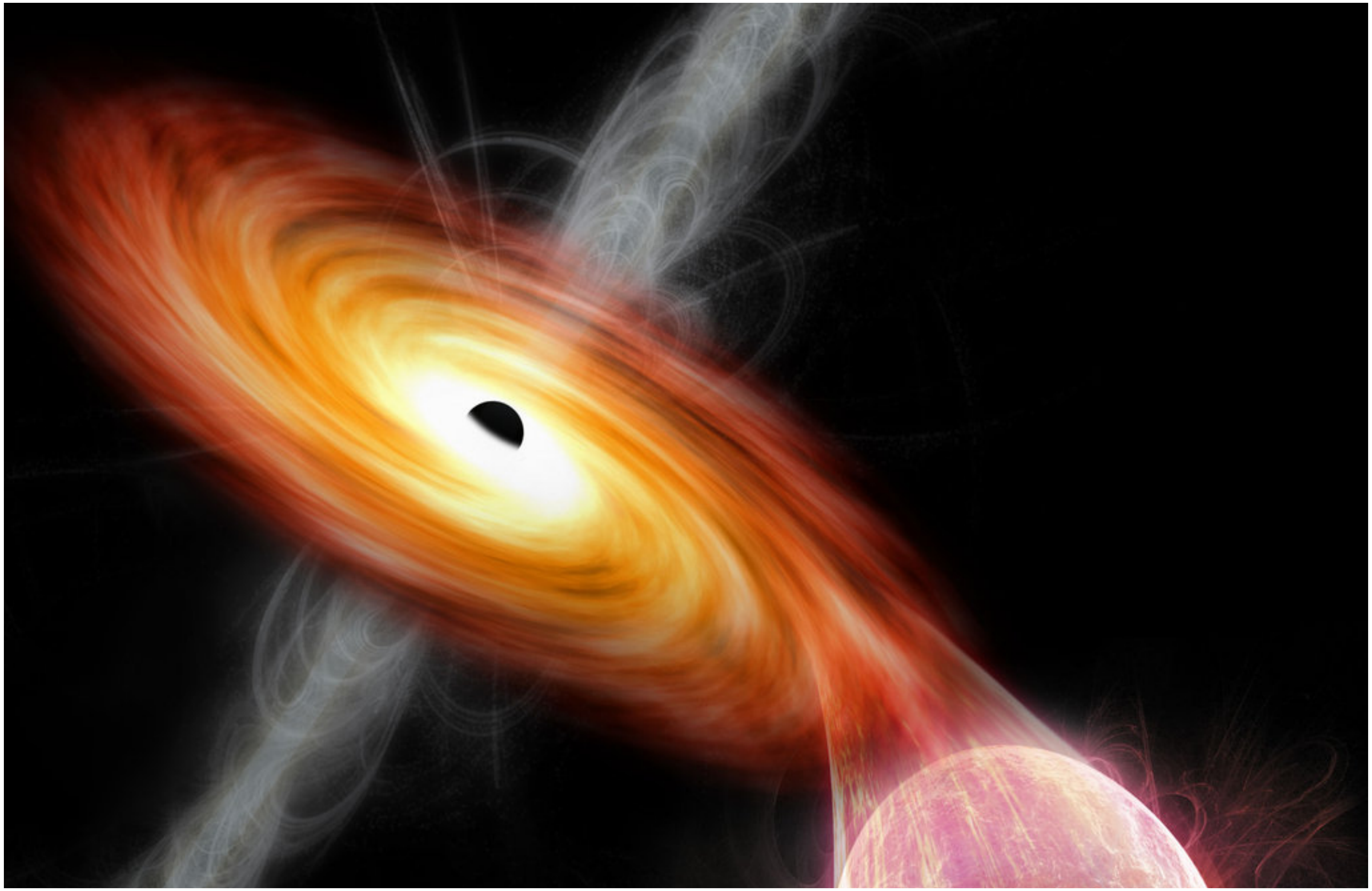
