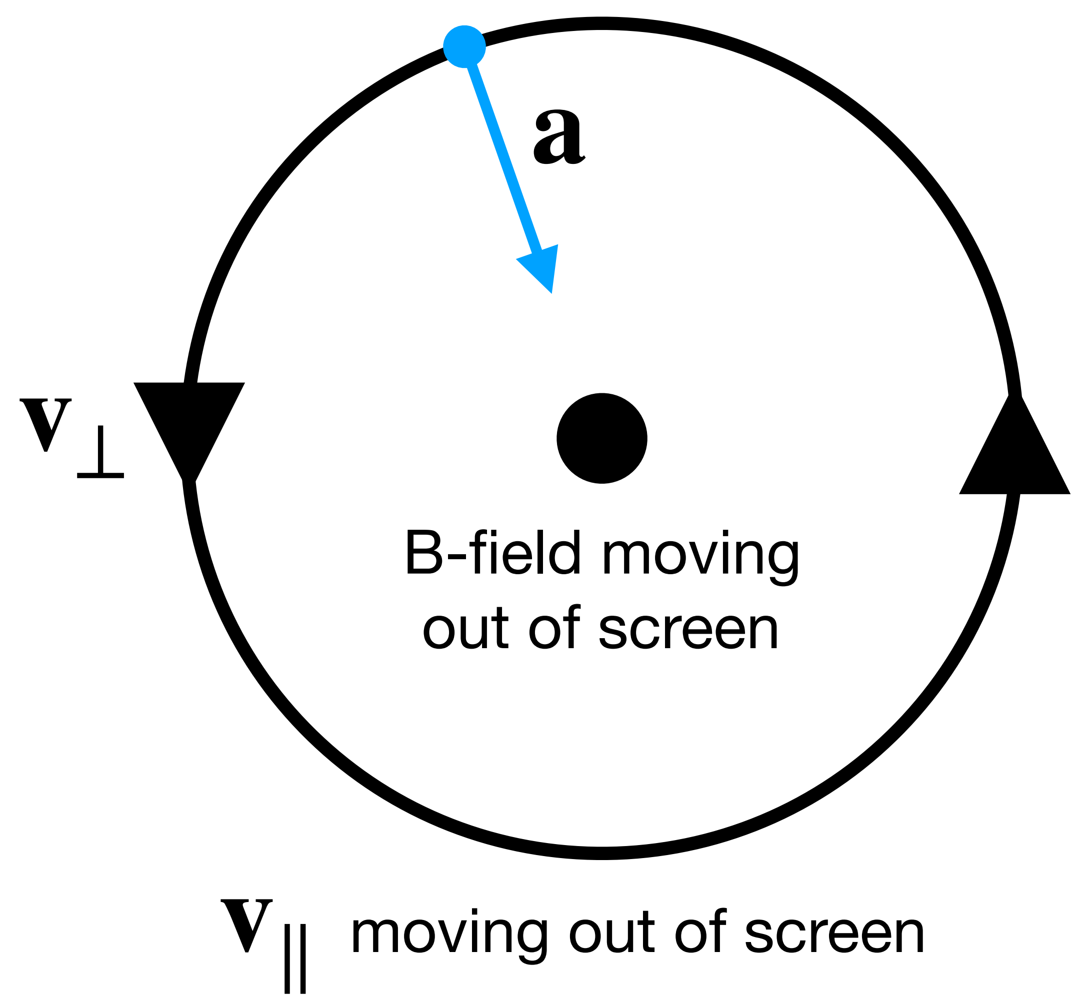


Acceleration of electron

- Lorentz force: $\mathbf{F} = q(\mathscr{E} + \mathbf{v} \times \mathbf{B}) = -e(\mathscr{E} + \mathbf{v} \times \mathbf{B})$
- Static B-field $\implies \mathbf{F} = -e\mathbf{v} \times \mathbf{B}$
- Therefore equation of motion: $\dfrac{d}{dt}(\gamma m \mathbf{v}) = -e\mathbf{v} \times \mathbf{B}$
- Therefore acceleration:

$$\mathbf{a} = -\frac{evB\sin\alpha}{\gamma m}\hat{\mathbf{r}}$$

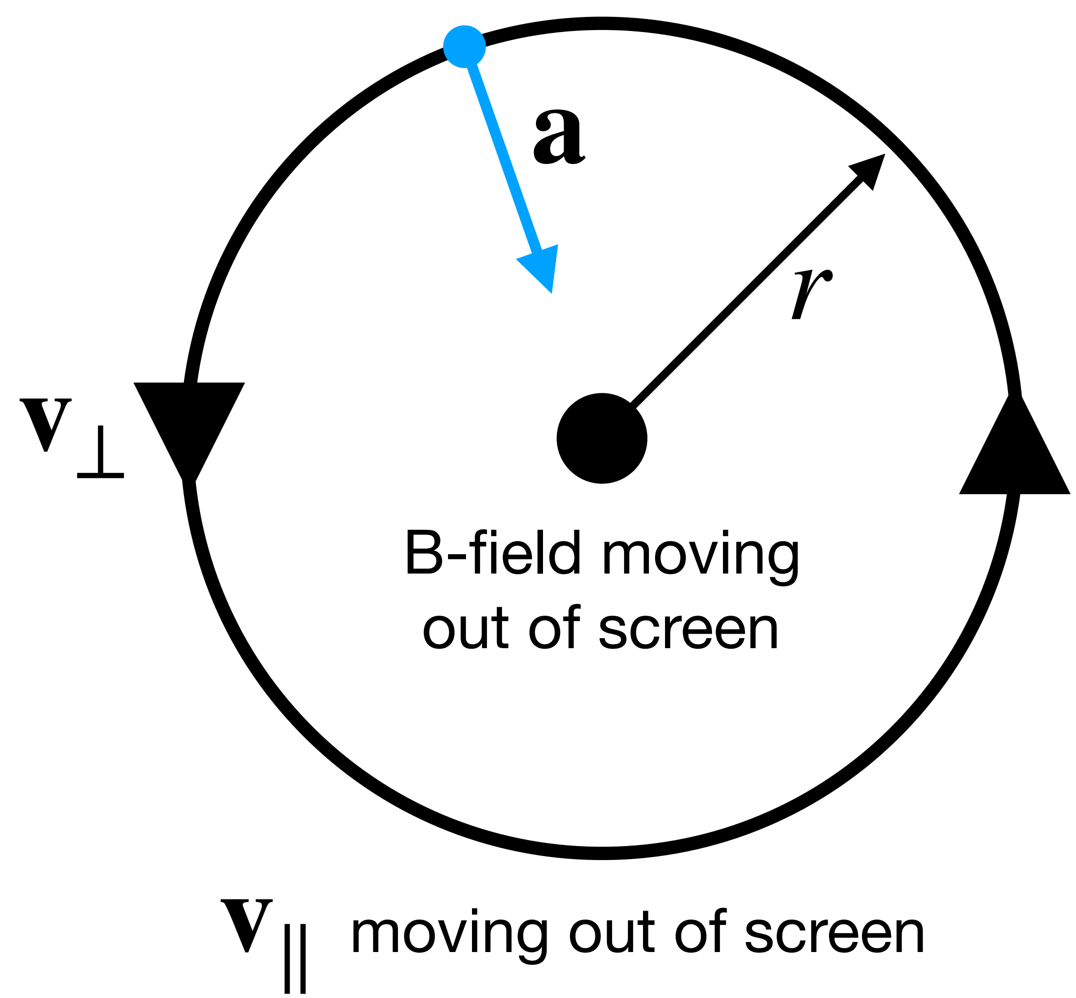




Gyroradius

- Calculate orbital radius of electron by setting a equal to centripetal acceleration:

$$a = \frac{evB\sin\alpha}{\gamma m} = \frac{v_\perp^2}{r} = \frac{v^2\sin^2\alpha}{r}$$



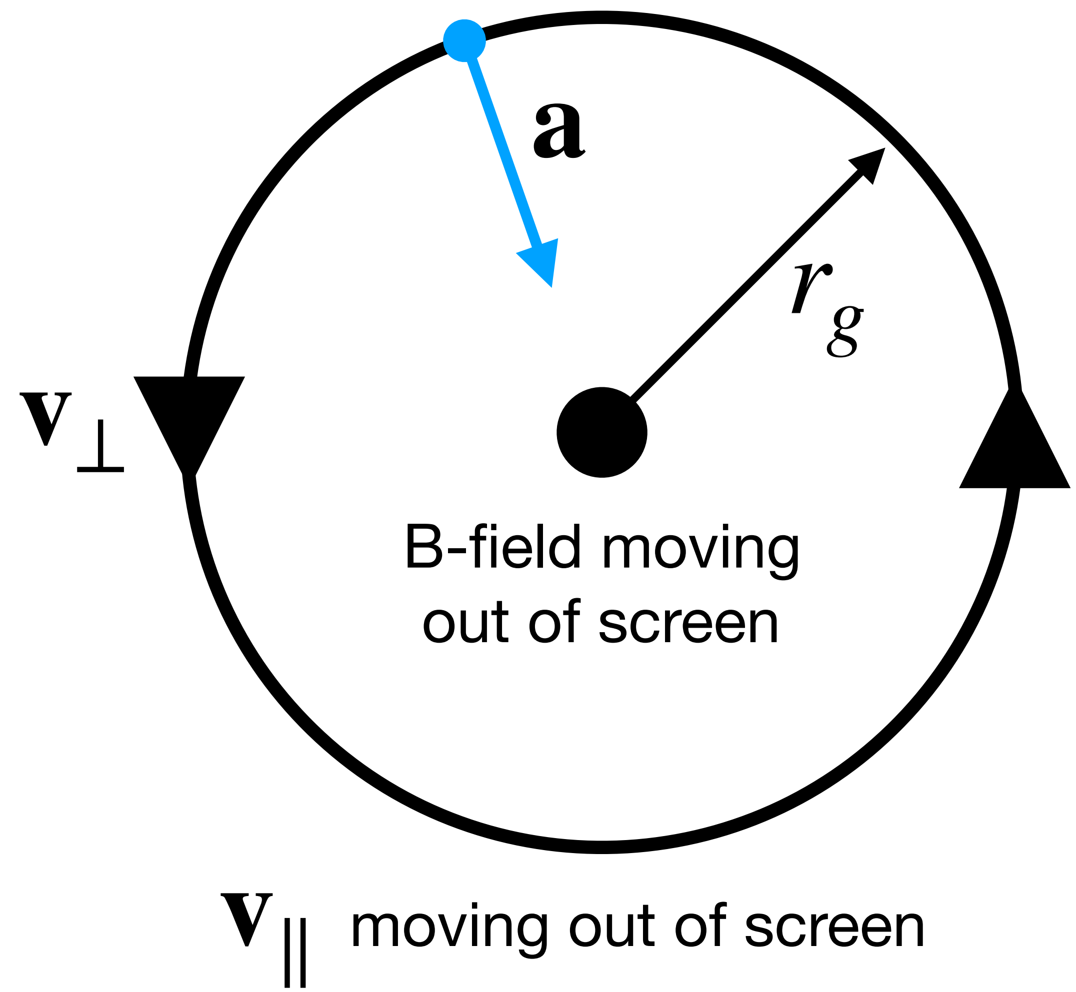
Gyroradius

- Calculate orbital radius of electron by setting a equal to centripetal acceleration:

$$a = \frac{evB \sin \alpha}{\gamma m} = \frac{v_\perp^2}{r} = \frac{v^2 \sin^2 \alpha}{r}$$

- Therefore *gyroradius*:

$$r_g = \frac{\gamma m v \sin \alpha}{eB}$$



Gyroradius

- Calculate orbital radius of electron by setting a equal to centripetal acceleration:

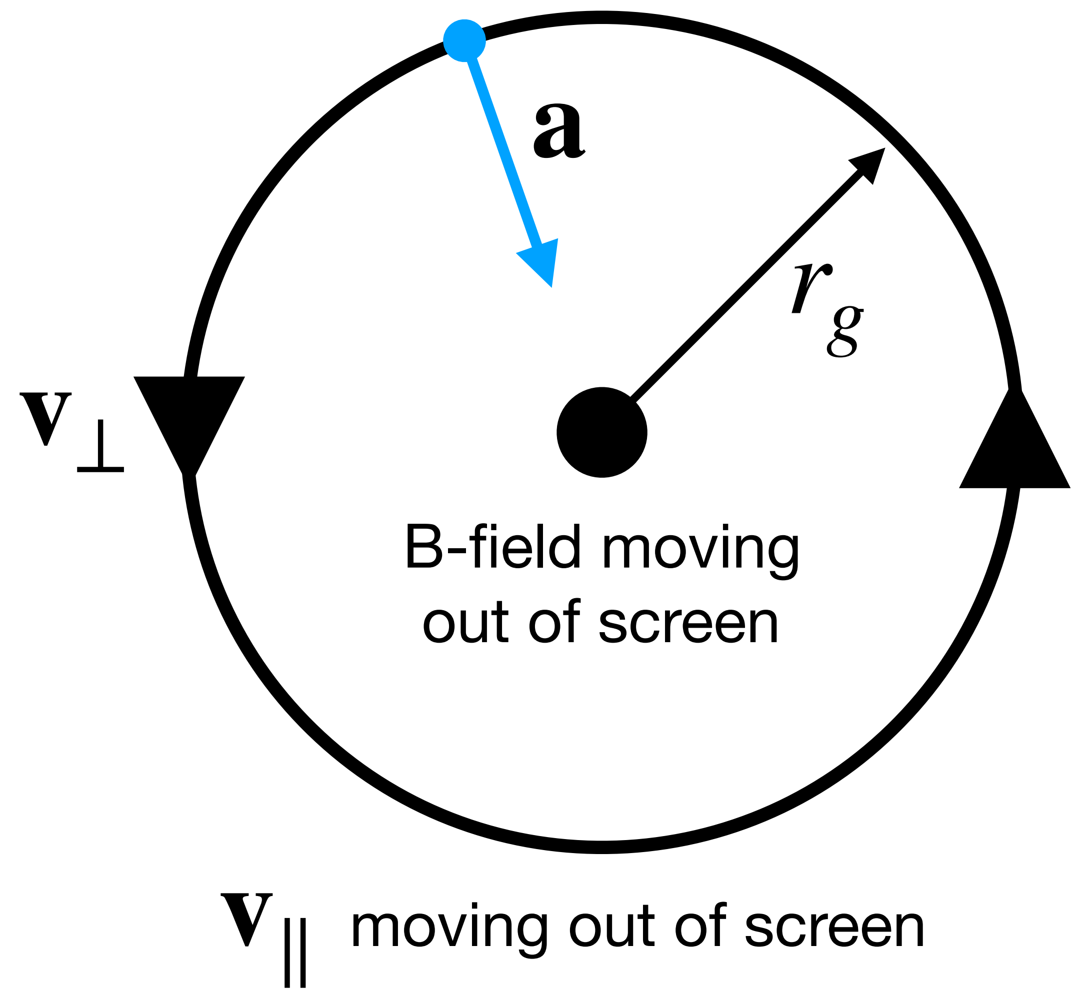
$$a = \frac{evB\sin\alpha}{\gamma m} = \frac{v_\perp^2}{r} = \frac{v^2\sin^2\alpha}{r}$$

- Therefore *gyroradius*:

$$r_g = \frac{\gamma m v \sin\alpha}{eB}$$

- *Gyro angular frequency*:

$$\omega_g = \frac{v_\perp}{r_g} = \frac{eB}{\gamma m}$$



Gyroradius

- Calculate orbital radius of electron by setting a equal to centripetal acceleration:

$$a = \frac{evB\sin\alpha}{\gamma m} = \frac{v_\perp^2}{r} = \frac{v^2\sin^2\alpha}{r}$$

- Therefore *gyroradius*:

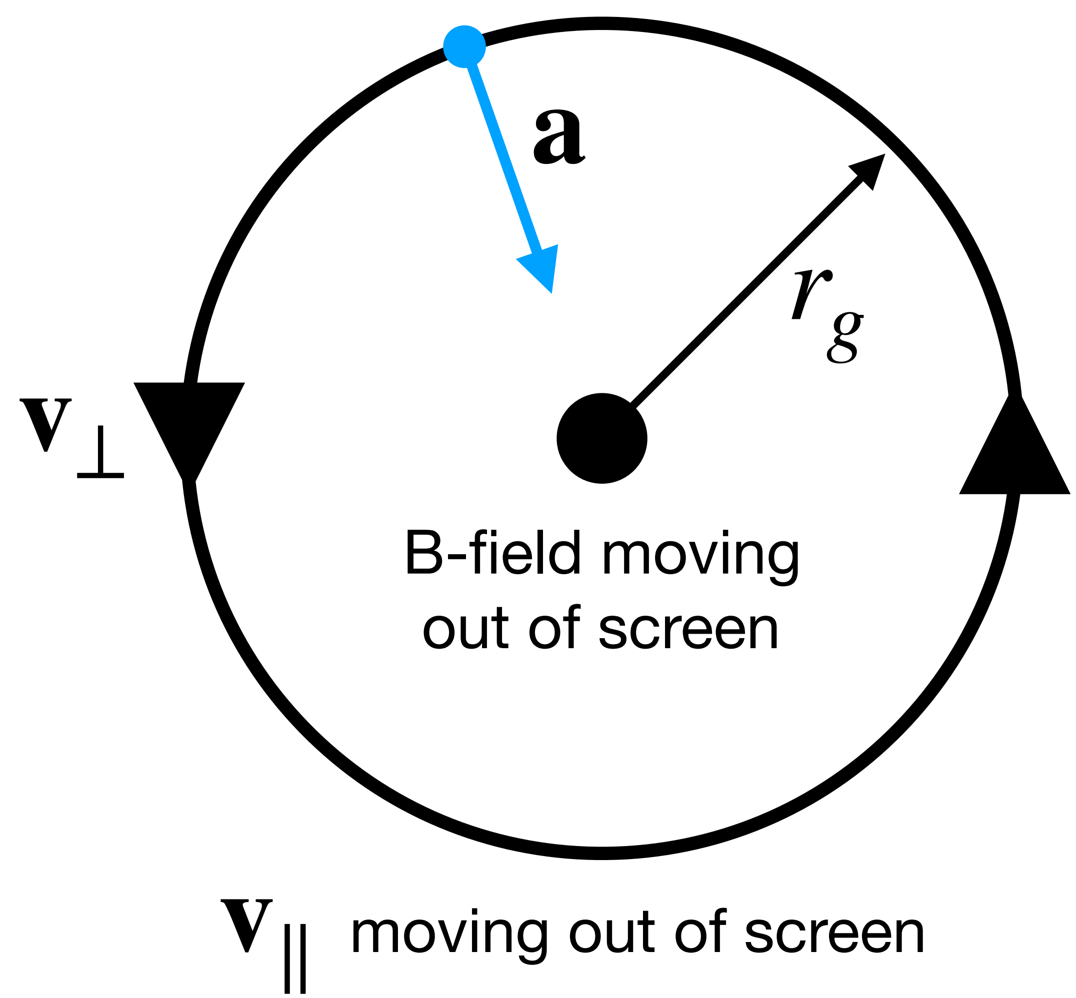
$$r_g = \frac{\gamma m v\sin\alpha}{eB}$$

- *Gyro angular frequency*:

$$\omega_g = \frac{v_\perp}{r_g} = \frac{eB}{\gamma m}$$

- *Gyrofrequency*:

$$\nu_g = \frac{v_\perp}{2\pi r_g} = \frac{eB}{2\pi\gamma m}$$



Gyroradius

- Calculate orbital radius of electron by setting a equal to centripetal acceleration:

$$a = \frac{evB\sin\alpha}{\gamma m} = \frac{v_\perp^2}{r} = \frac{v^2\sin^2\alpha}{r}$$

- Therefore *gyroradius*:

$$r_g = \frac{\gamma m v\sin\alpha}{eB}$$

- *Gyro angular frequency*:

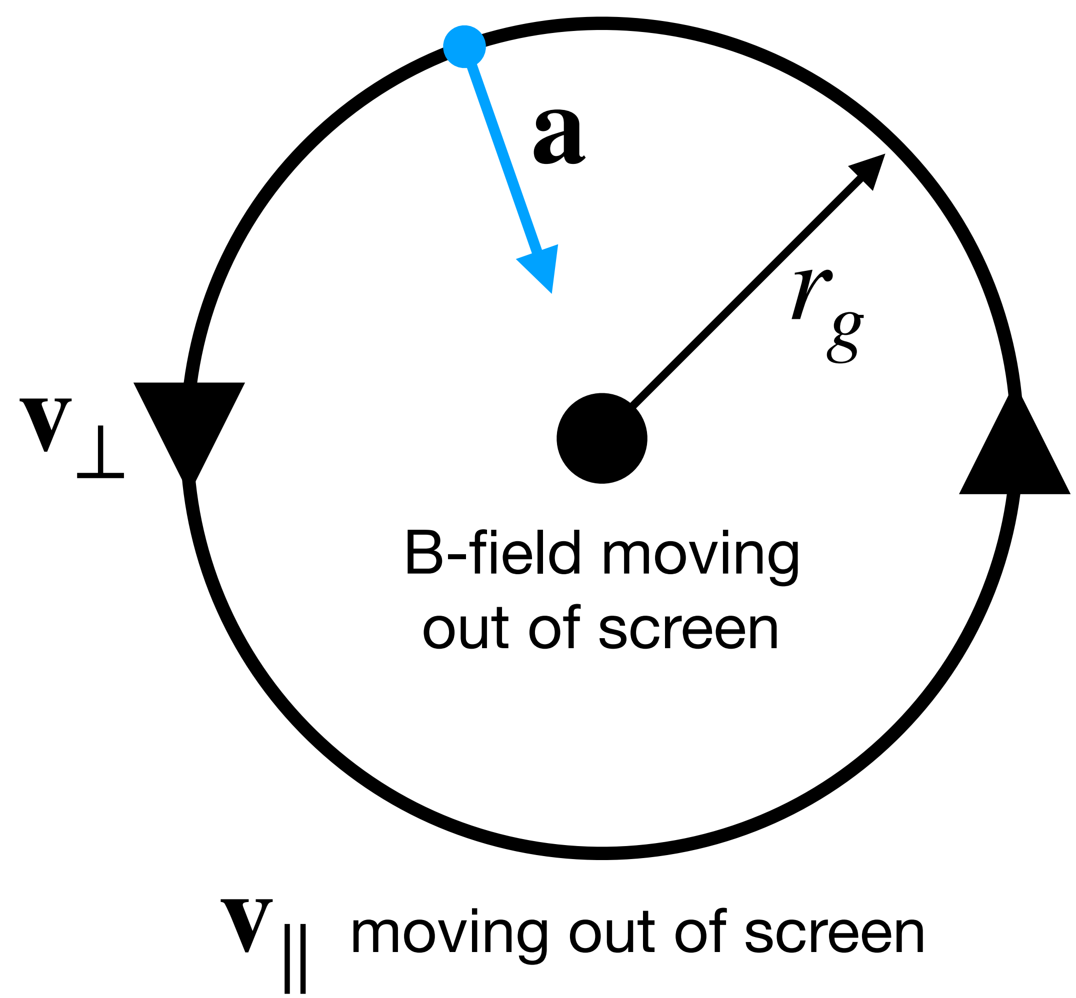
$$\omega_g = \frac{v_\perp}{r_g} = \frac{eB}{\gamma m}$$