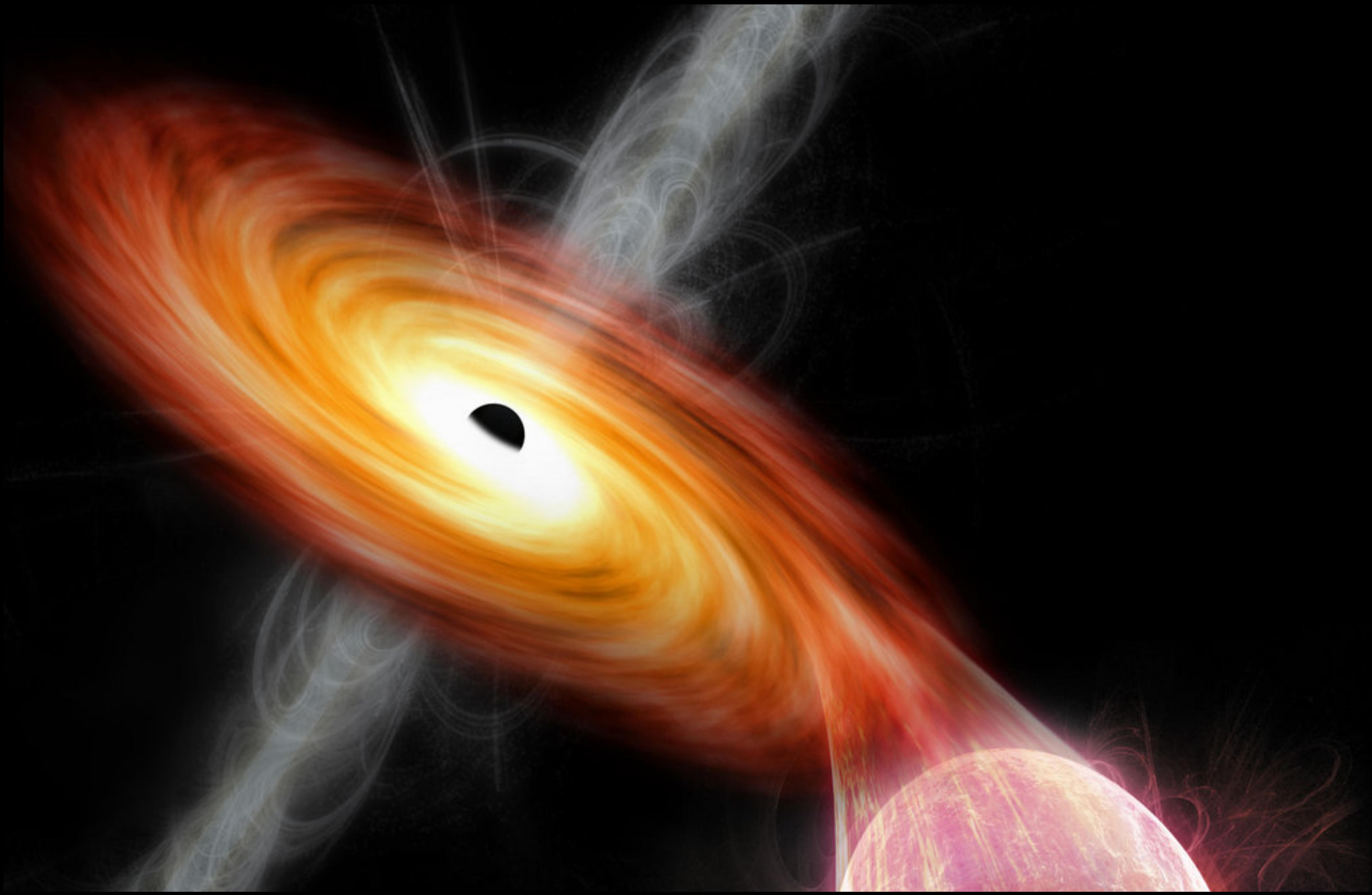
High Energy Astrophysics

Dr. Adam Ingram

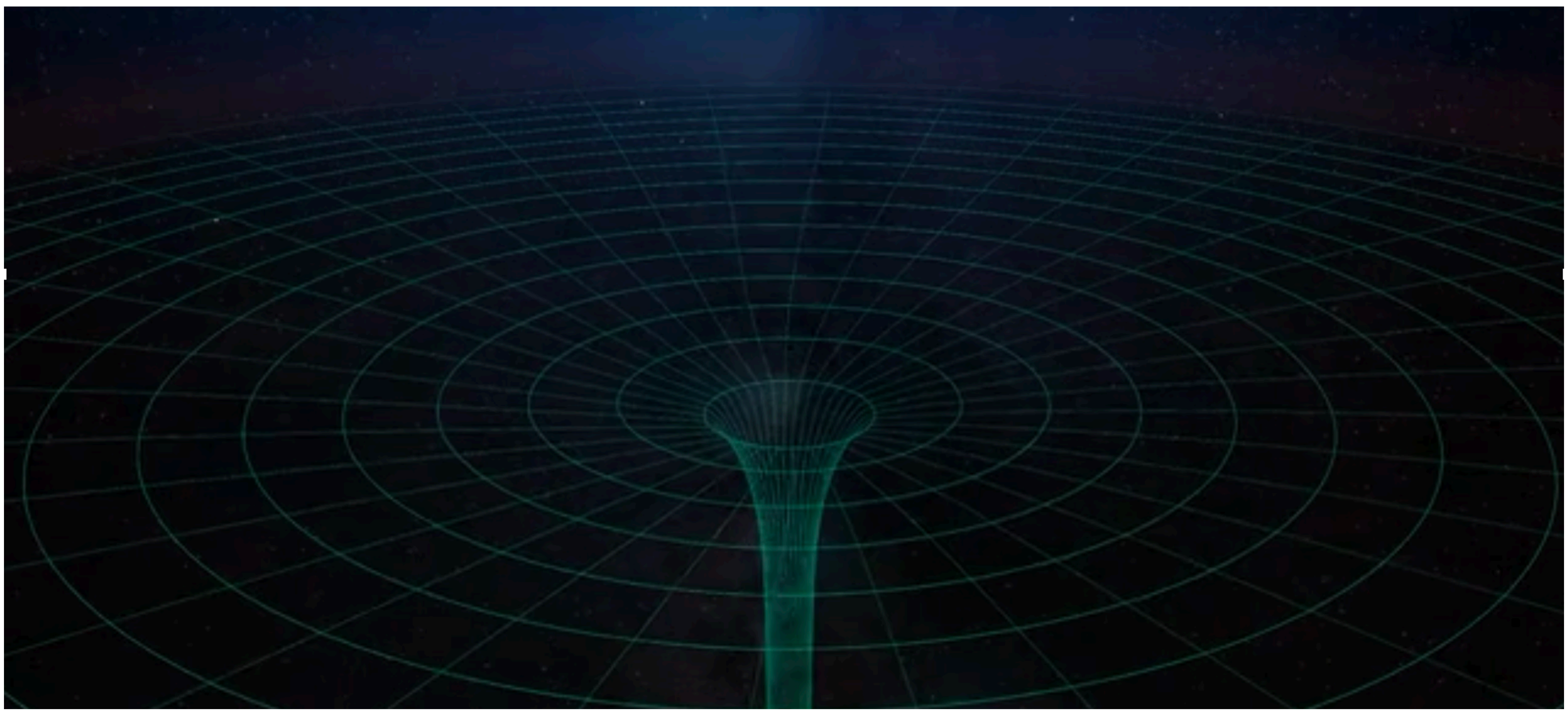


Lecture 4
Black Hole Accretion Discs



Black Holes

- All mass, M, in a singularity
- Event Horizon: $v_{\mathrm{esc}} = c$!
- Newtonian approx: $v_{\mathrm{esc}}^2 = 2GM/r \implies r_h = 2GM/c^2$
- This is correct for a non-spinning black hole!
- Size scale: gravitational radius: $r_g = GM/c^2$