- *Gyrofrequency*:

$$\nu_g = \frac{v_\perp}{2\pi r_g} = \frac{eB}{2\pi\gamma m}$$

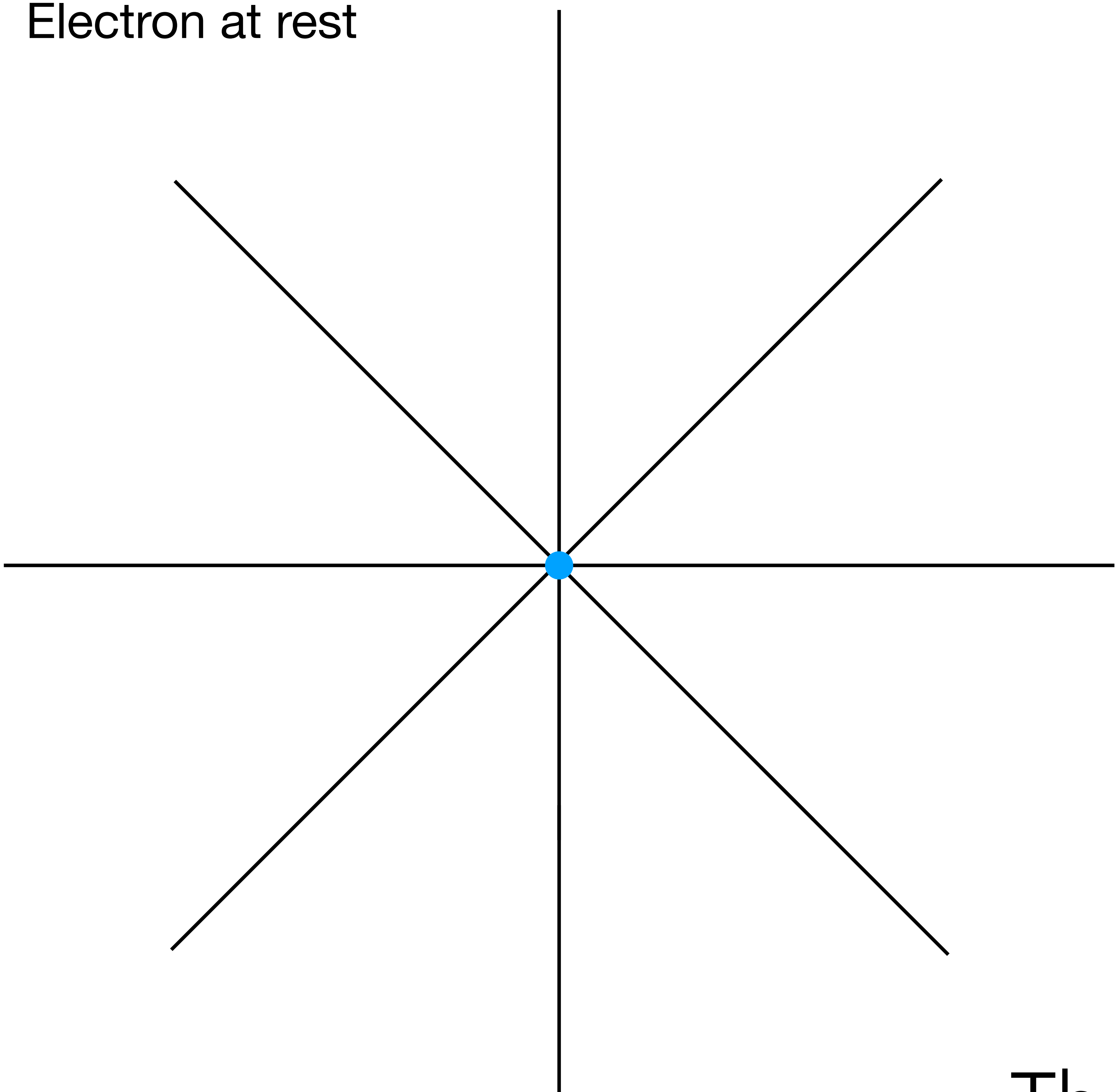
- *Gyroperiod*:

$$t_g = \frac{1}{\nu_g} = \frac{2\pi\gamma m}{eB}$$



Radiation generated

Electron at rest



From Coulomb's law,
electric field is:

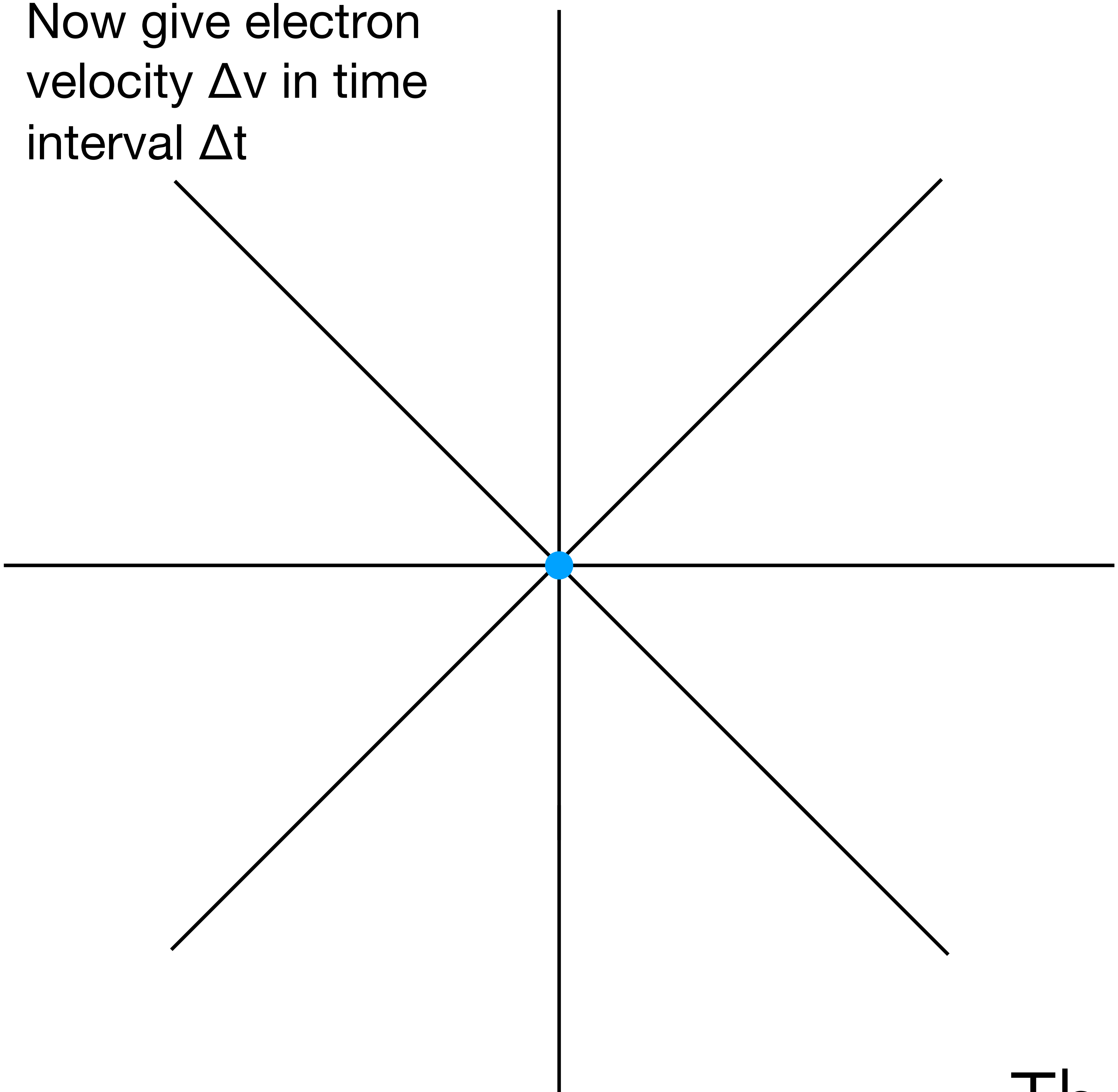
$$\mathscr{E}_r = \frac{e}{4\pi\epsilon_0 r^2}$$

$$\mathscr{E}_\theta = 0$$

Thomson's reasoning

Radiation generated

Now give electron velocity Δv in time interval Δt



From Coulomb's law, electric field is:

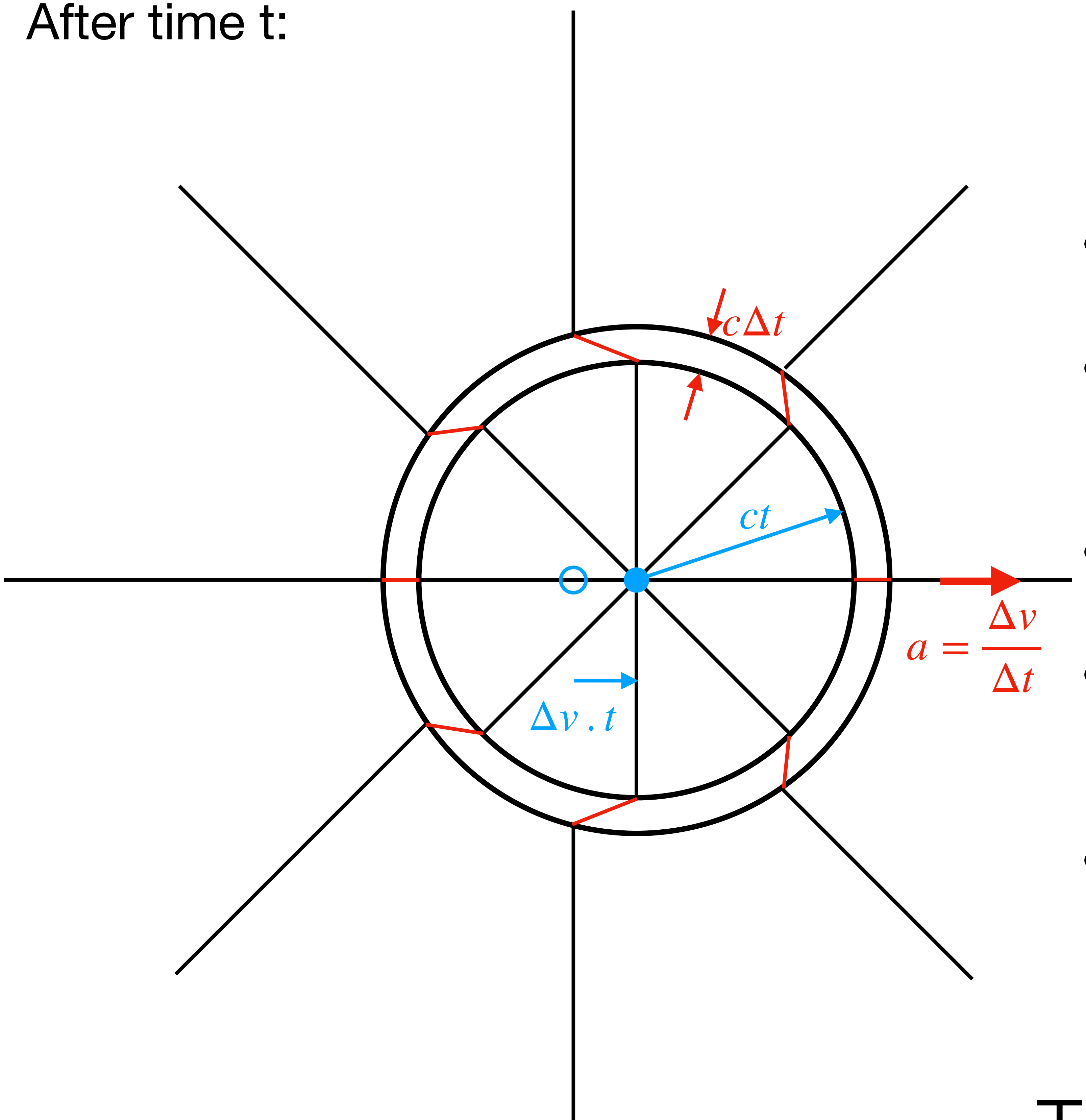
$$\mathscr{E}_r = \frac{e}{4\pi\epsilon_0 r^2}$$

$$\mathscr{E}_\theta = 0$$

Thomson's reasoning

Radiation generated

After time t:

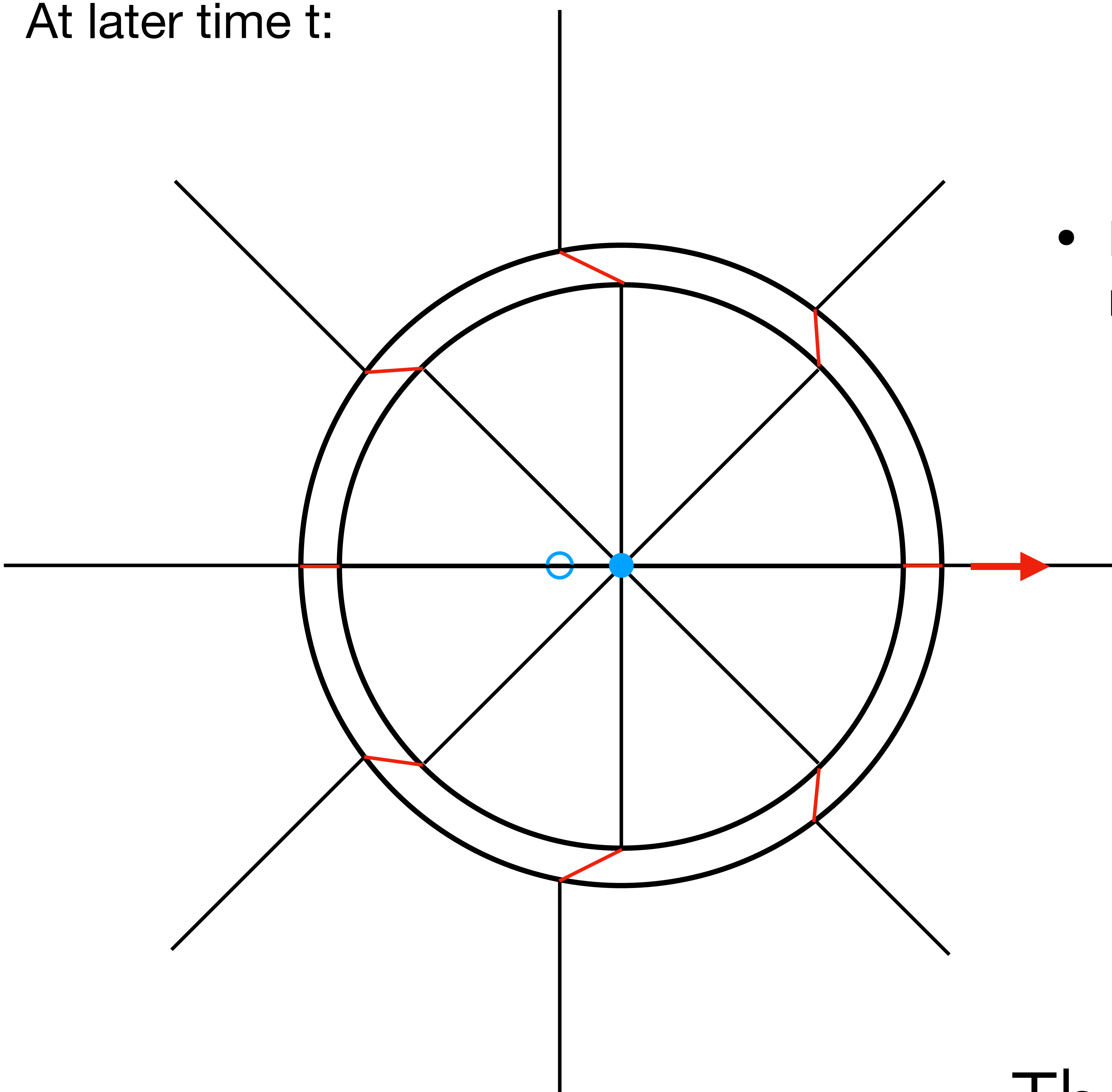


- Electron has travelled distance Δv t
- Electric field is radial centred on electron in sphere of radius ct
- Field outside of this sphere hasn't adjusted yet
- Kink in field in shell of radius cΔt where inner and outer fields join up
- This kink corresponds to non-zero $\mathscr{E}_\theta$

Thomson's reasoning

Radiation generated

At later time t:

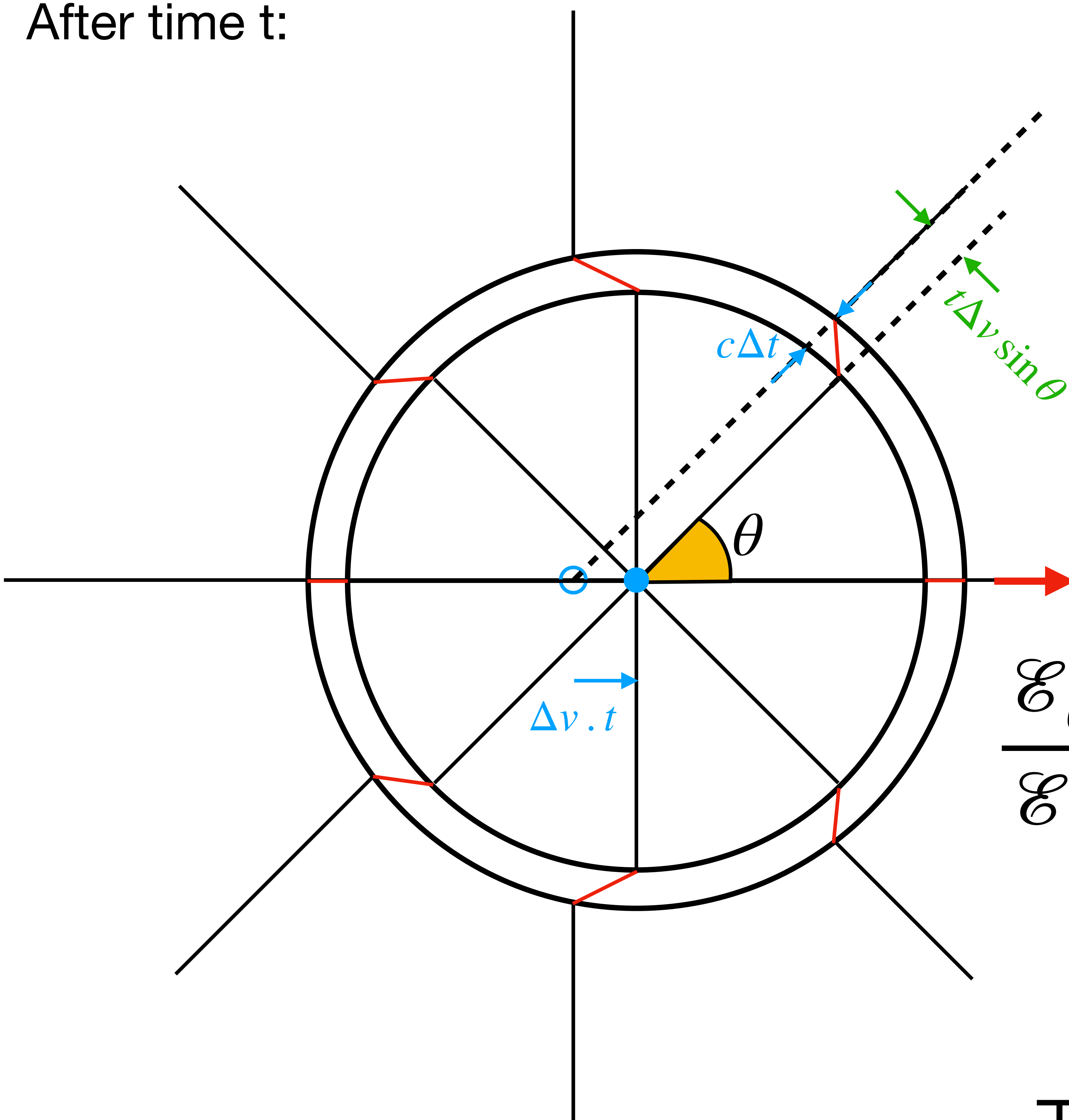


- Pulse of non-zero $\mathscr{E}_\theta$ has moved out

Thomson's reasoning

Radiation generated

After time t:



- Pulse of non-zero $\mathscr{E}_\theta$ has moved out
- Pulse strength (in electron rest frame) depends on angle to acceleration θ:

$$\frac{\mathscr{E}_\theta}{\mathscr{E}_r} = \frac{t\Delta v\sin\theta}{c\Delta t} = \frac{ar}{c^2}\sin\theta$$

Thomson’s reasoning

Power radiated
(non-relativistic)

$$\mathscr{E}_r = \frac{e}{4\pi\epsilon_0 r^2} \qquad \frac{\mathscr{E}_\theta}{\mathscr{E}_r} = \frac{ar}{c^2}\sin\theta \qquad c^2 = \frac{1}{\epsilon_0\mu_0}$$

Energy flow in pulse / time / area at distance r = modulus of Poynting vector:

$$S = \left(\epsilon_0/\mu_0\right)^{1/2}\mathscr{E}_\theta^2 = \frac{e^2a^2\sin^2\theta}{(4\pi)^2\epsilon_0 c^3 r^2}$$

Therefore power radiated into full sphere (remember dA = r 2π sinθ dθ):

$$P = \frac{e^2a^2}{6\pi\epsilon_0 c^3}$$

Power radiated

$$P = \frac{e^2 a^2}{6\pi\epsilon_0 c^3}$$

- We want to deal with ultra-relativistic electrons, so need relativistic limit! Luckily, can use non-relativistic formula in the instantaneous electron rest frame S' and then use Lorentz invariance of dE/dt to trivially move back to the observer's frame S (in which the B-field is at rest)

Power radiated

$$P = \frac{e^2 a^2}{6\pi\epsilon_0 c^3}$$

- We want to deal with ultra-relativistic electrons, so need relativistic limit! Luckily, can use non-relativistic formula in the instantaneous electron rest frame S' and then use Lorentz invariance of dE/dt to trivially move back to the observer's frame S (in which the B-field is at rest)

- This comes about because dE and dt Lorentz transform in the same way:

$$dE = \gamma dE'; \quad dt = \gamma dt' \;\; \therefore (dE/dt) = (dE/dt)'$$

Power radiated

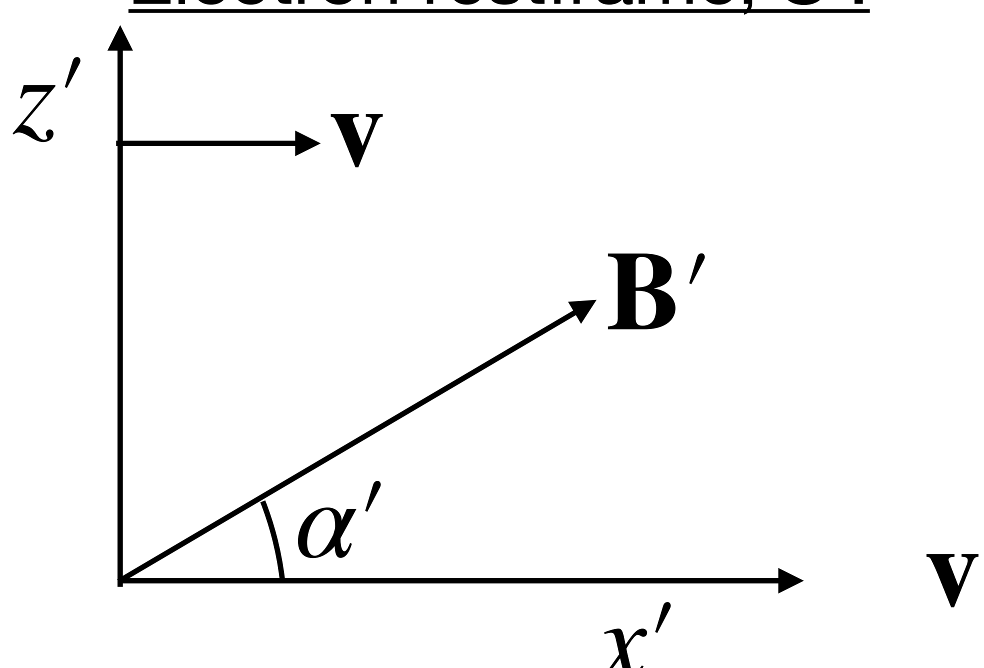
$$P = \frac{e^2 a^2}{6\pi\epsilon_0 c^3}$$

- We want to deal with ultra-relativistic electrons, so need relativistic limit! Luckily, can use non-relativistic formula in the instantaneous electron rest frame S’ and then use Lorentz invariance of dE/dt to trivially move back to the observer’s frame S (in which the B-field is at rest)