Non-spinning black hole: Spacetime described by Schwarzschild metric

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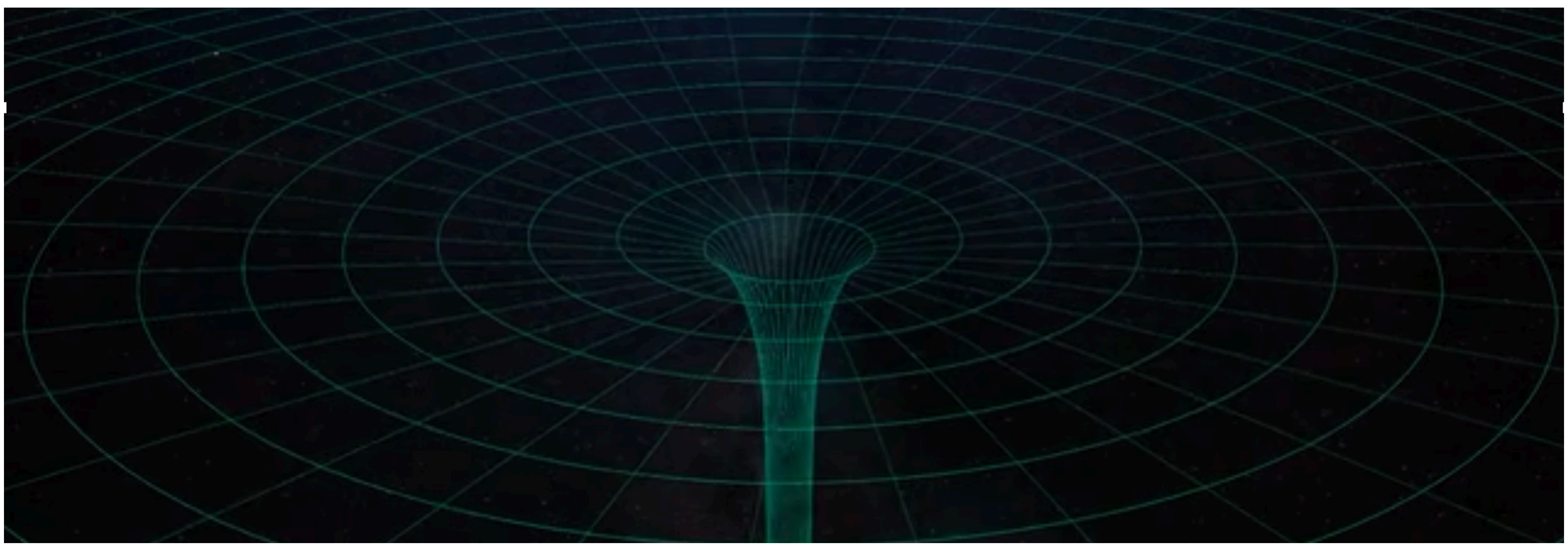
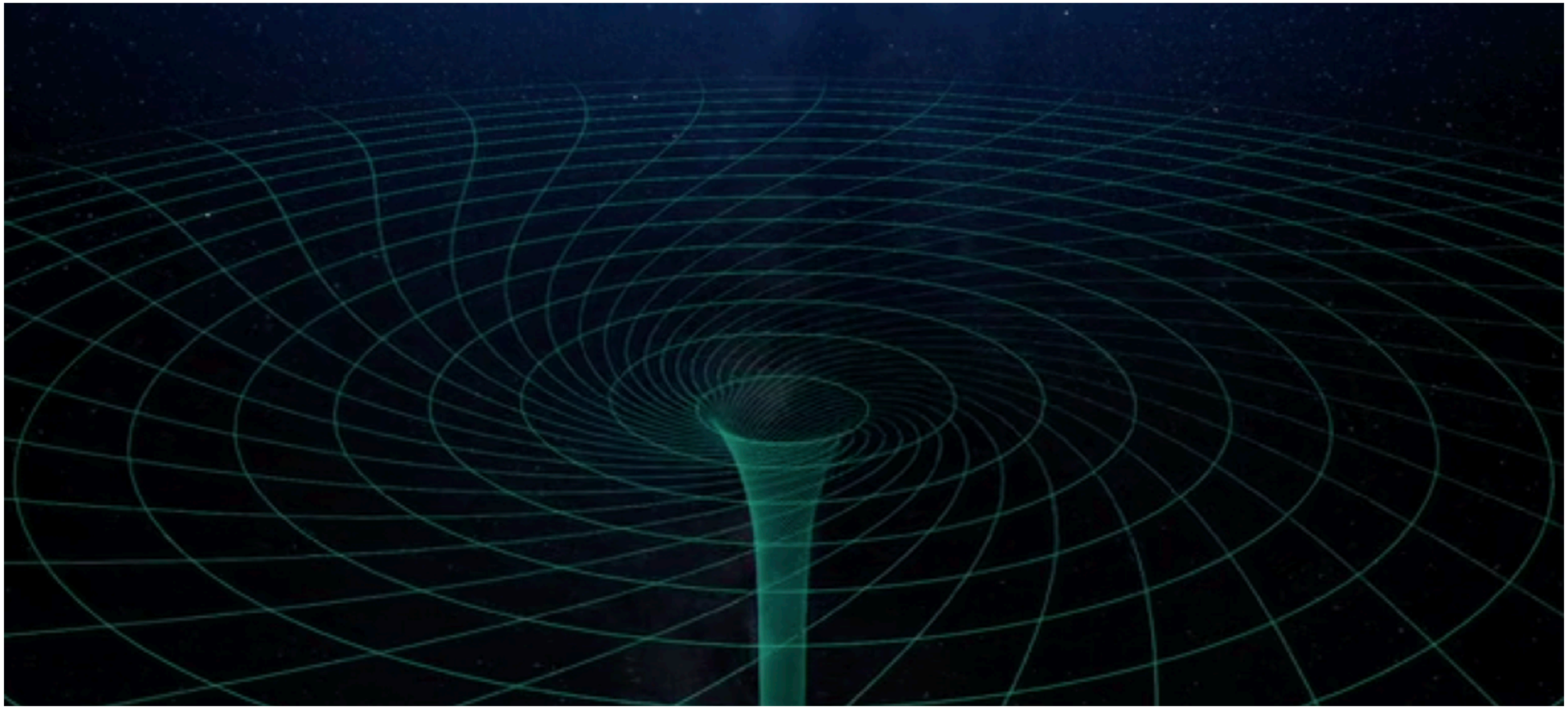
Symbol r_g was gyroradius; is now gravitational radius!