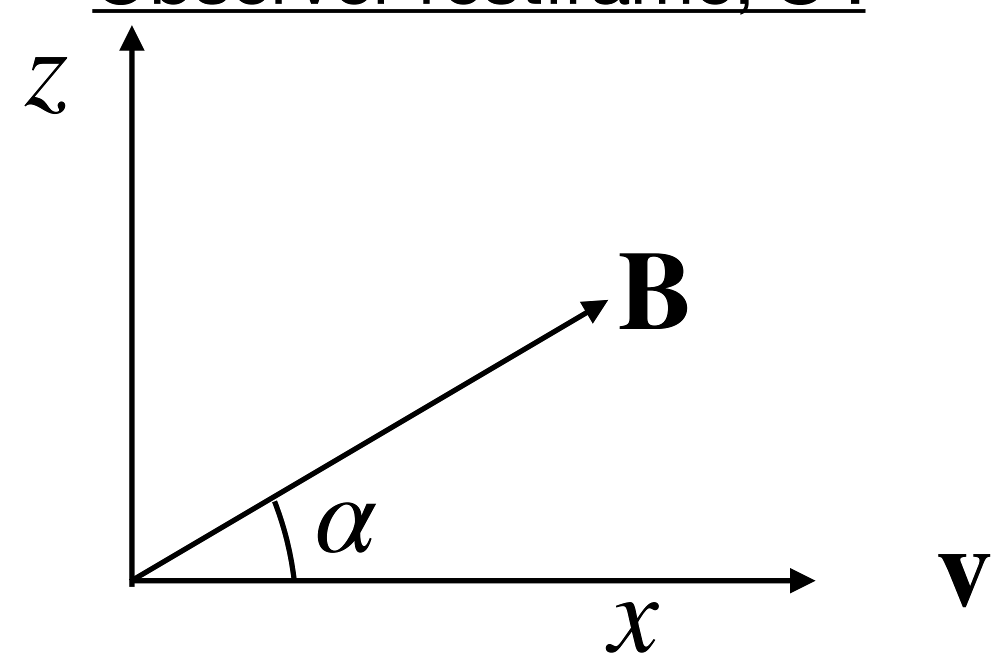
- This comes about because dE and dt Lorentz transform in the same way:

$$dE = \gamma dE'; \quad dt = \gamma dt' \ \therefore (dE/dt) = (dE/dt)'$$

<u>Electron restframe, S’:</u>



<u>Observer restframe, S’:</u>



Power radiated

- In S frame: $B_x = B\cos\alpha;\ \ B_y = 0;\ \ B_z = B\sin\alpha$

  $\mathscr{E} = 0$

Power radiated

- In S frame: $B_x = B\cos\alpha; \;\; B_y = 0; \;\; B_z = B\sin\alpha$

  $\mathscr{E} = 0$

- In S' frame:

$$P' = \frac{e^2 a'}{6\pi\epsilon_0 c^3}$$

$$\mathbf{F}' = m\mathbf{a}' = e(\mathscr{E}' + \mathbf{v}' \times \mathbf{B}')$$

Power radiated

- In S frame: $B_x = B\cos\alpha;\ \ B_y = 0;\ \ B_z = B\sin\alpha$

  $\mathscr{E} = 0$

- In S' frame:

$$P' = \frac{e^2 a'}{6\pi\epsilon_0 c^3}$$

$$\mathbf{F}' = m\mathbf{a}' = e(\mathscr{E}' + \mathbf{v}' \times \mathbf{B}')$$

- By design of the rest frame, v'=0, therefore: $m\mathbf{a}' = e\mathscr{E}'$

Power radiated

- In S frame: $B_x = B\cos\alpha;\;\; B_y = 0;\;\; B_z = B\sin\alpha$

  $\mathscr{E} = 0$

- In S' frame:

$$P' = \frac{e^2 a'}{6\pi\epsilon_0 c^3}$$

$$\mathbf{F}' = m\mathbf{a}' = e(\mathscr{E}' + \mathbf{v}' \times \mathbf{B}')$$

- By design of the rest frame, v'=0, therefore: $m\mathbf{a}' = e\mathscr{E}'$

- Lorentz transforms to get electric field in S':

$$\mathscr{E}'_x = \mathscr{E}_x$$
$$\mathscr{E}'_y = \gamma(\mathscr{E}_y - vB_z)$$
$$\mathscr{E}'_z = \gamma(\mathscr{E}_z - vB_y)$$

Power radiated

- In S frame: $B_x = B\cos\alpha;\ \ B_y = 0;\ \ B_z = B\sin\alpha$

  $\mathscr{E} = 0$

- In S' frame:

$$P' = \frac{e^2 a'}{6\pi\epsilon_0 c^3}$$

$$\mathbf{F}' = m\mathbf{a}' = e(\mathscr{E}' + \mathbf{v}' \times \mathbf{B}')$$

- By design of the rest frame, v'=0, therefore: $m\mathbf{a}' = e\mathscr{E}'$

- Lorentz transforms to get electric field in S':

$$\mathscr{E}'_x = \mathscr{E}_x = 0$$
$$\mathscr{E}'_y = \gamma(\mathscr{E}_y - vB_z) = -\gamma v B_z = -\gamma v B\sin\alpha$$
$$\mathscr{E}'_z = \gamma(\mathscr{E}_z - vB_y) = 0$$

Power radiated

- Therefore: $$a' = -\frac{e\gamma v B \sin\alpha}{m}$$

Power radiated

- Therefore:

$$a' = -\frac{e\gamma vB\sin\alpha}{m}$$

- Therefore:

$$P = P' = \frac{e^2a'}{6\pi\epsilon_0c^3} = \frac{e^4\gamma^2B^2v^2\sin^2\alpha}{6\pi\epsilon c^3m_e^2}$$

Power radiated

- Therefore: $$a' = -\frac{e\gamma vB\sin\alpha}{m}$$
- Therefore: $$P = P' = \frac{e^2a'}{6\pi\epsilon_0c^3} = \frac{e^4\gamma^2B^2v^2\sin^2\alpha}{6\pi\epsilon c^3m_e^2}$$
- Re-arrange:

$$c^2 = \frac{1}{\epsilon_0\mu_0}$$ $$U_{\text{mag}} = \frac{B^2}{2\mu_0}$$ = energy density of magnetic field

$$\sigma_T = \frac{e^4}{6\pi\epsilon_0^2c^4m^2}$$ = Thomson cross-section

$$\implies \quad P = 2\sigma_T c U_{\text{mag}}\left(\frac{v}{c}\right)^2\gamma^2\sin^2\alpha$$

Power radiated

- Therefore: $$a' = -\frac{e\gamma vB\sin\alpha}{m}$$
- Therefore:

$$P = P' = \frac{e^2 a'}{6\pi\epsilon_0 c^3} = \frac{e^4\gamma^2 B^2 v^2 \sin^2\alpha}{6\pi\epsilon c^3 m_e^2}$$

- Re-arrange:

$$c^2 = \frac{1}{\epsilon_0\mu_0} \qquad U_{\mathrm{mag}} = \frac{B^2}{2\mu_0}$$ = energy density of magnetic field

$$\sigma_T = \frac{e^4}{6\pi\epsilon_0^2 c^4 m^2}$$ = Thomson cross-section

$$\Longrightarrow \qquad P = 2\sigma_T c U_{\mathrm{mag}}\left(\frac{v}{c}\right)^2\gamma^2\sin^2\alpha$$

i.e. Tighter helixes with the same v radiate more (because more of velocity is in circular motion)

Power radiated

$$P = 2\sigma_T c U_{\mathrm{mag}} \left(\frac{v}{c}\right)^2 \gamma^2 \sin^2\alpha$$

- Average over isotropic distribution of pitch angles:

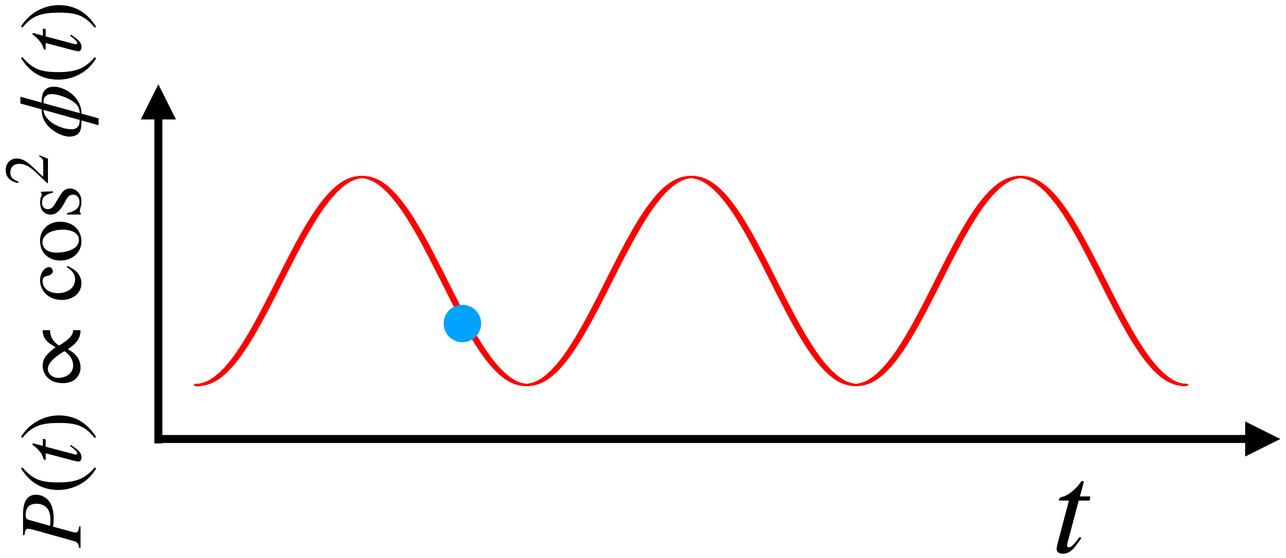
$$\boxed{\therefore \langle P \rangle = \frac{4}{3}\sigma_T c U_{\mathrm{mag}} \left(\frac{v}{c}\right)^2 \gamma^2}$$

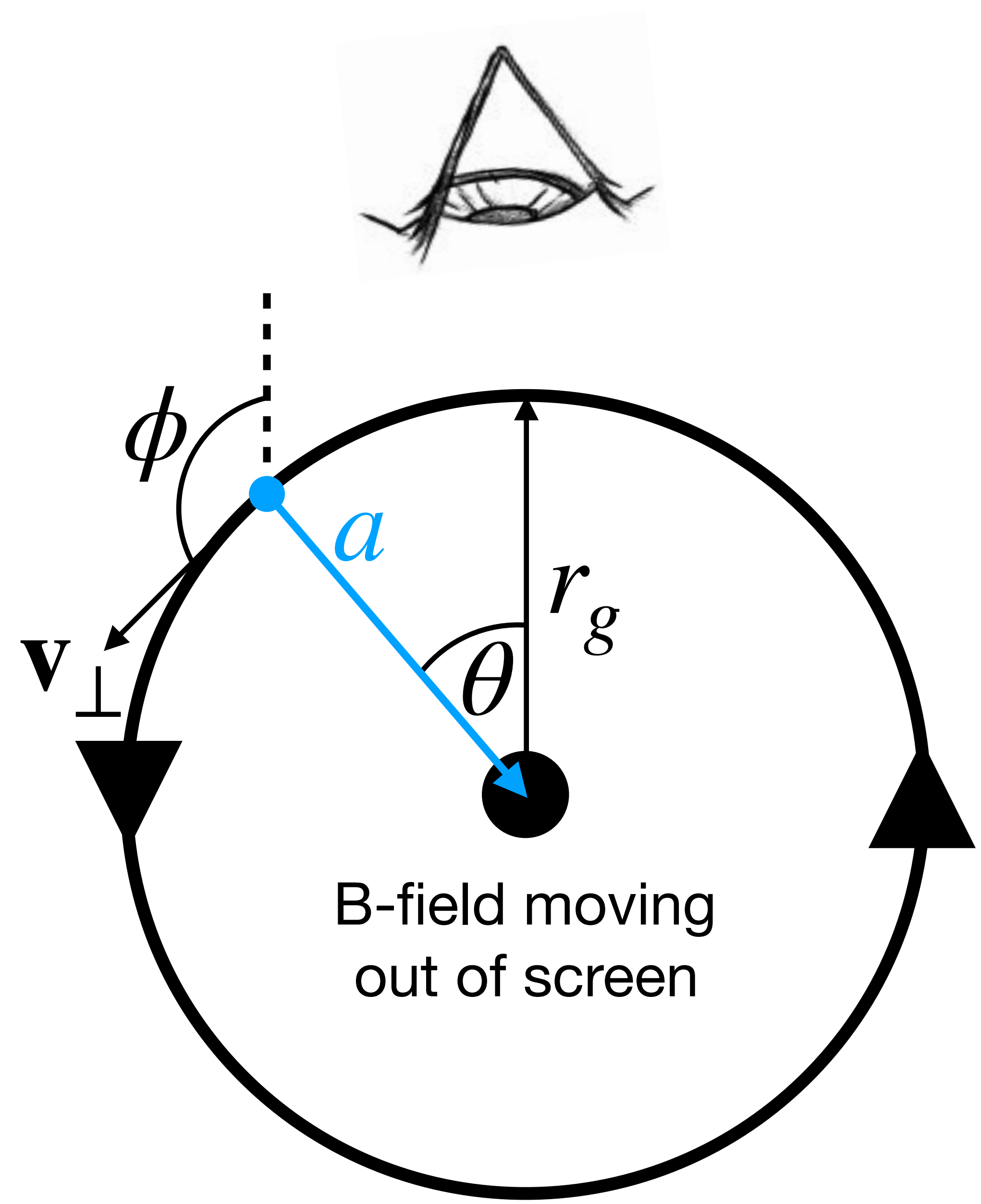
i.e. For a population of electrons travelling in random initial directions but all with the same speed (and therefore the same Lorentz factor and same energy).

Spectrum

γ=1: gyroradiation

Linearly polarised

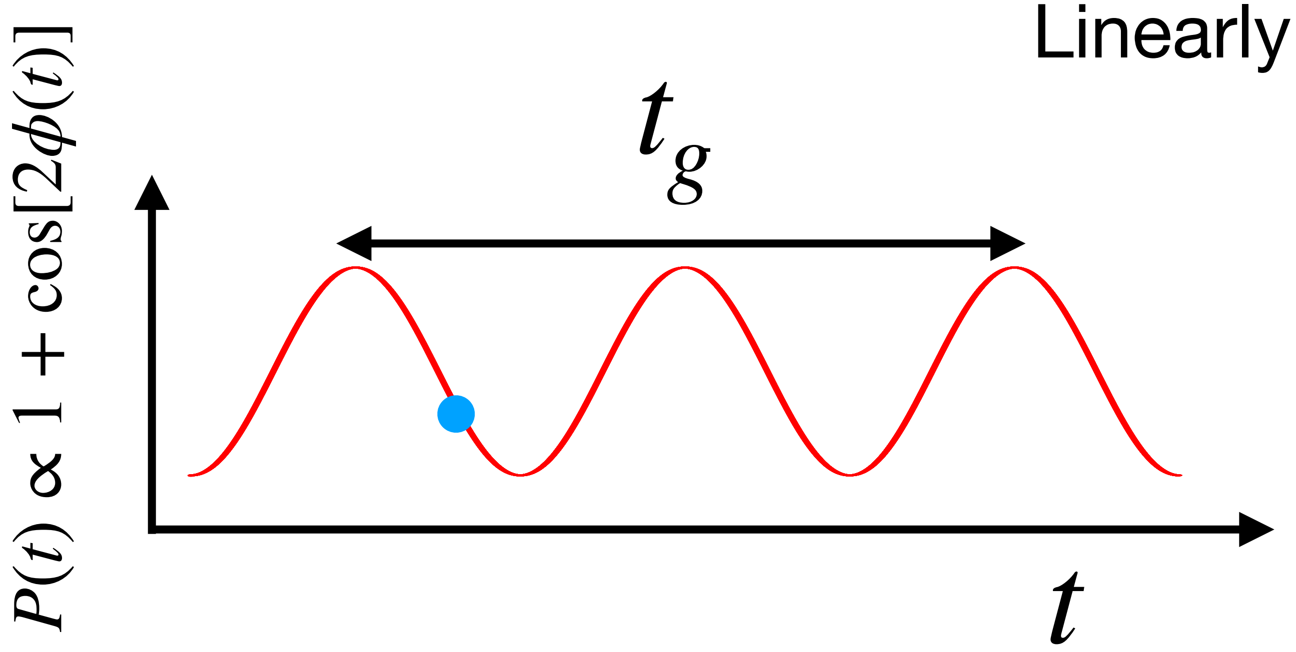


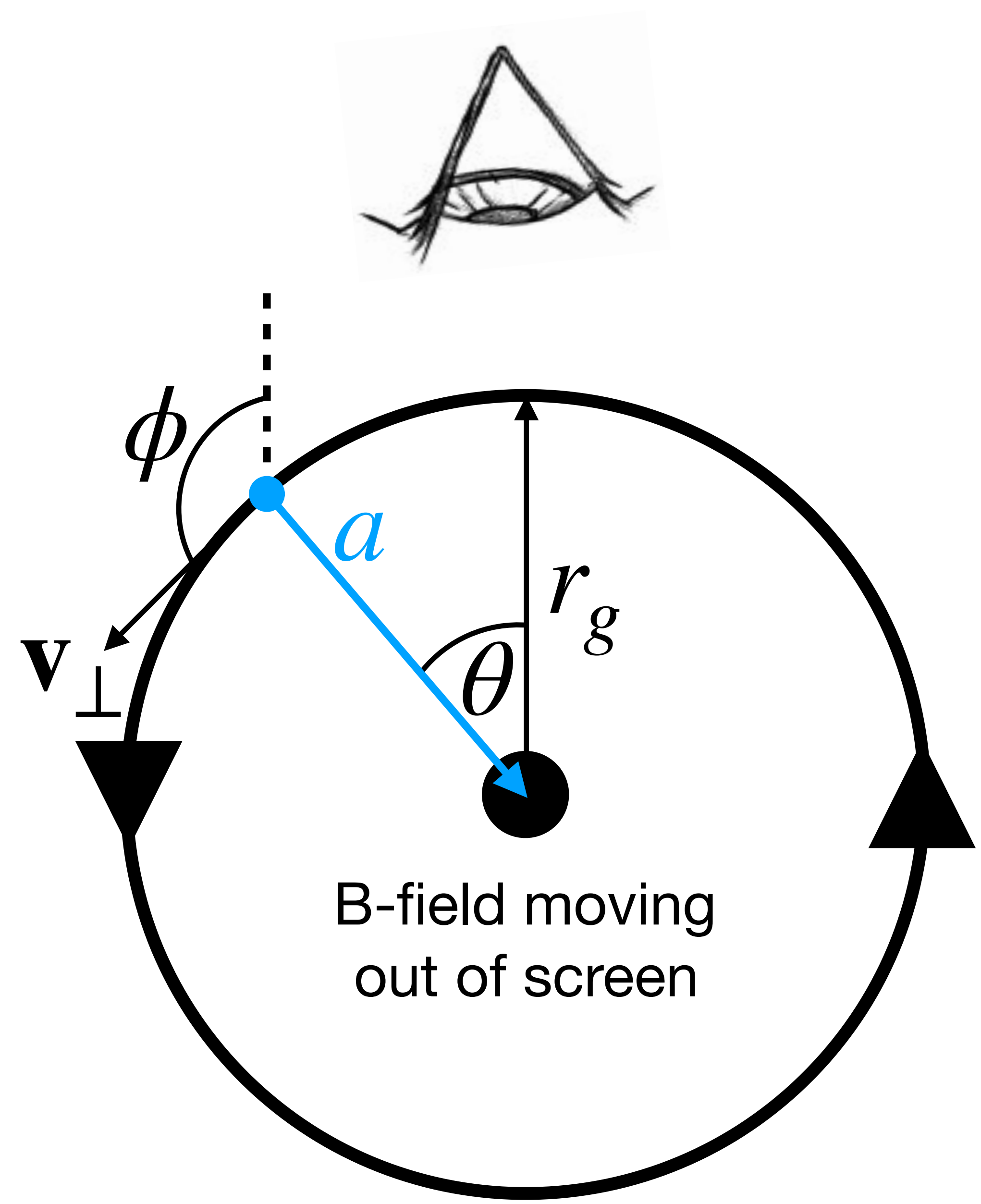


Spectrum

γ=1: gyroradiation

Linearly polarised

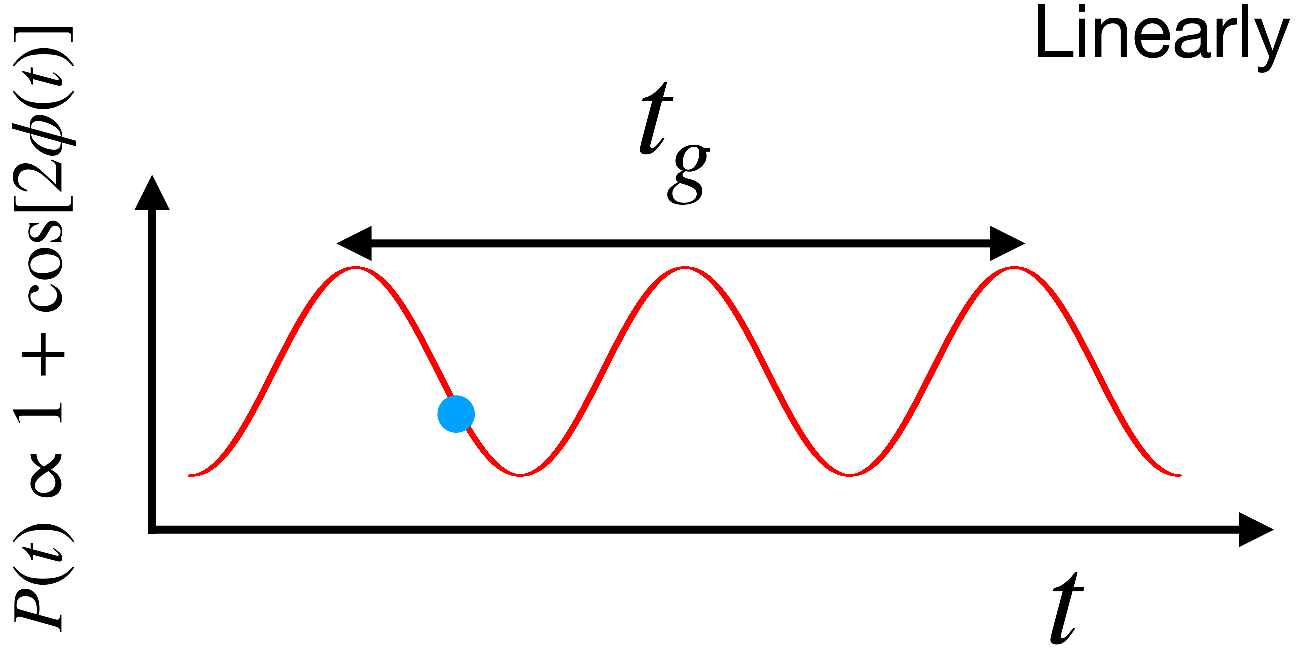




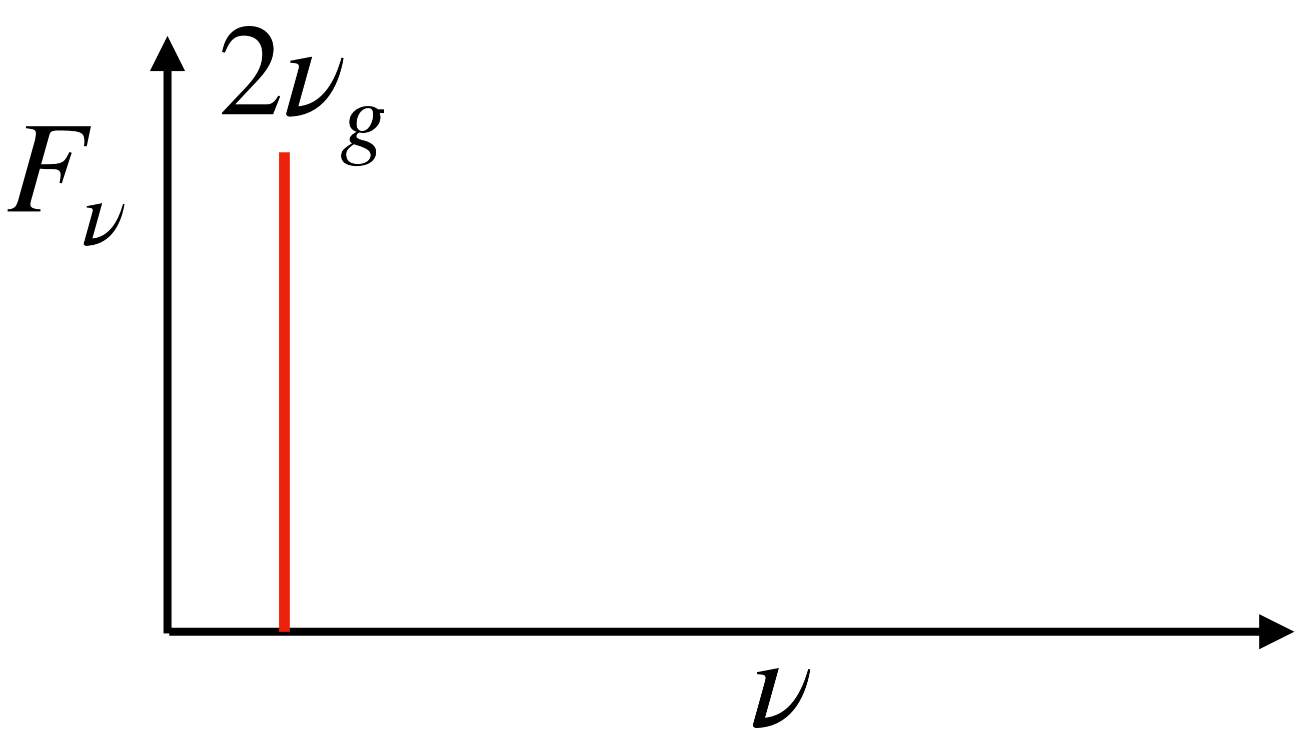
Spectrum

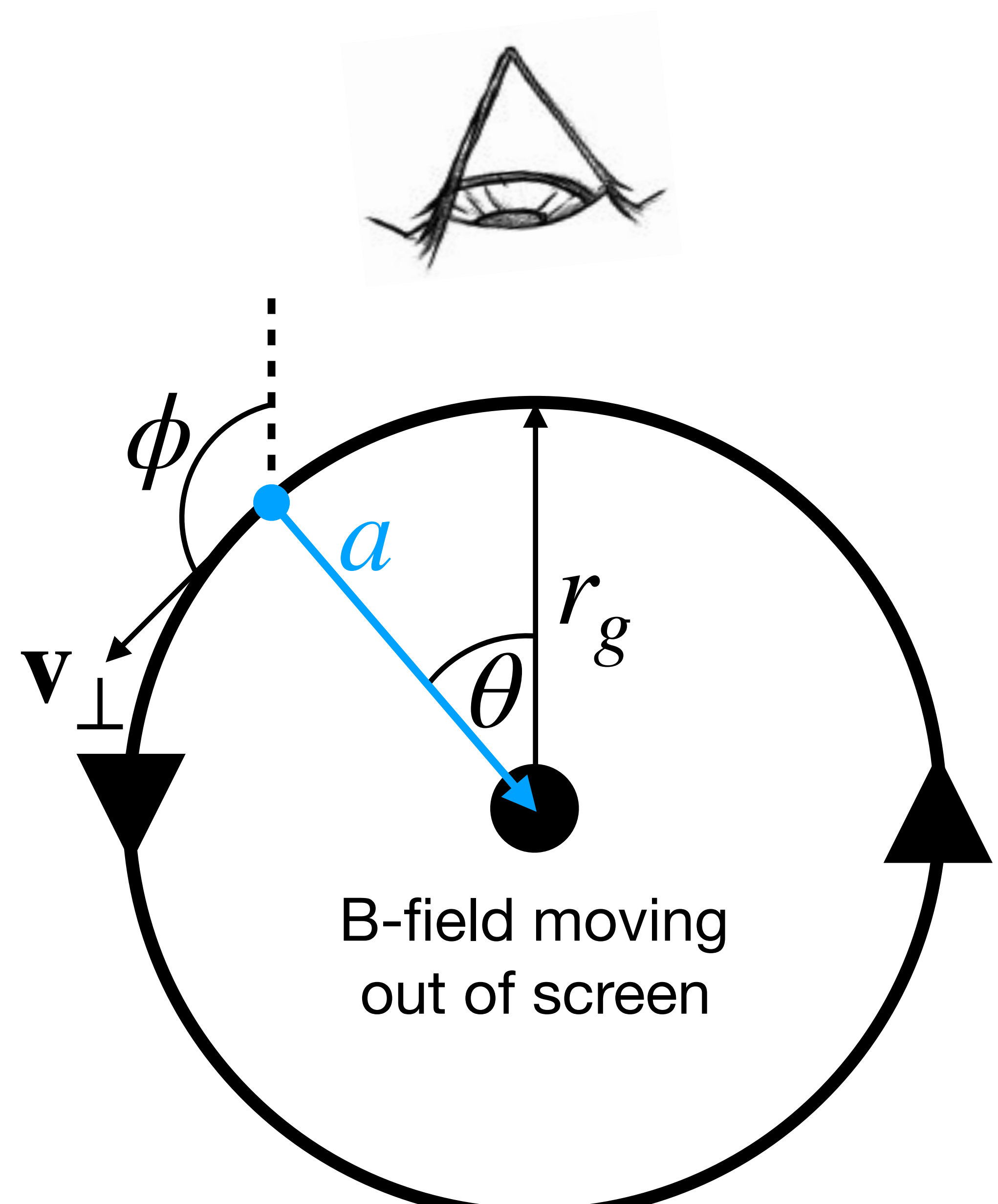
γ=1: gyroradiation

Linearly polarised



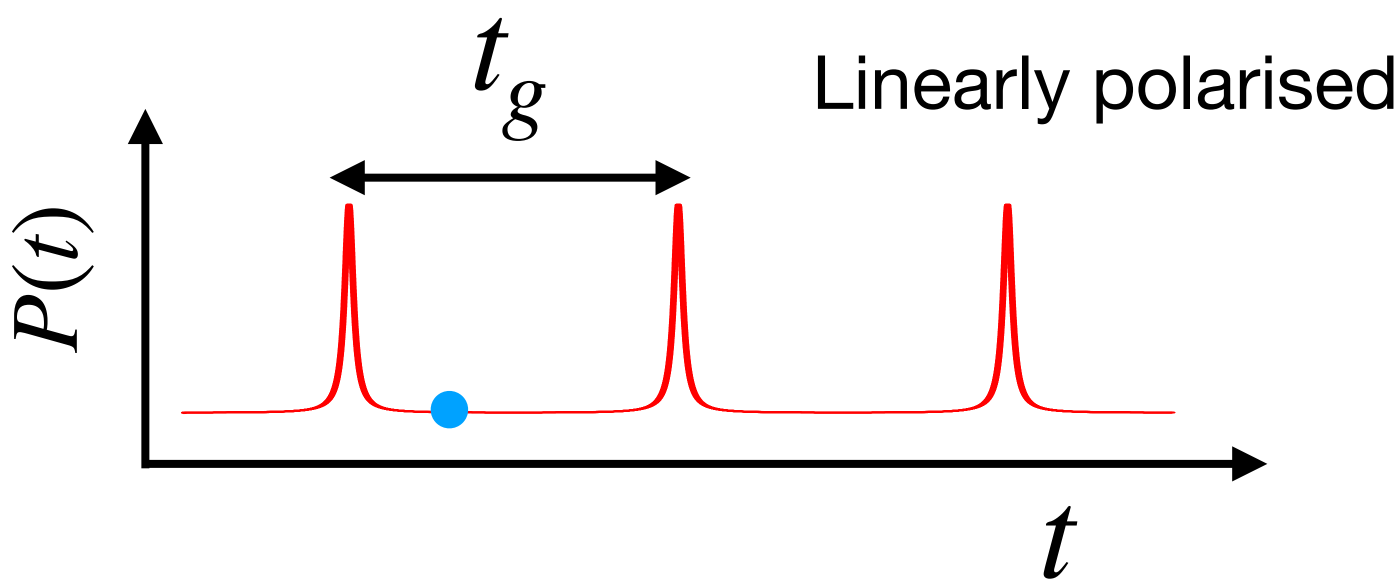
Fourier transform to get spectrum:

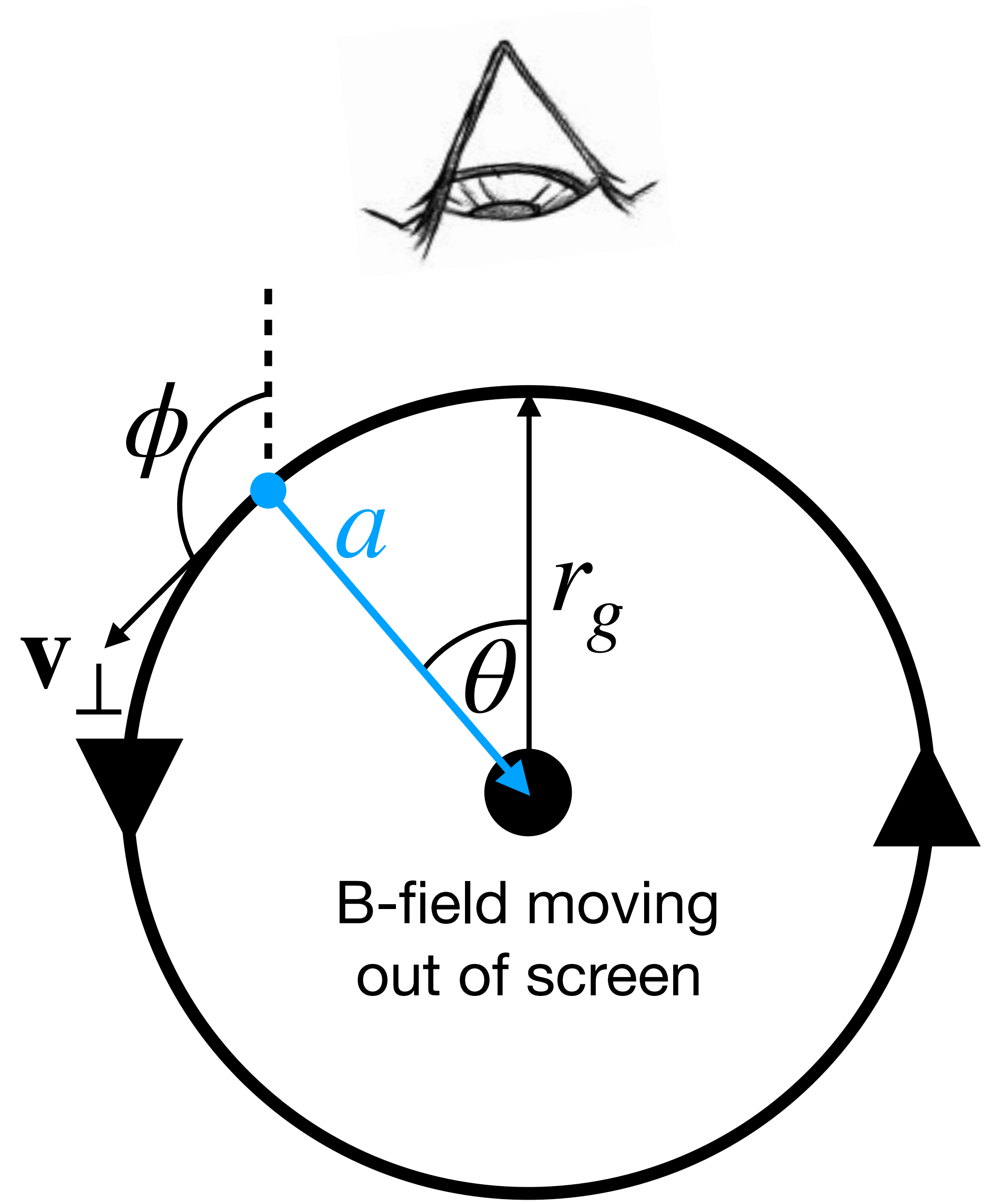




Spectrum

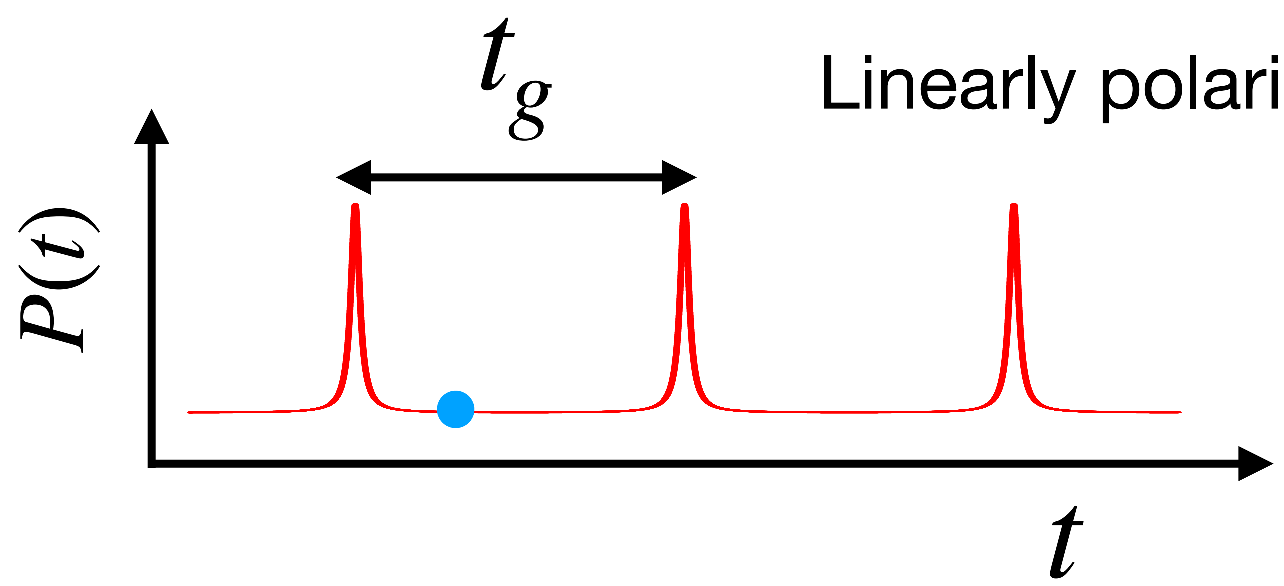
γ>>1: Relativistic beaming





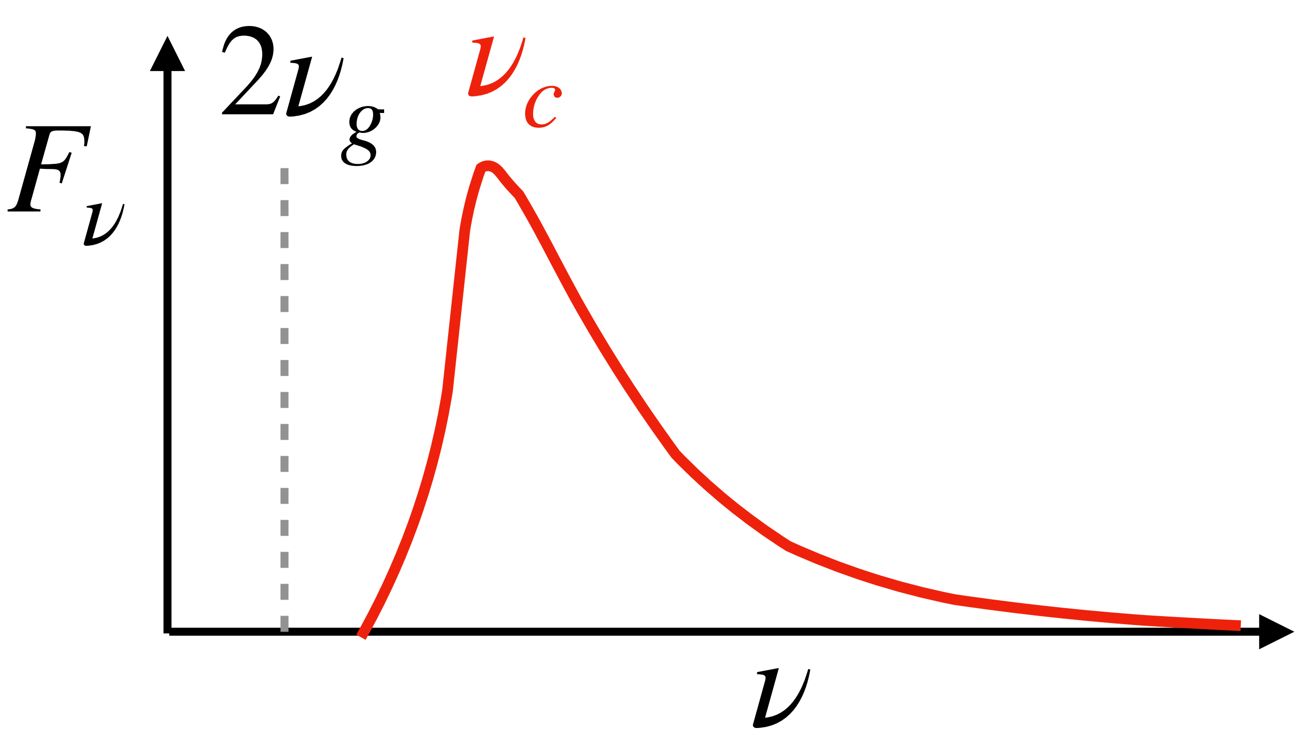
Spectrum

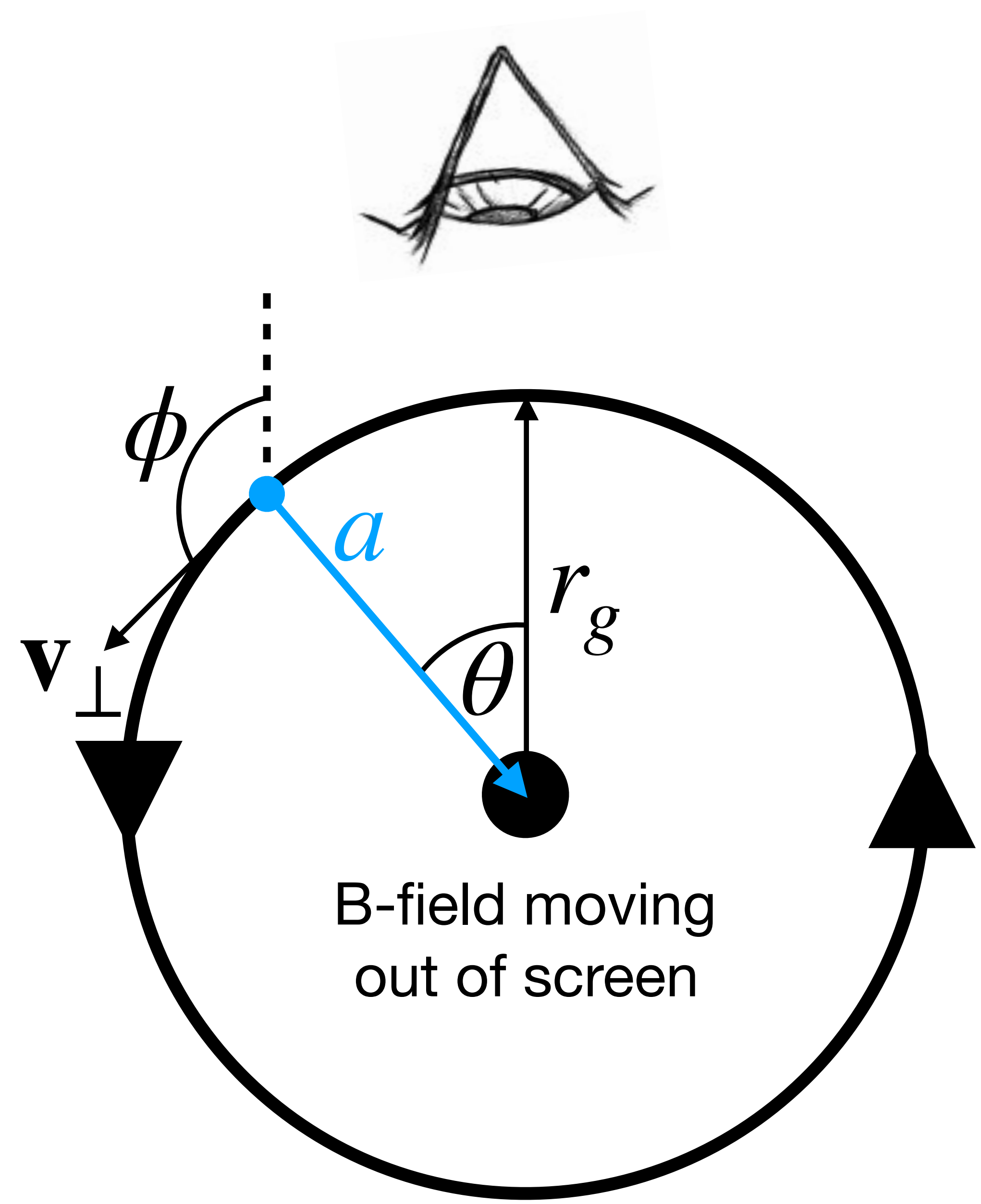
γ>>1: Relativistic beaming



Linearly polarised

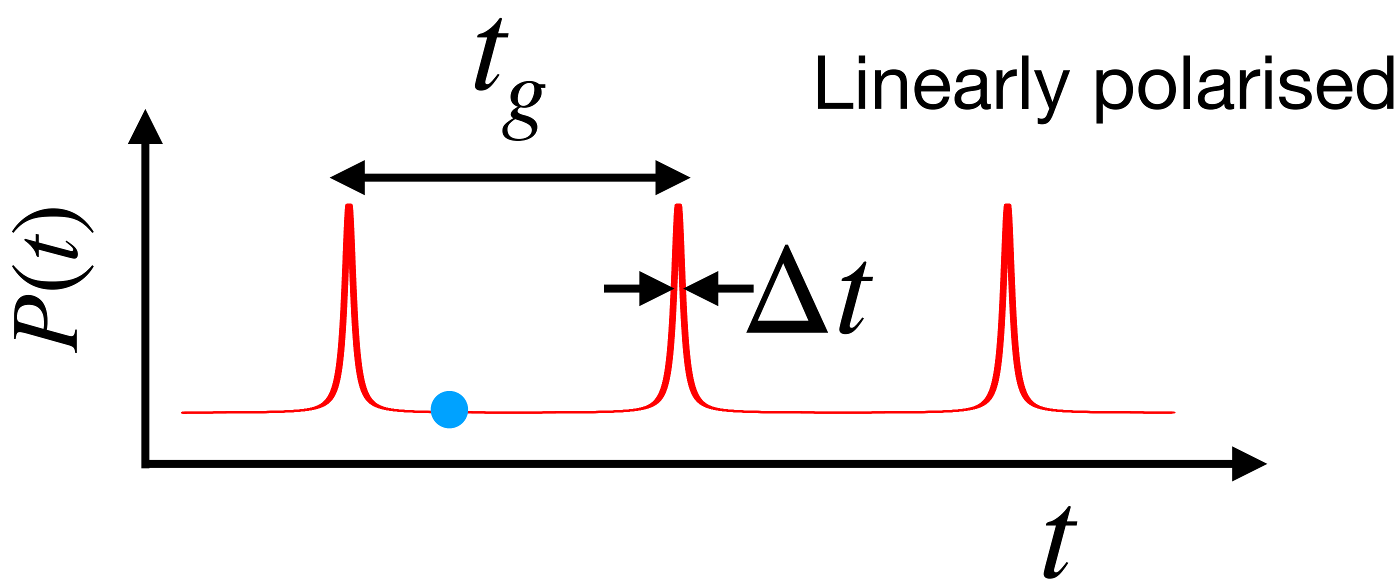
Fourier transform to get spectrum:



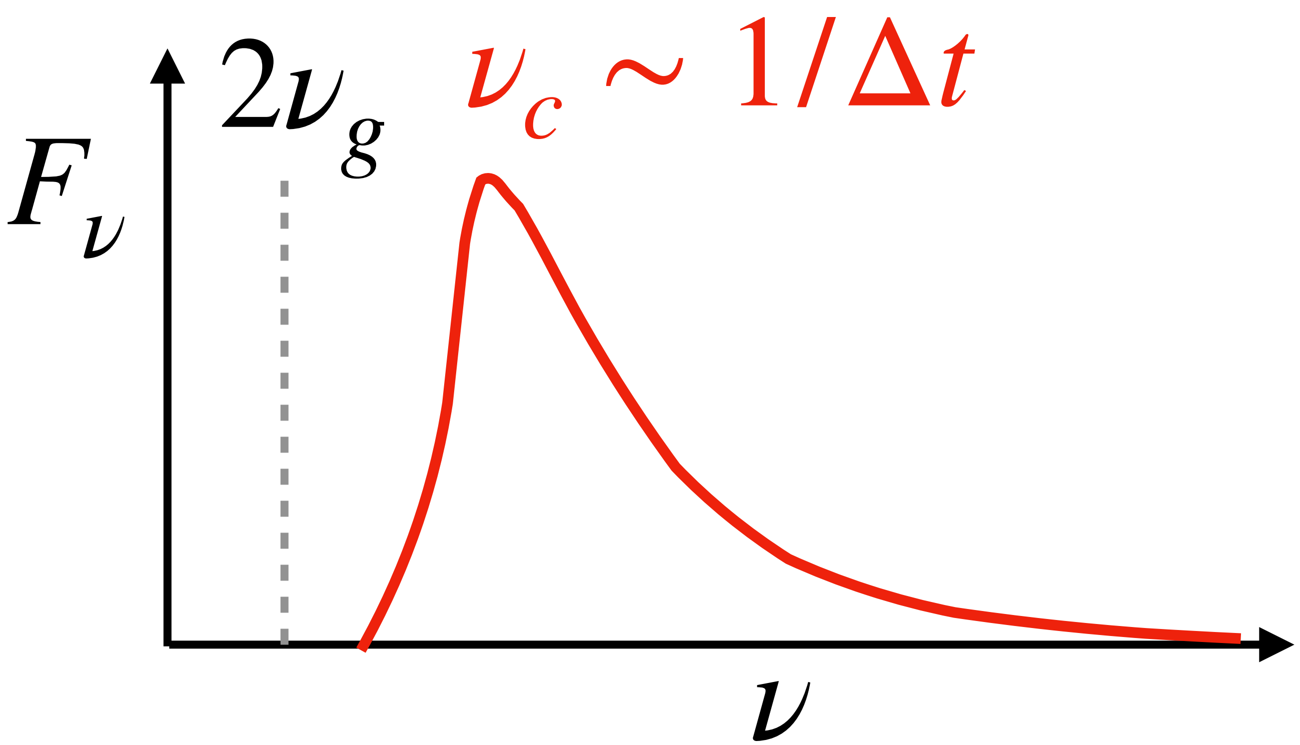


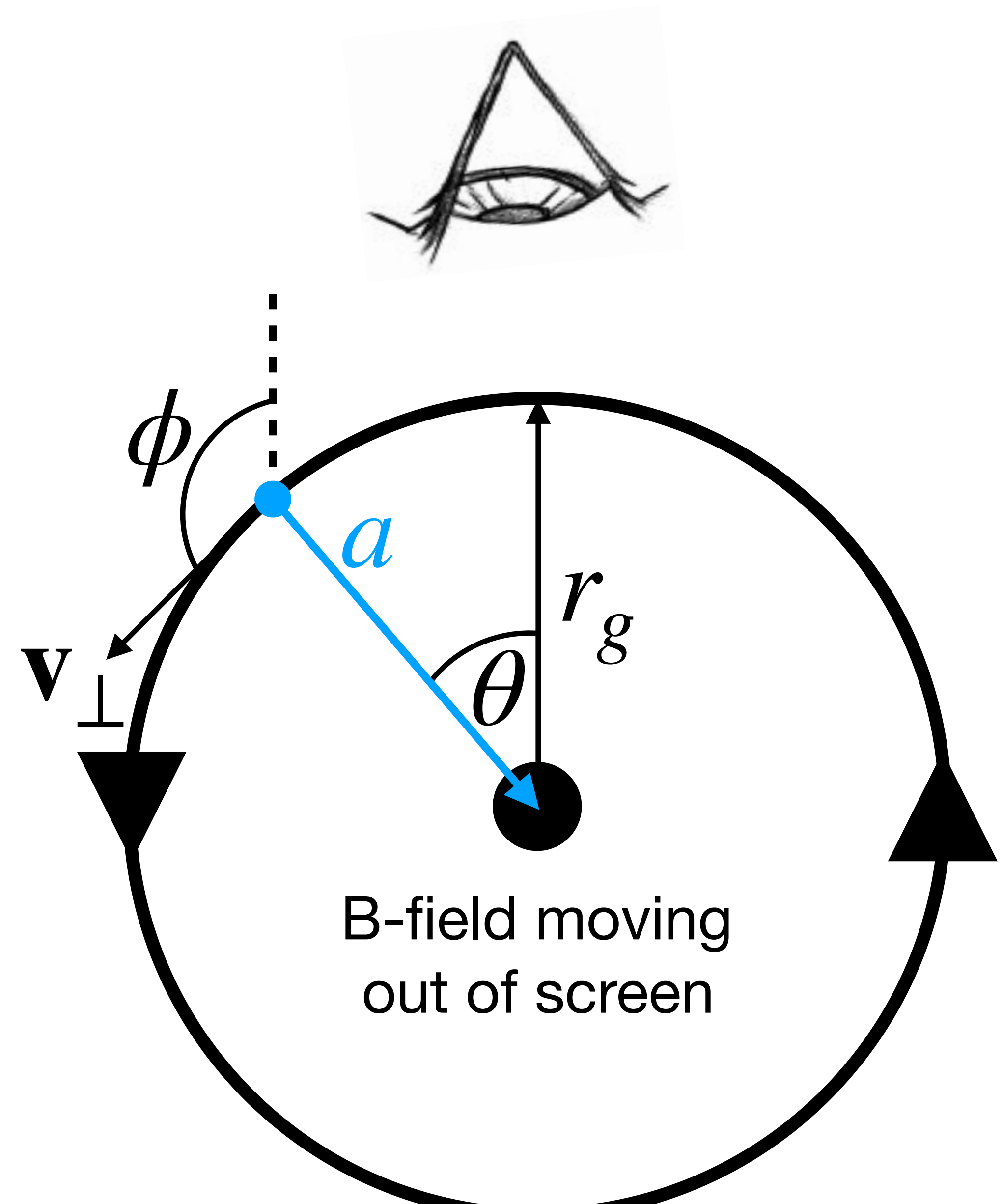
Spectrum

γ>>1: Relativistic beaming



Fourier transform to get spectrum:





Spectrum

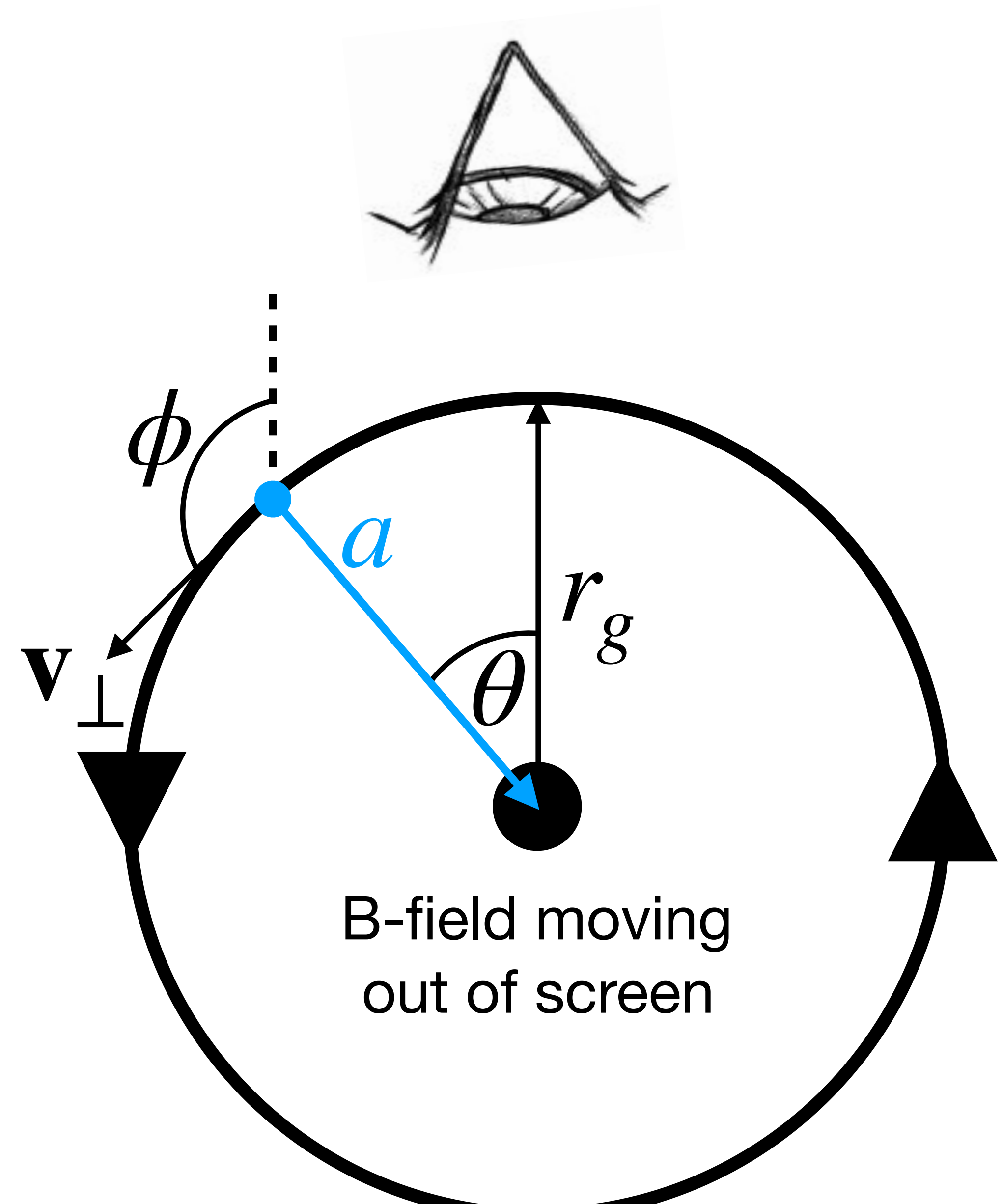
γ>>1: Relativistic beaming

Relativistic aberration formula:

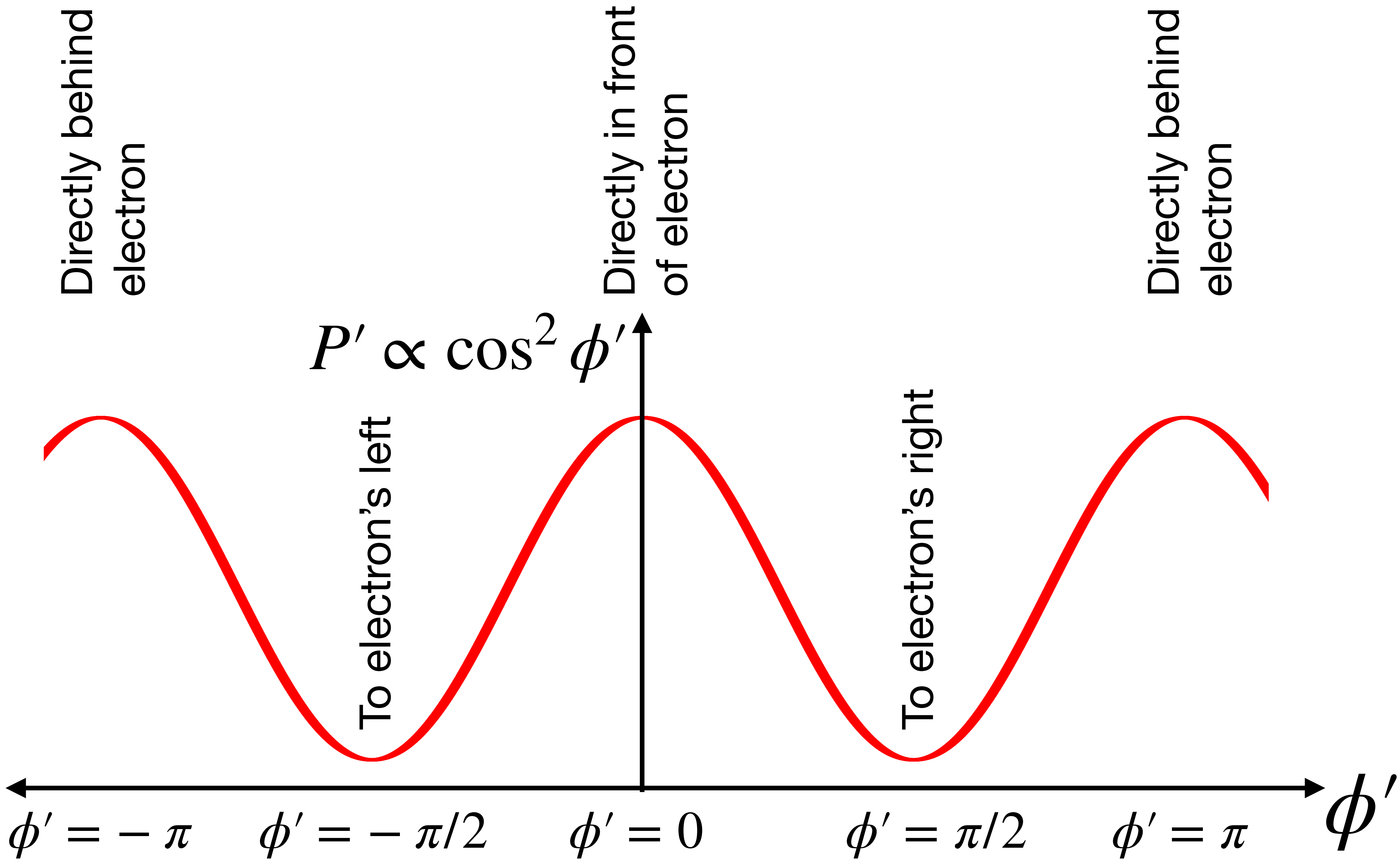
$$\cos\phi = \frac{\cos\phi' + v/c}{1 + (v/c)\cos\phi'}$$

Observer's rest frame: ϕ

Electron's rest frame: ϕ'

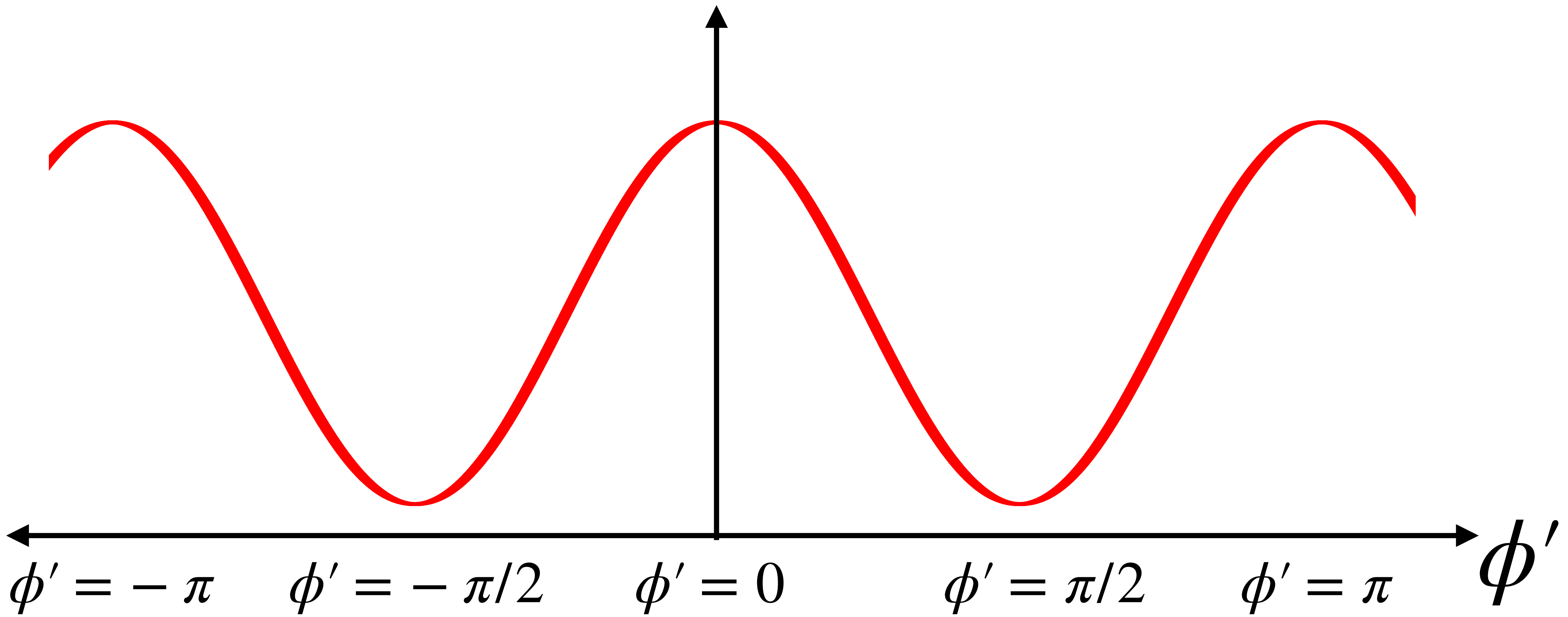


Relativistic aberration



Relativistic aberration

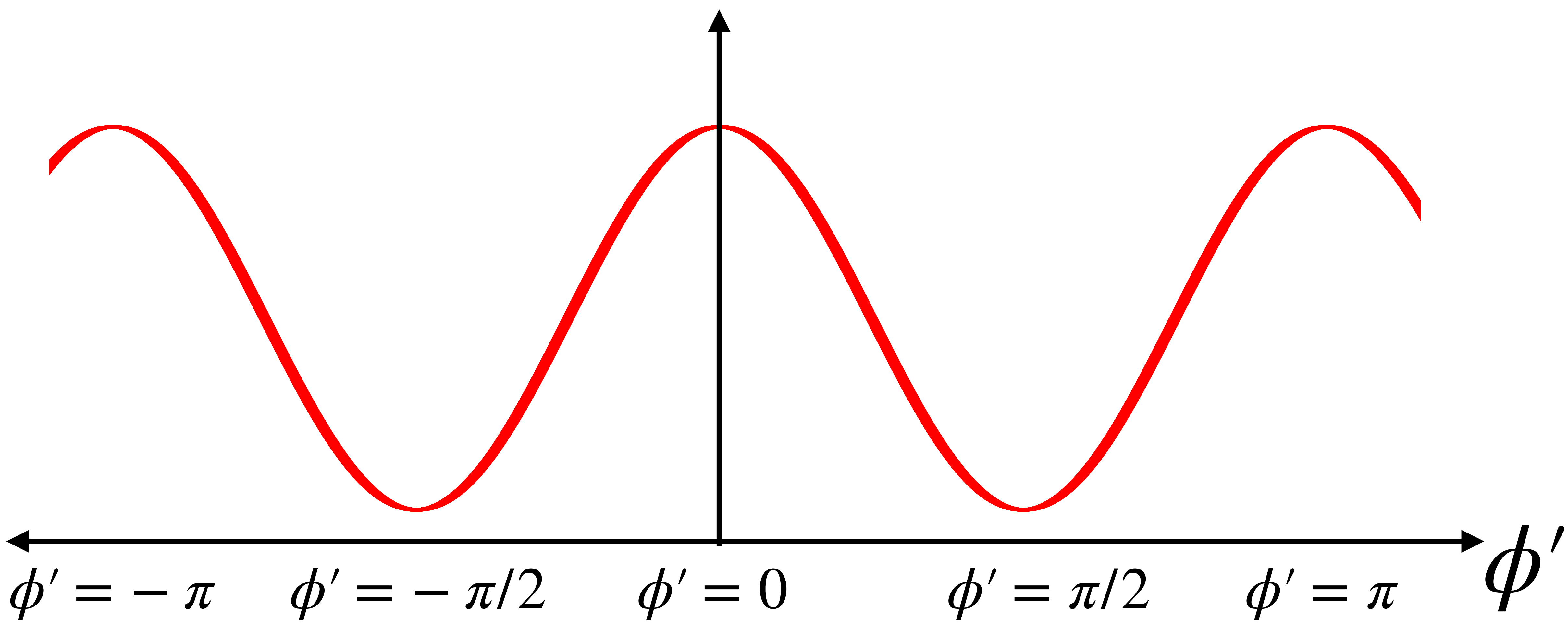
- P(t) peaks at $\phi' = 0$ and is zero at $\phi' = \pi/2$ and $\phi' = -\pi/2$