Non-spinning black hole: Spacetime described by Schwarzschild metric

Black Holes

- Black hole can have angular momentum: $J_{\rm bh}$
- Dimensionless spin parameter: $a = J_{\rm bh}/(Mcr_g)$
- Causality: $-1 \leq a \leq 1$ …-ve for retrograde orbits.
- Horizon depends on spin: $r_h/r_g = 1 + \sqrt{1 - a^2}$
- Rotating spacetime provides centrifugal barrier, so can get closer to BH

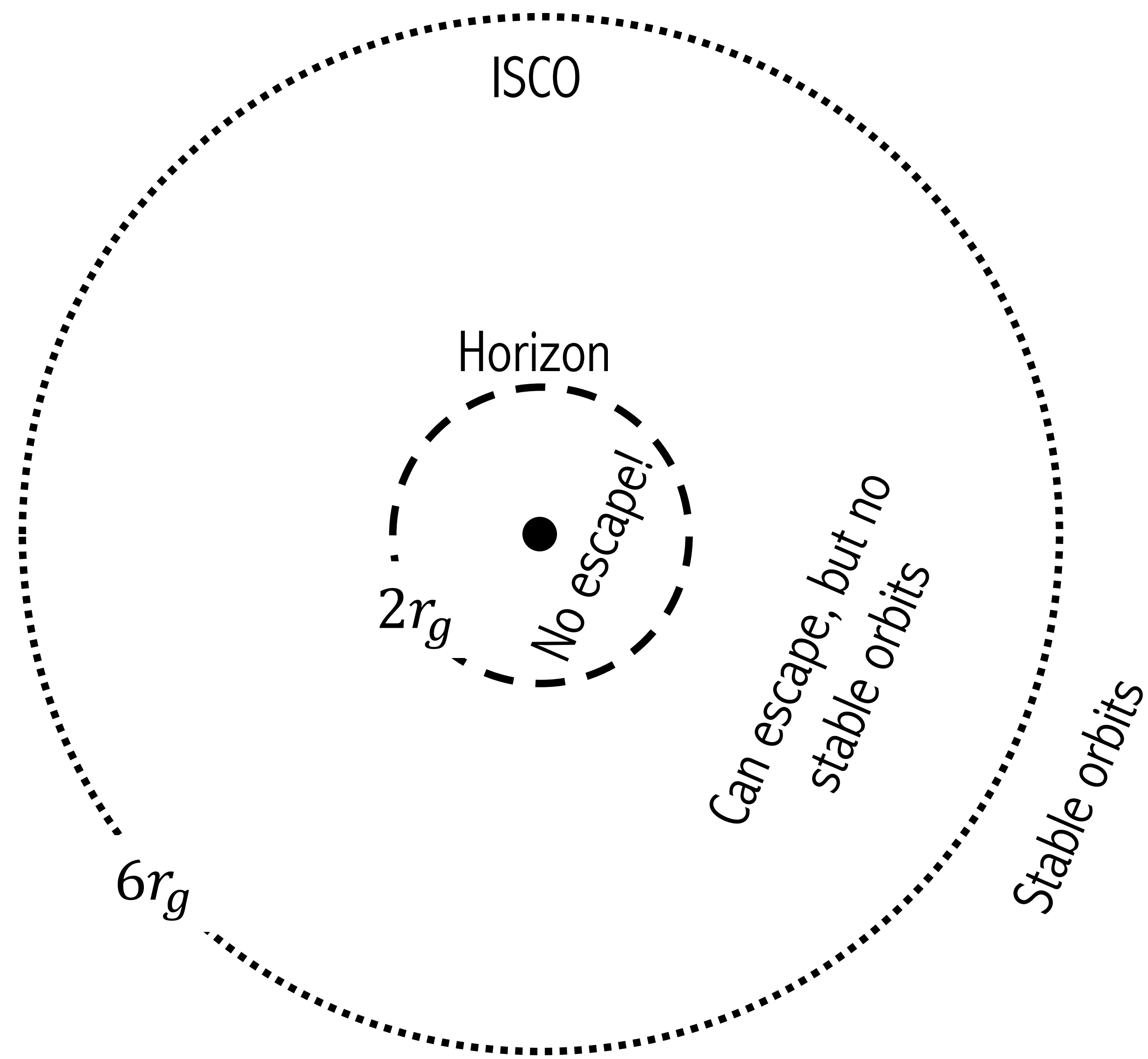


Non-spinning black hole: Spacetime described by Kerr metric

Innermost Stable Circular Orbit (ISCO)