Relativistic aberration

- P(t) peaks at $\phi' = 0$ and is zero at $\phi' = \pi/2$ and $\phi' = -\pi/2$
- These zero points to observer are at:

$$\cos\phi = \frac{\cos\phi' + v/c}{1 + (v/c)\cos\phi'} = v/c$$

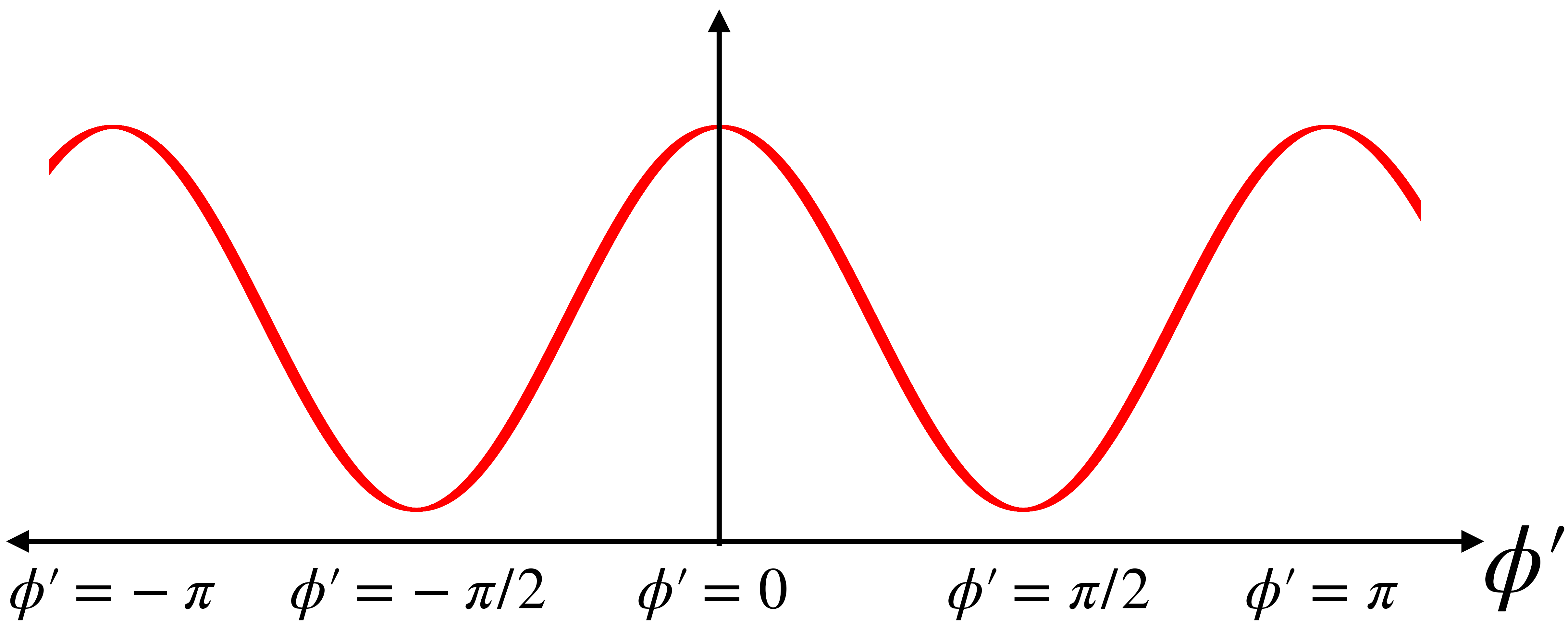


Relativistic aberration

- P(t) peaks at $\phi' = 0$ and is zero at $\phi' = \pi/2$ and $\phi' = -\pi/2$
- These zero points to observer are at:

$$\cos\phi = \frac{\cos\phi' + v/c}{1 + (v/c)\cos\phi'} = v/c$$

$$\sin\phi = \pm\sqrt{1 - (v/c)^2} = \pm 1/\gamma$$

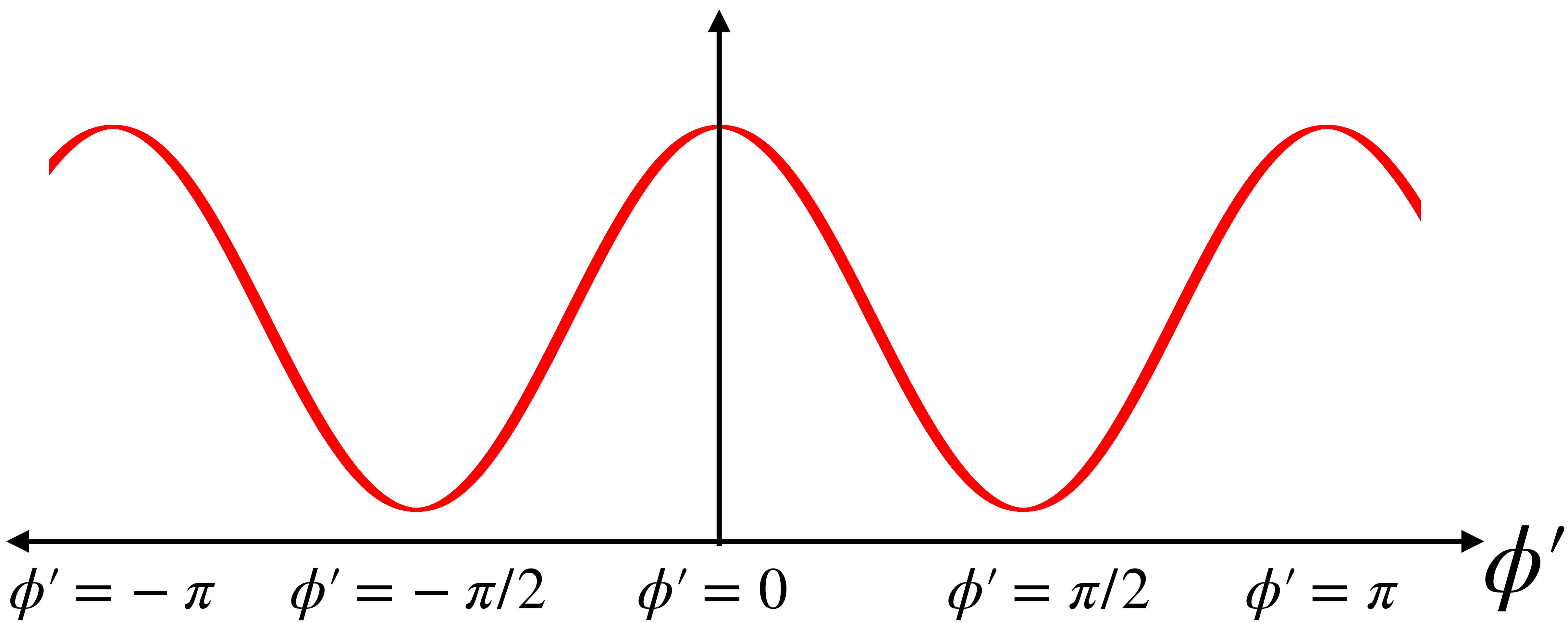


Relativistic aberration

- P(t) peaks at $\phi' = 0$ and is zero at $\phi' = \pi/2$ and $\phi' = -\pi/2$
- These zero points to observer are at:

$$\cos\phi = \frac{\cos\phi' + v/c}{1 + (v/c)\cos\phi'} = v/c$$

$$\sin\phi = \pm\sqrt{1 - (v/c)^2} = \pm 1/\gamma \implies \phi \approx \pm 1/\gamma$$

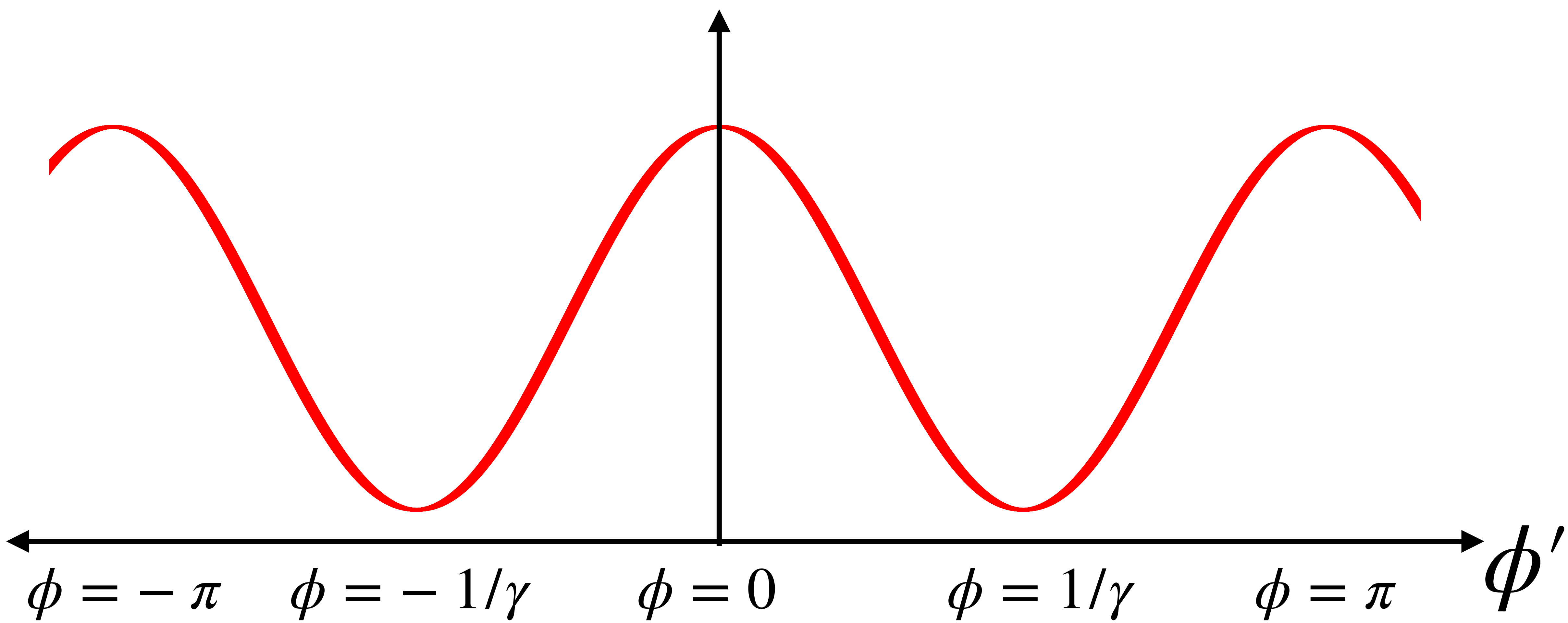


Relativistic aberration

- P(t) peaks at $\phi' = 0$ and is zero at $\phi' = \pi/2$ and $\phi' = -\pi/2$
- These zero points to observer are at:

$$\cos\phi = \frac{\cos\phi' + v/c}{1 + (v/c)\cos\phi'} = v/c$$

$$\sin\phi = \pm\sqrt{1 - (v/c)^2} = \pm 1/\gamma \implies \phi \approx \pm 1/\gamma$$

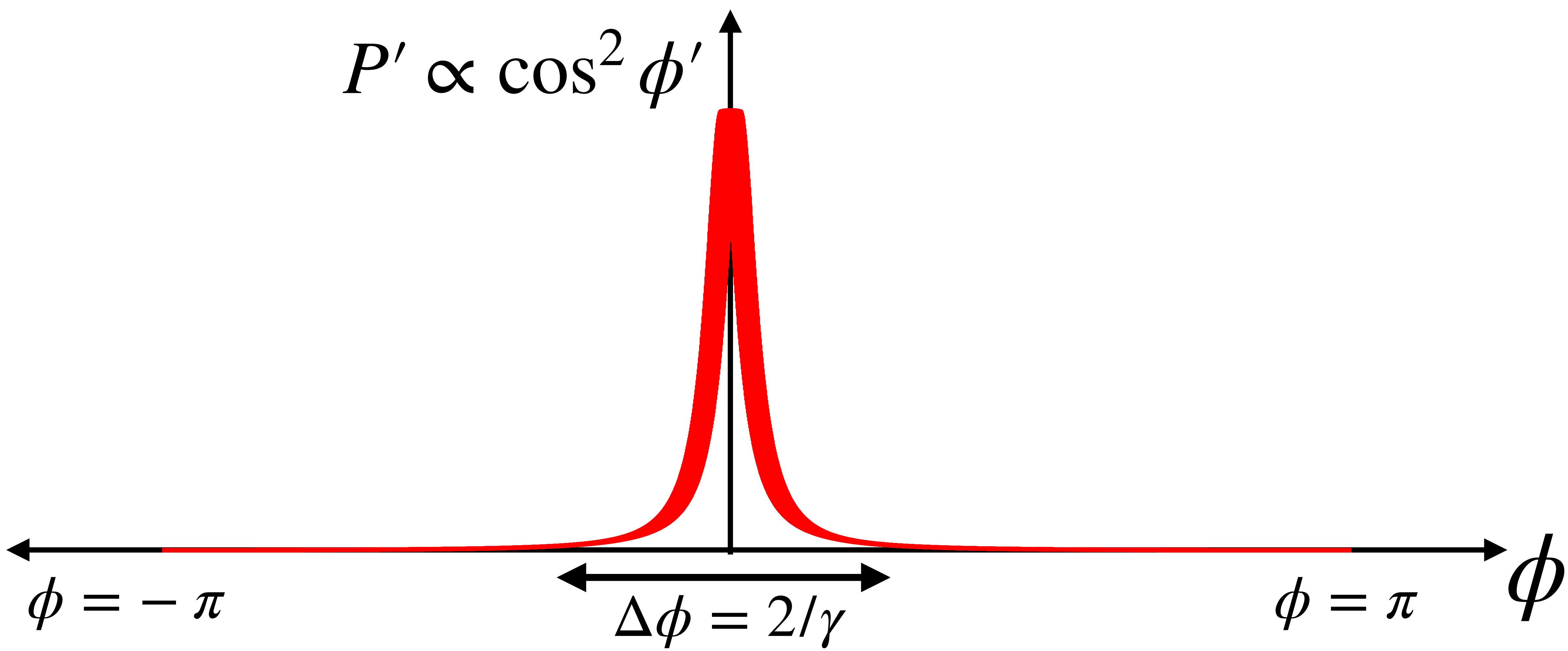


Relativistic aberration

- P(t) peaks at $\phi' = 0$ and is zero at $\phi' = \pi/2$ and $\phi' = -\pi/2$
- These zero points to observer are at:

$$\cos\phi = \frac{\cos\phi' + v/c}{1 + (v/c)\cos\phi'} = v/c$$

$$\sin\phi = \pm\sqrt{1 - (v/c)^2} = \pm 1/\gamma \implies \phi \approx \pm 1/\gamma$$

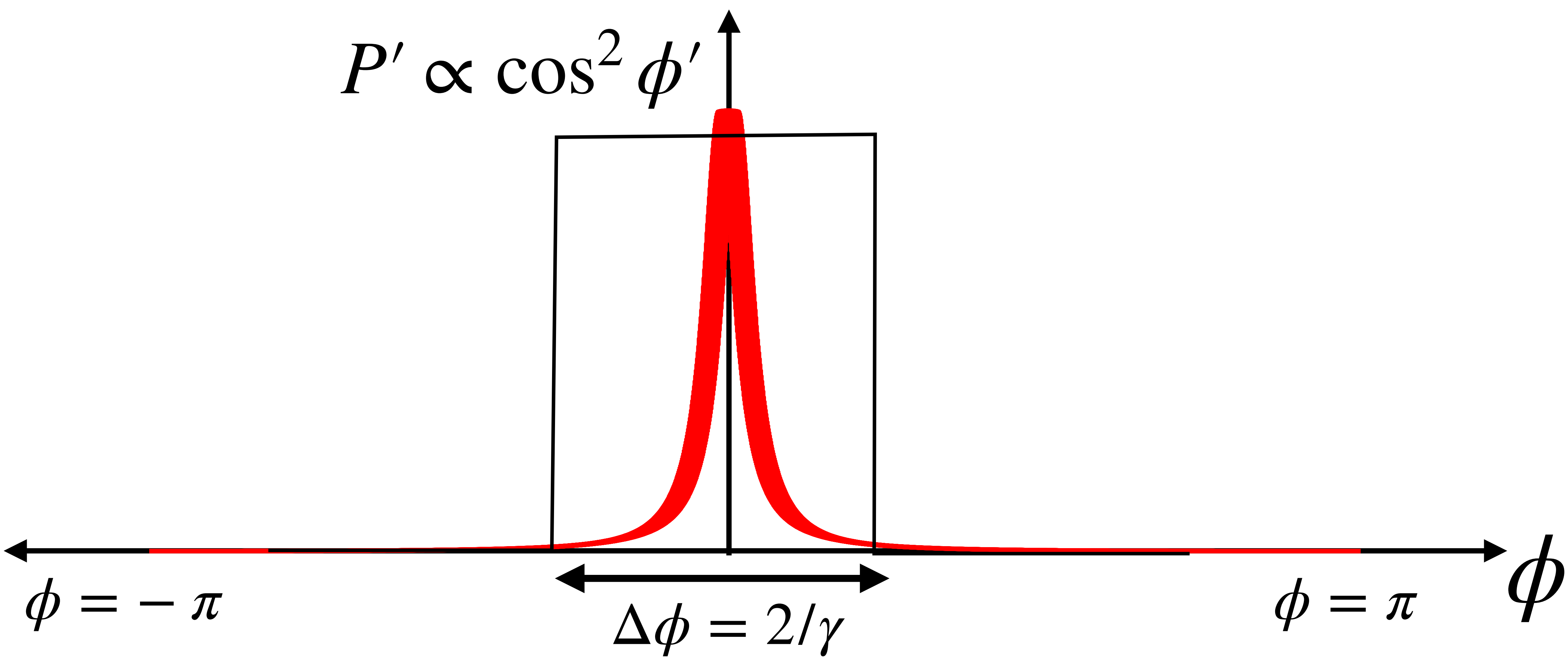


Relativistic aberration

- P(t) peaks at $\phi' = 0$ and is zero at $\phi' = \pi/2$ and $\phi' = -\pi/2$
- These zero points to observer are at:

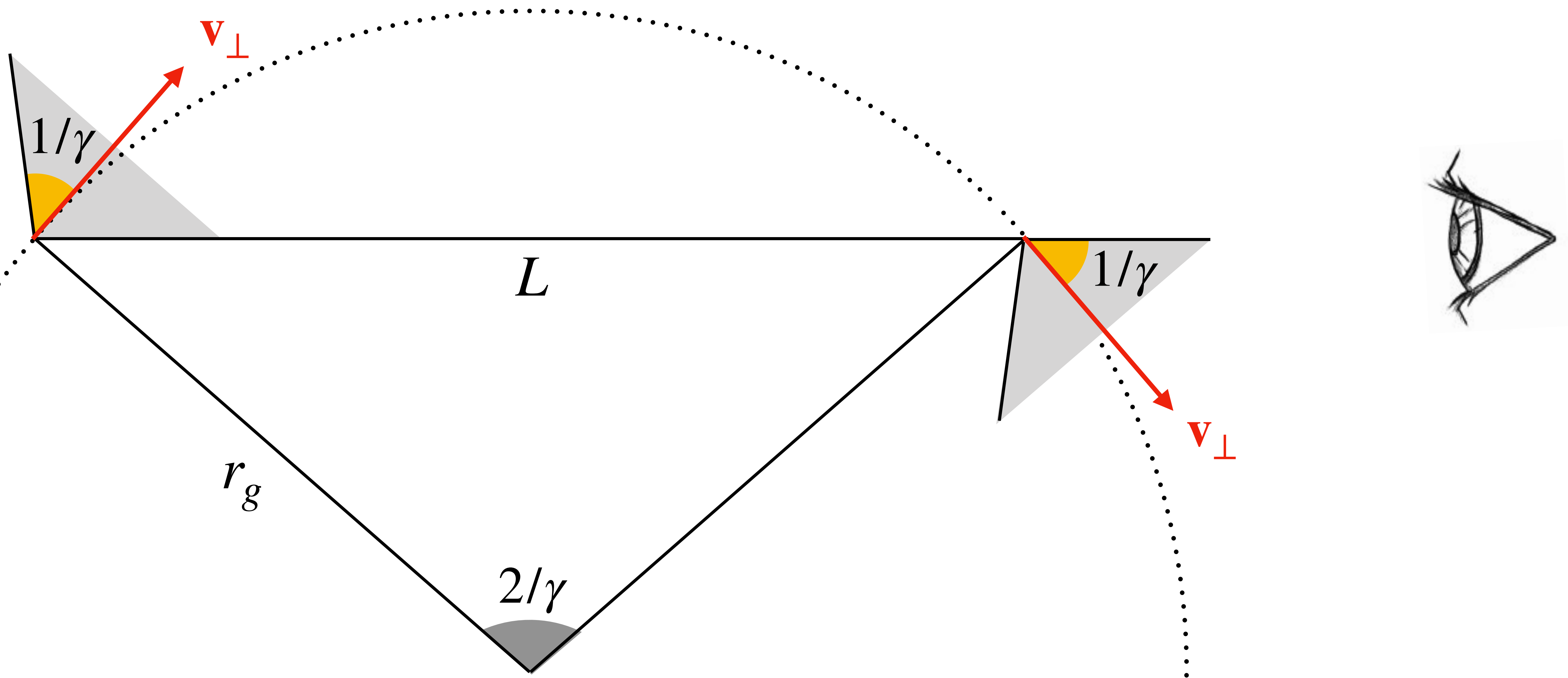
$$\cos\phi = \frac{\cos\phi' + v/c}{1 + (v/c)\cos\phi'} = v/c$$

$$\sin\phi = \pm\sqrt{1 - (v/c)^2} = \pm 1/\gamma \implies \phi \approx \pm 1/\gamma$$



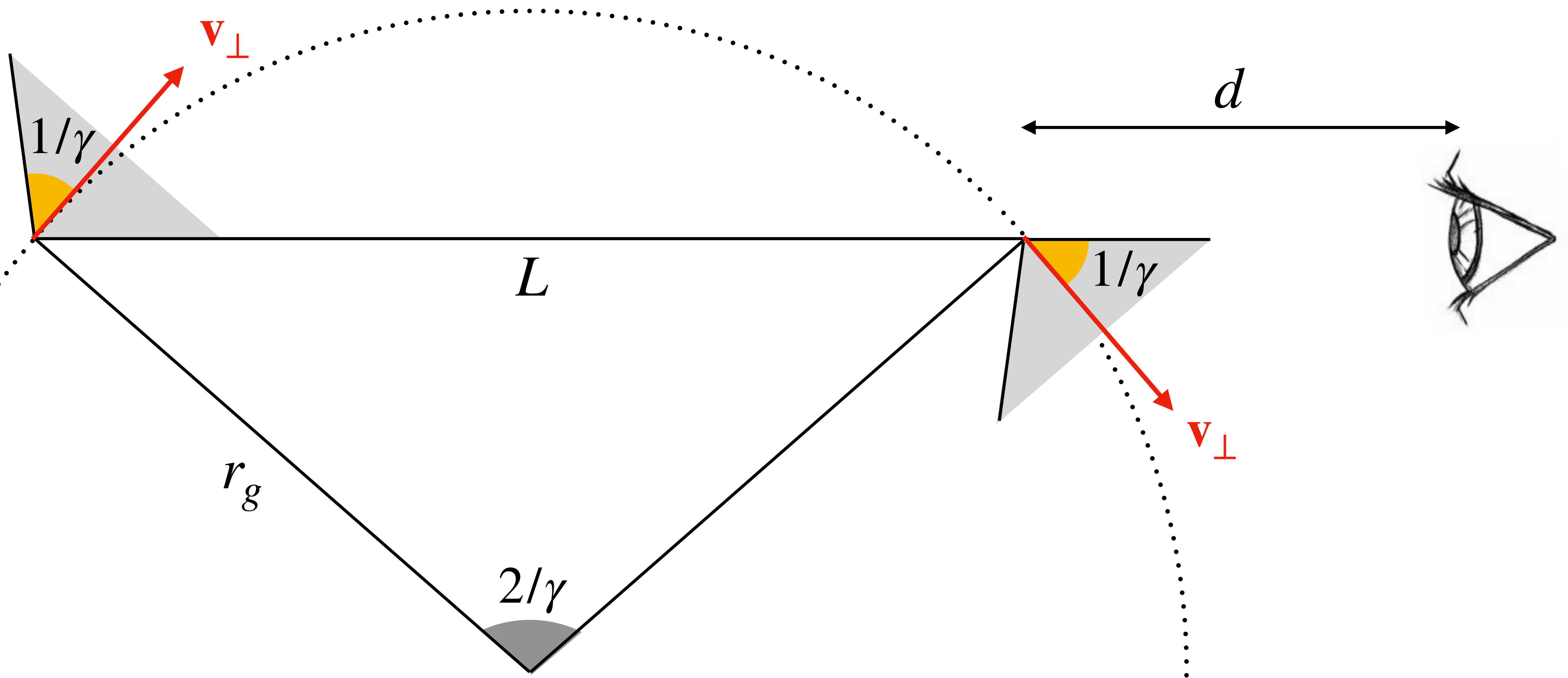
Relativistic aberration

- Electron orbits an angle 2/γ from our sightline first cutting the cone to our sightline first not cutting the cone.
- Electron travels this distance in: $\delta t = (2/\gamma)/\omega_g$



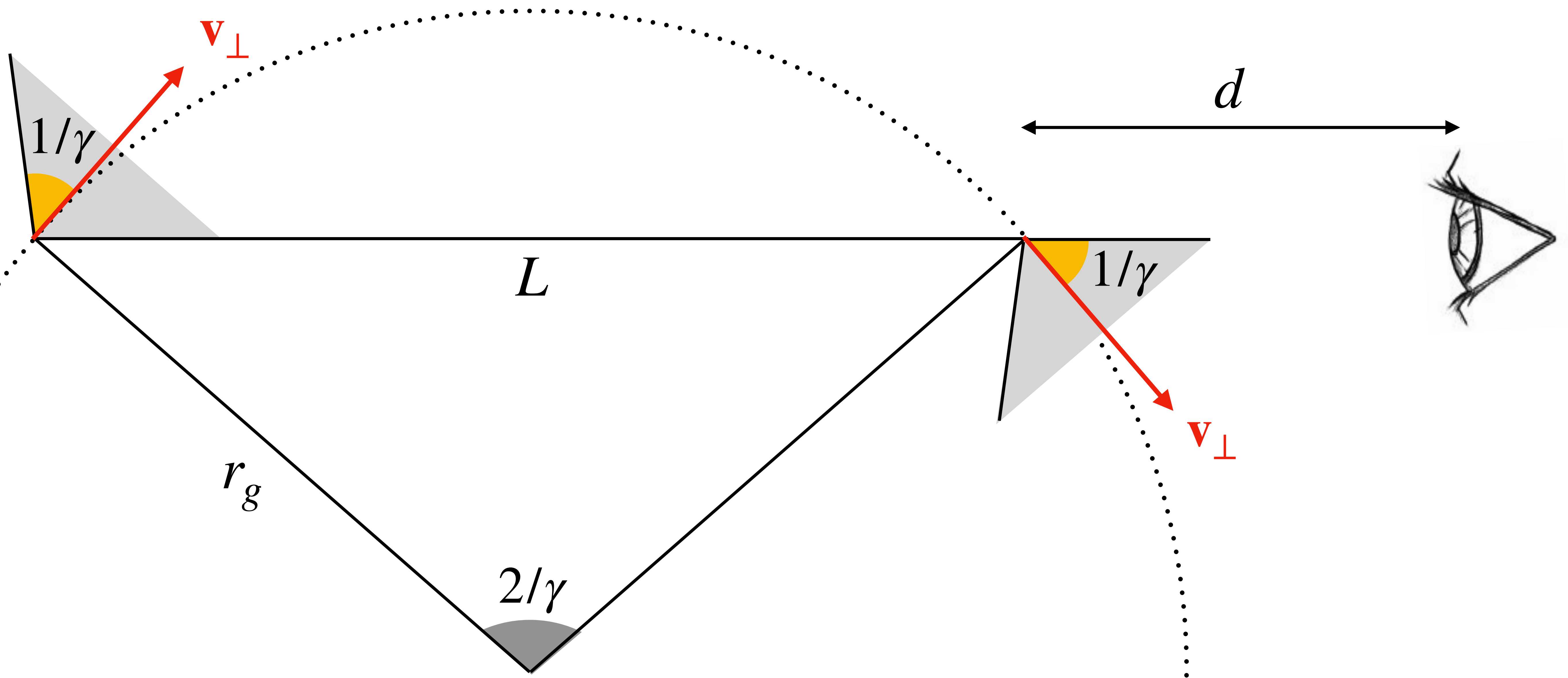
Relativistic aberration

- Electron orbits an angle 2/γ from our sightline first cutting the cone to our sightline first not cutting the cone.
- Electron travels this distance in: $\delta t = (2/\gamma)/\omega_g$
- We first <u>*see*</u> the pulse at time: $t_1 = L/c + d/c$



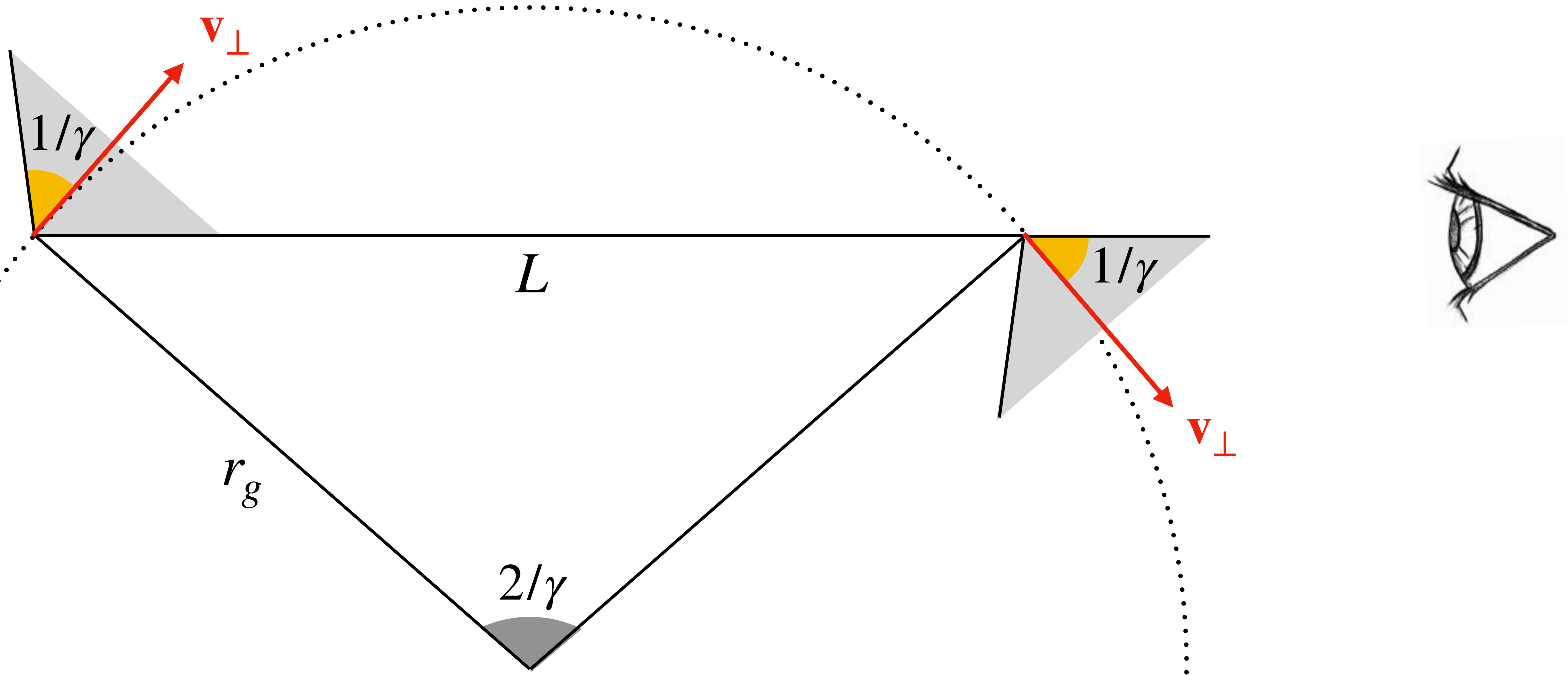
Relativistic aberration

- Electron orbits an angle 2/γ from our sightline first cutting the cone to our sightline first not cutting the cone.
- Electron travels this distance in: $\delta t = (2/\gamma)/\omega_g$
- We first *see* the pulse at time: $t_1 = L/c + d/c$
- We stop seeing the pulse at time: $t_2 = \delta t + d/c$



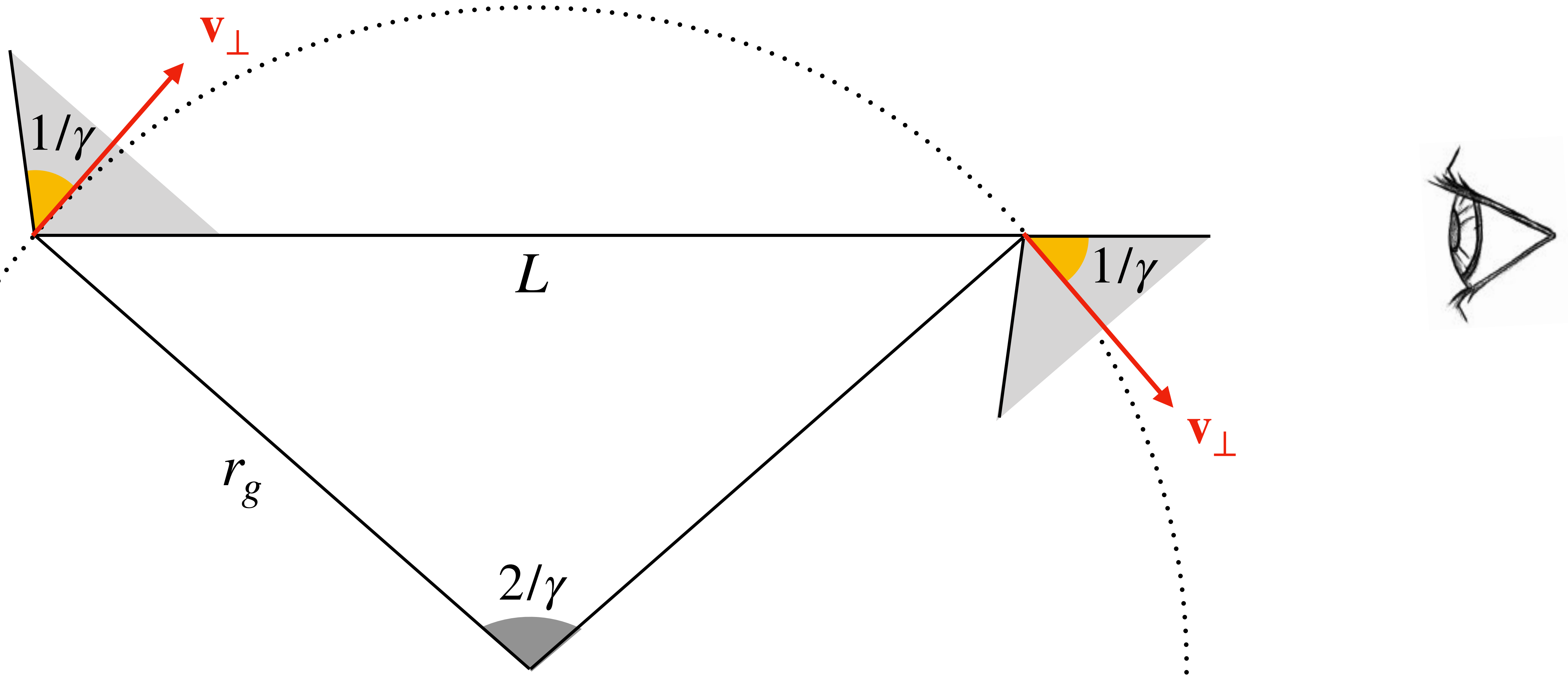
Relativistic aberration

- Electron orbits an angle 2/γ from our sightline first cutting the cone to our sightline first not cutting the cone.
- Electron travels this distance in: $\delta t = (2/\gamma)/\omega_g$
- We first *see* the pulse at time: $t_1 = L/c + d/c$
- We stop seeing the pulse at time: $t_2 = \delta t + d/c$
- Therefore length of pulse is: $\Delta t = t_2 - t_1 = \delta t - L/c$



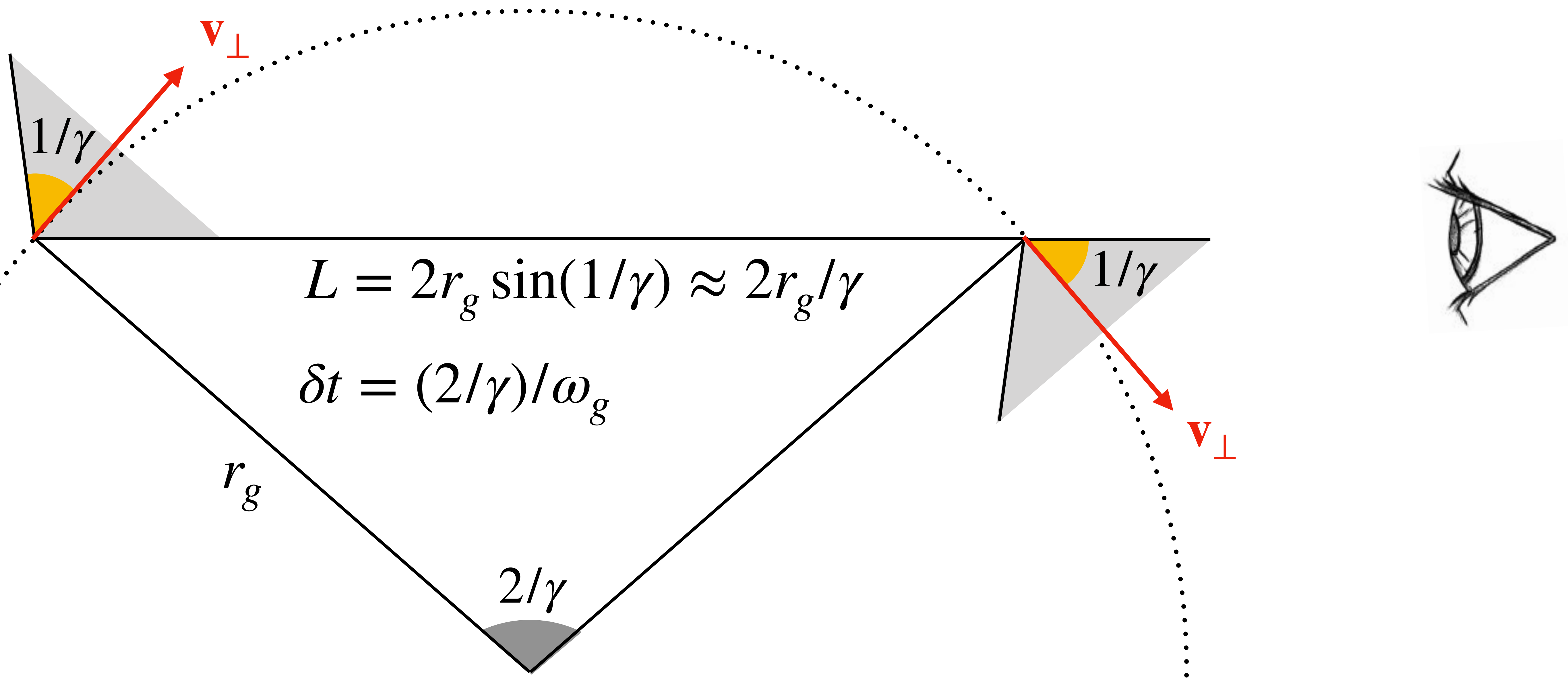
Relativistic aberration

$$\Delta t = \delta t - L/c$$



Relativistic aberration

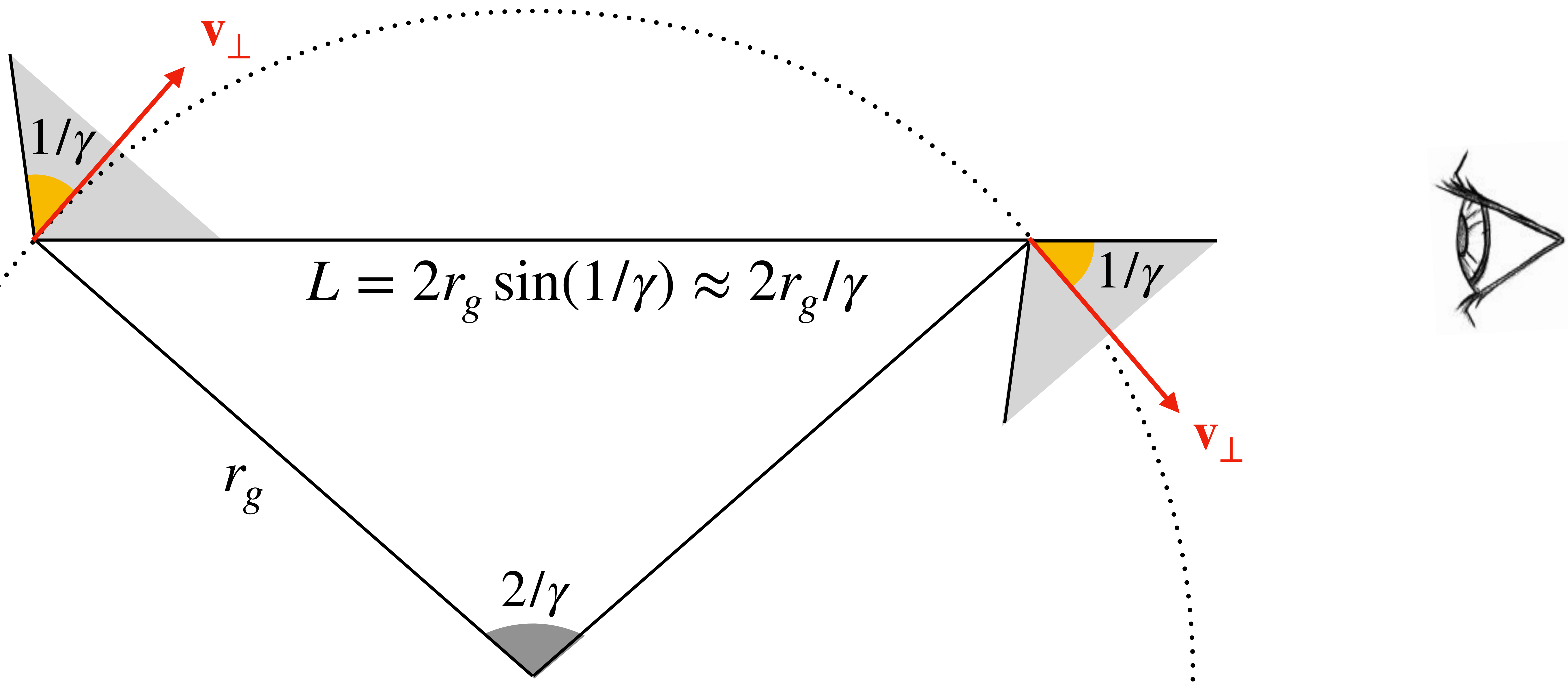
$$\Delta t = \delta t - L/c = \frac{2}{\gamma}\left[\frac{1}{\omega_g} - \frac{r_g}{c}\right]$$



Relativistic aberration

$$\Delta t = \delta t - L/c = \frac{2}{\gamma}\left[\frac{1}{\omega_g} - \frac{r_g}{c}\right]$$

$$\Delta t \approx \frac{1}{\gamma^3 \omega_g} \qquad (v_\perp \approx v;\ \ \gamma^2 \gg 1)$$



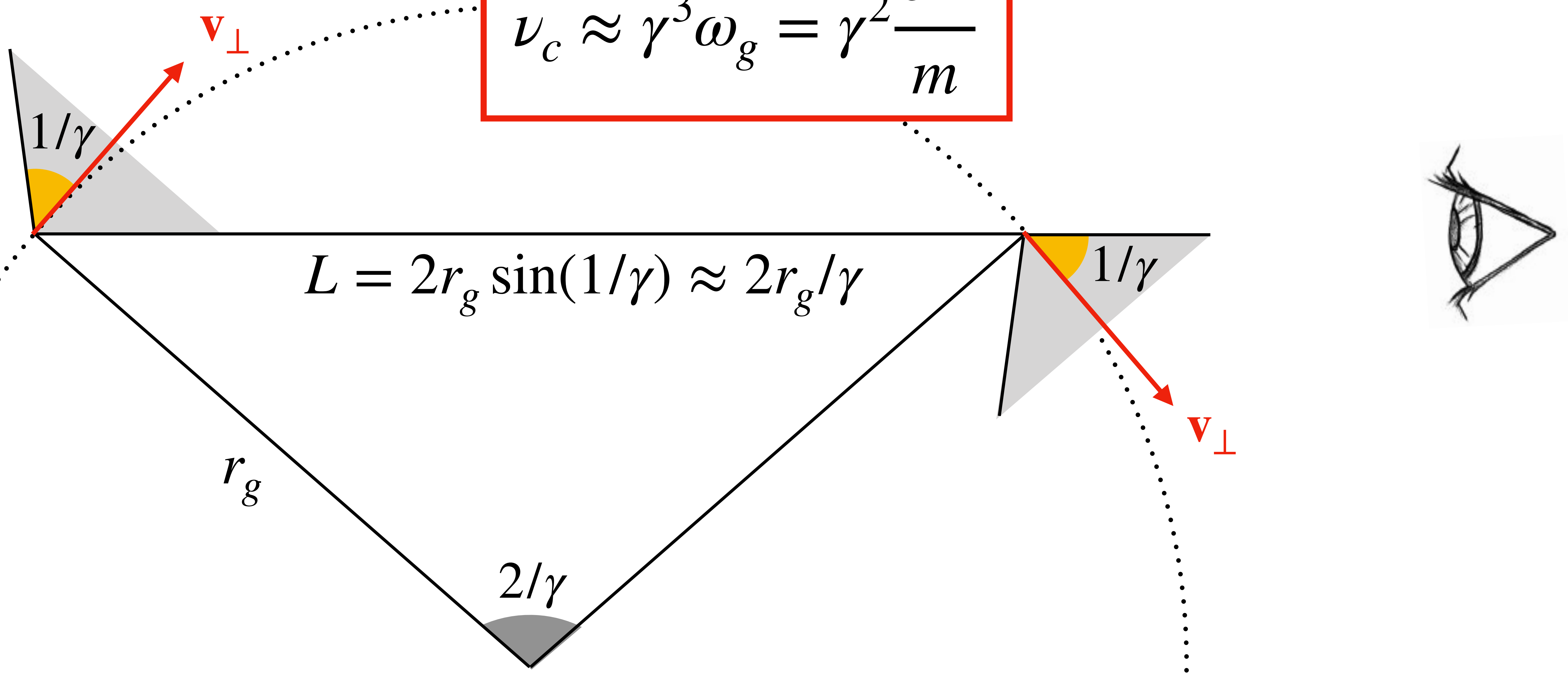
Relativistic aberration

$$\Delta t = \delta t - L/c = \frac{2}{\gamma}\left[\frac{1}{\omega_g} - \frac{r_g}{c}\right]$$

$$\Delta t \approx \frac{1}{\gamma^3 \omega_g}$$

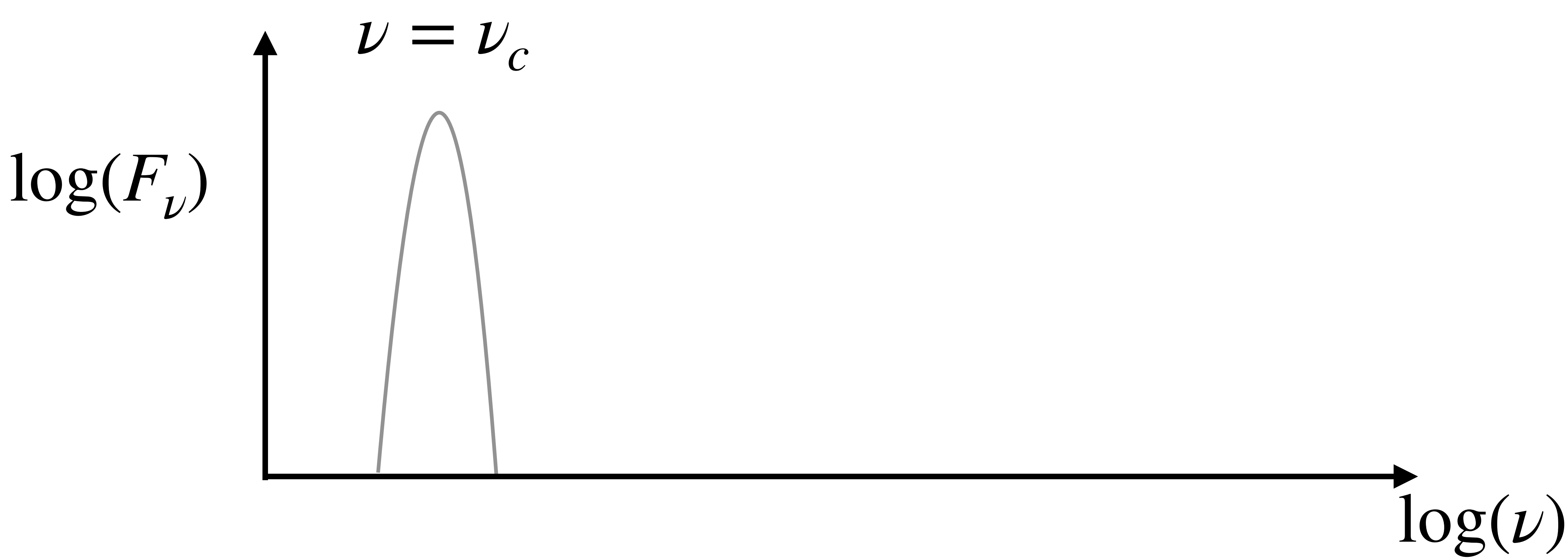
$$(v_\perp \approx v;\;\; \gamma^2 \gg 1)$$

$$\boxed{\nu_c \approx \gamma^3 \omega_g = \gamma^2 \frac{eB}{m}}$$



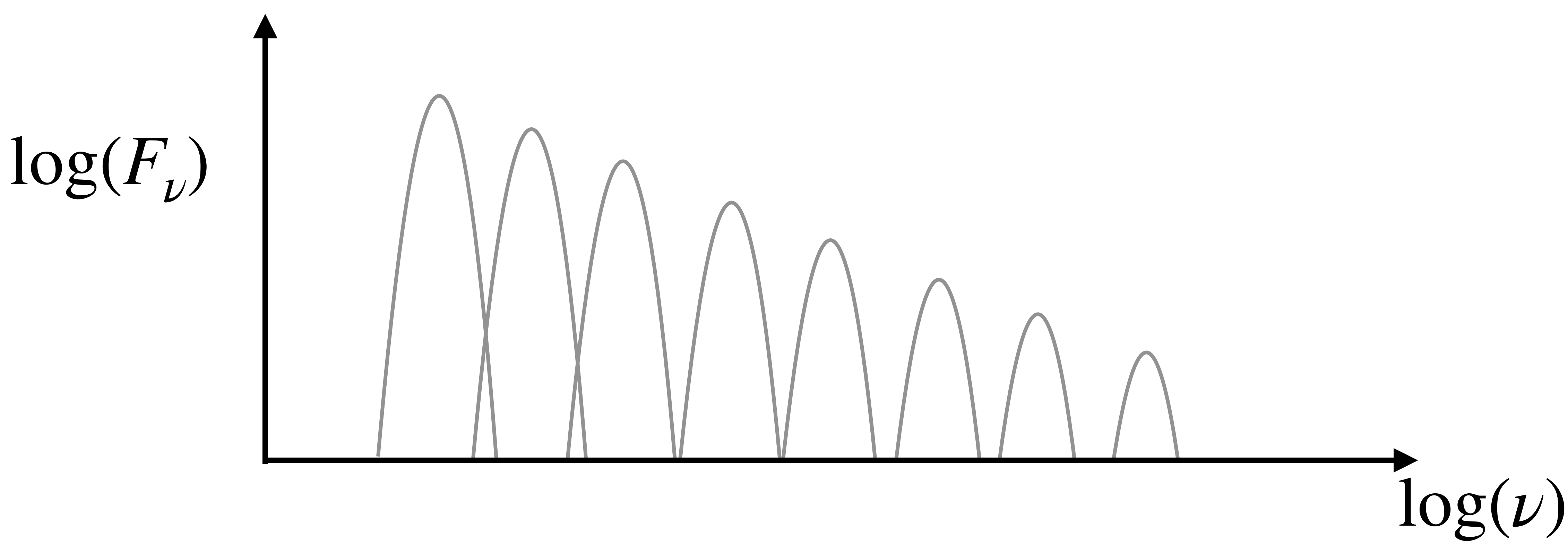
Population of electrons

- Spectrum for electrons with energy $E \propto \gamma$ is narrow and peaked at $\nu \propto \gamma^2$; peak value is $P \propto \gamma^2$



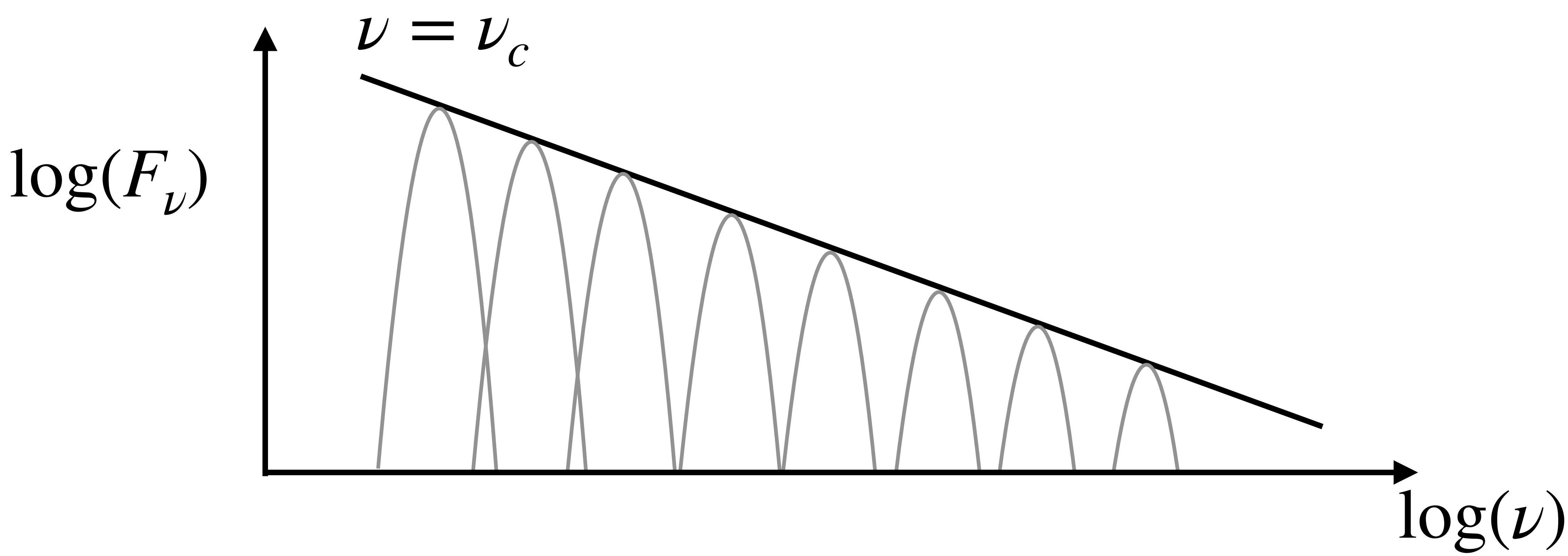
Population of electrons

- Spectrum for electrons with energy $E \propto \gamma$ is narrow and peaked at $\nu \propto \gamma^2$; peak value is $P \propto \gamma^2$
- Population of electrons: $dN/d\gamma \propto \gamma^{-k}$ (previous lecture).



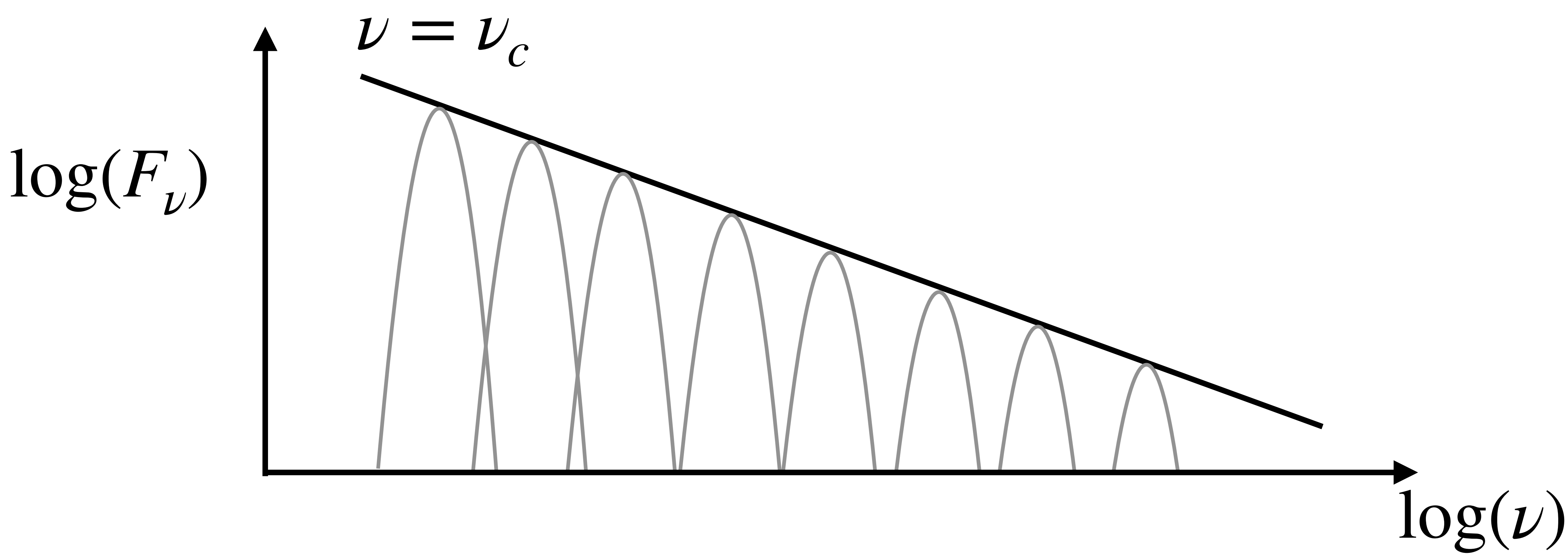
Population of electrons

- Spectrum for electrons with energy $E \propto \gamma$ is narrow and peaked at $\nu \propto \gamma^2$; peak value is $P \propto \gamma^2$
- Population of electrons: $dN/d\gamma \propto \gamma^{-k}$ (previous lecture).
- Overall spectrum: $F_\nu \propto \nu^{-\alpha}$ (α is spectral index, _not_ pitch angle).



Population of electrons

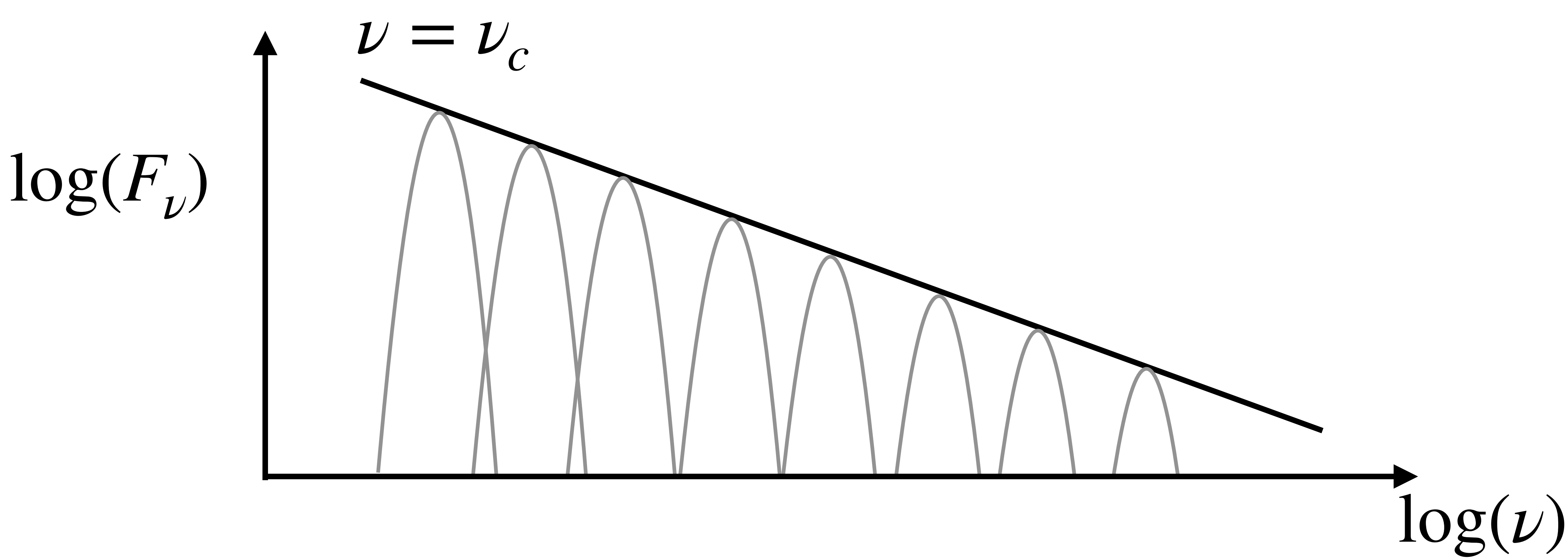
- Spectrum for electrons with energy $E \propto \gamma$ is narrow and peaked at $\nu \propto \gamma^2$; peak value is $P \propto \gamma^2$
- Population of electrons: $dN/d\gamma \propto \gamma^{-k}$ (previous lecture).
- Overall spectrum: $F_\nu \propto \nu^{-\alpha}$ (α is spectral index, _not_ pitch angle).
- Homework problem to show that: $\alpha = \dfrac{k-1}{2}$



Population of electrons

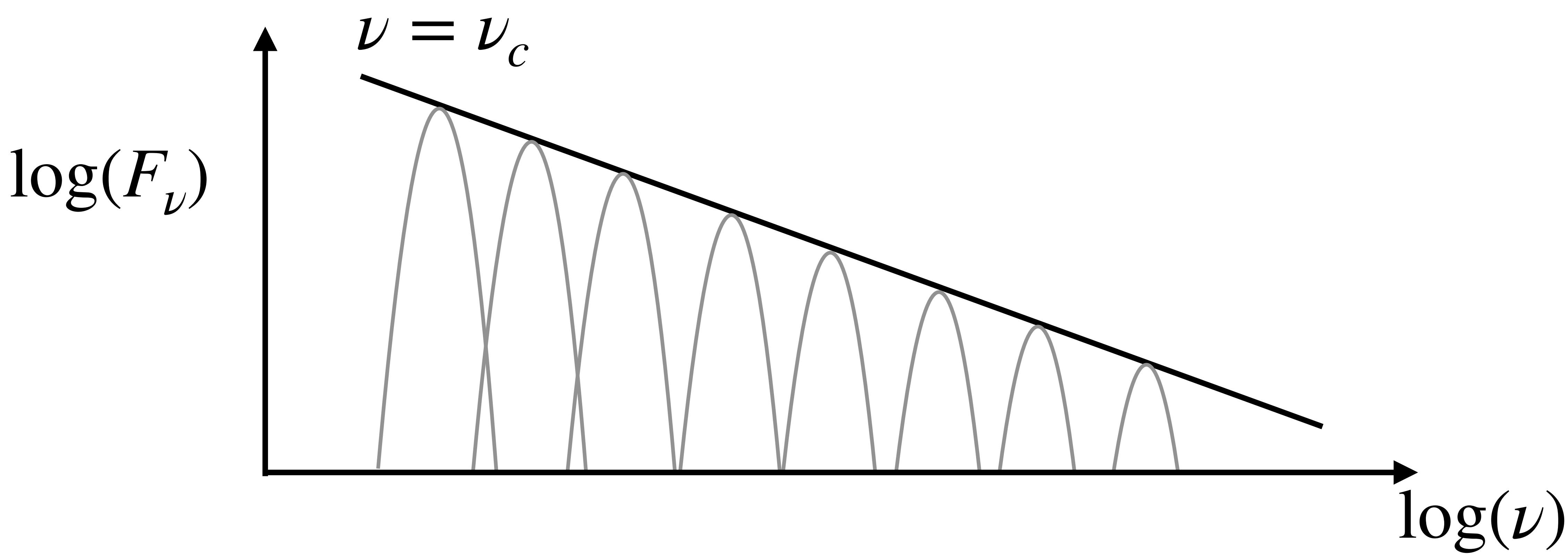
- Spectrum for electrons with energy $E \propto \gamma$ is narrow and peaked at $\nu \propto \gamma^2$; peak value is $P \propto \gamma^2$
- Population of electrons: $dN/d\gamma \propto \gamma^{-k}$ (previous lecture).
- Overall spectrum: $F_\nu \propto \nu^{-\alpha}$ (α is spectral index, _not_ pitch angle).
- Homework problem to show that: $\alpha = \dfrac{k-1}{2}$

Hint: $F_\nu = \dfrac{dF}{d\nu} \propto P\dfrac{dN}{d\nu} \propto P\dfrac{dN}{d\gamma}\dfrac{d\gamma}{d\nu}$



Population of electrons

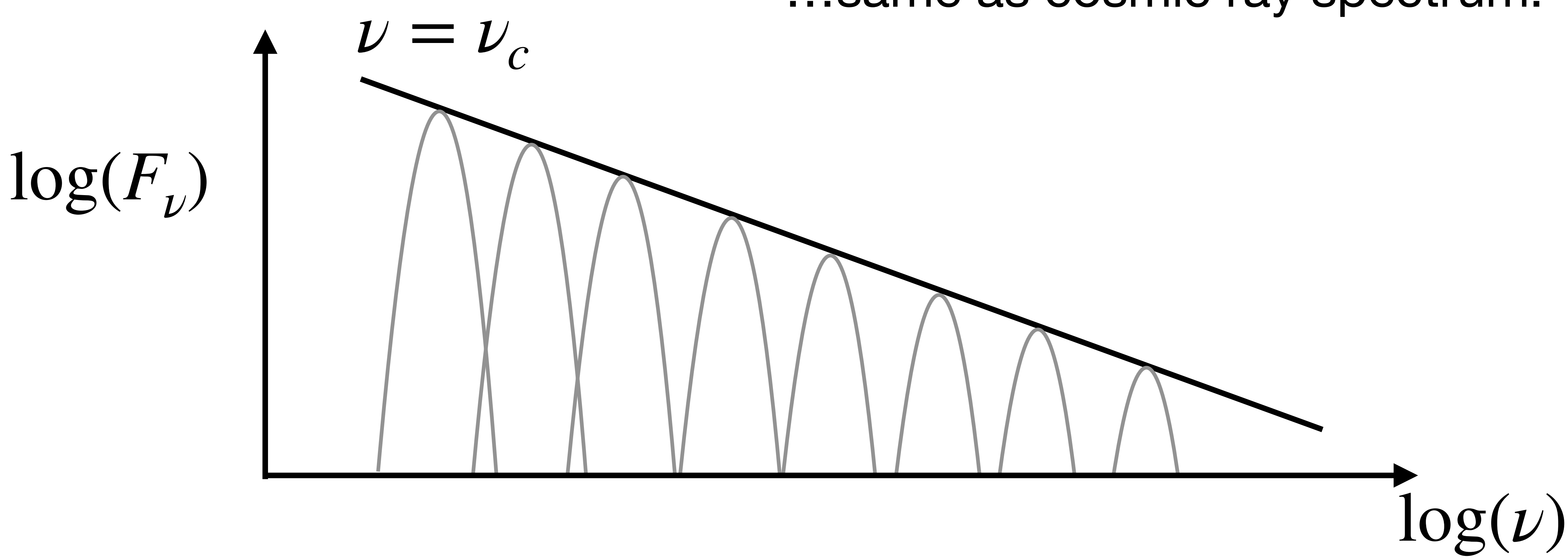
- Spectrum for electrons with energy $E \propto \gamma$ is narrow and peaked at $\nu \propto \gamma^2$; peak value is $P \propto \gamma^2$
- Population of electrons: $dN/d\gamma \propto \gamma^{-k}$ (previous lecture).
- Overall spectrum: $F_\nu \propto \nu^{-\alpha}$ (α is spectral index, *not* pitch angle).
- Homework problem to show that: $\alpha = \dfrac{k-1}{2}$
- $k = 2 \implies \alpha = 0.5$ …we sometimes measure this for radio sources!



Population of electrons

- Spectrum for electrons with energy $E \propto \gamma$ is narrow and peaked at $\nu \propto \gamma^2$; peak value is $P \propto \gamma^2$
- Population of electrons: $dN/d\gamma \propto \gamma^{-k}$ (previous lecture).
- Overall spectrum: $F_\nu \propto \nu^{-\alpha}$ (α is spectral index, *not* pitch angle).
- Homework problem to show that: $\alpha = \dfrac{k-1}{2}$
- $k = 2 \implies \alpha = 0.5$ …we sometimes measure this for radio sources!
- But often measure $\alpha > 0.5 \implies k > 2$

…same as cosmic ray spectrum!



Minimum energy

- By integrating the spectrum of a synchrotron emitting object, we can measure the total energy density emitted by the plasma (need as estimate for distance and size).

Minimum energy

- By integrating the spectrum of a synchrotron emitting object, we can measure the total energy density emitted by the plasma (need as estimate for distance and size).
- But energy density is $U_{\rm tot} = U_{\rm e} + U_{\rm mag}$

Minimum energy

- By integrating the spectrum of a synchrotron emitting object, we can measure the total energy density emitted by the plasma (need as estimate for distance and size).
- But energy density is $U_{\mathrm{tot}} = U_{\mathrm{e}} + U_{\mathrm{mag}}$
- To break this degeneracy, and therefore measure B-field, we can use the *minimum energy method* (Burbidge 1956)

$$U_{\mathrm{mag}} = \frac{B^2}{2\mu_0}$$

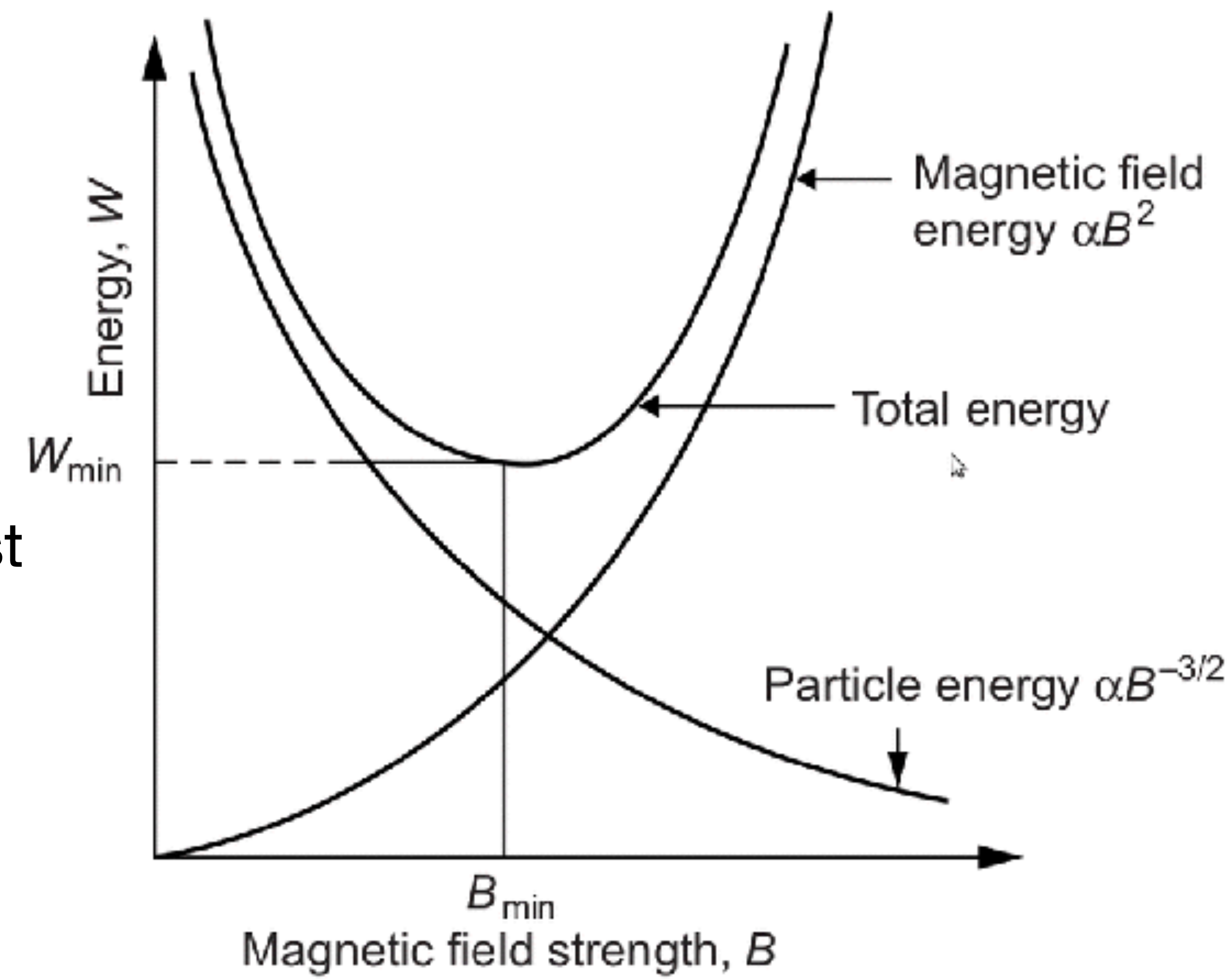
$$U_{\mathrm{e}} = \int E(\gamma) \frac{dN}{d\gamma} d\gamma$$

Minimum energy

- By integrating the spectrum of a synchrotron emitting object, we can measure the total energy density emitted by the plasma (need as estimate for distance and size).
- But energy density is $U_{\rm tot} = U_{\rm e} + U_{\rm mag}$
- To break this degeneracy, and therefore measure B-field, we can use the *minimum energy method* (Burbidge 1956)

$$U_{\rm mag} = \frac{B^2}{2\mu_0}$$

$$U_{\rm e} = \int E(\gamma)\frac{dN}{d\gamma}d\gamma \propto B^{-3/2}$$



- Plasma presumably finds ~lowest energy state in equilibrium.

Minimum energy

- Minimum energy requirement: $U_{\mathrm{e}} \sim U_{\mathrm{mag}}$ (equipartition).
- Will calculate true minimum energy relation for homework problem.
- Roughly equipartition values expected from particles jiggling around and sharing energy in the plasma.
- See https://github.com/robfender/ThunderBooks/blob/master/Equipartition%20analysis.ipynb