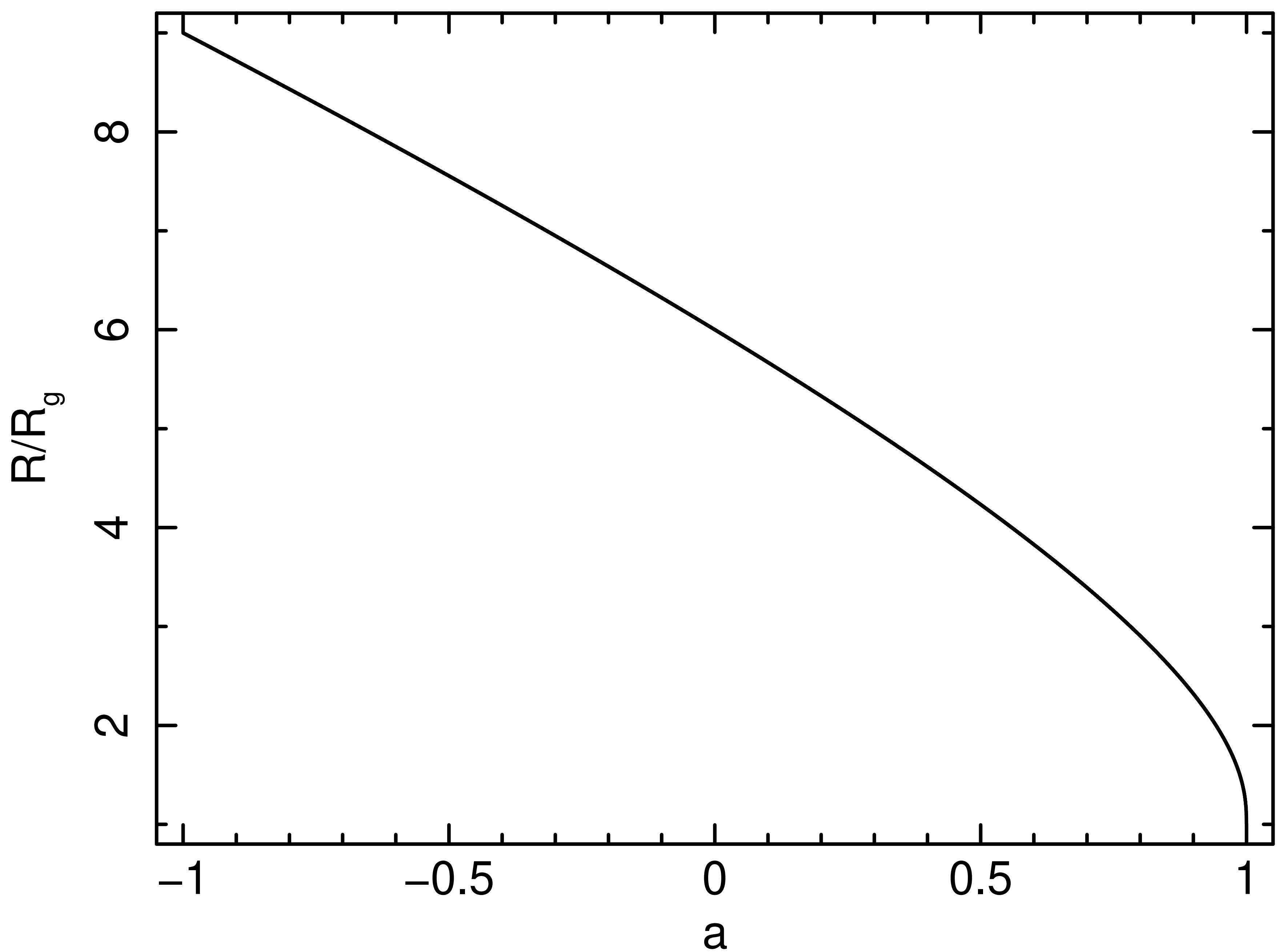
- Stable circular orbits: gravitational attraction balanced by centrifugal barrier.
- Always have stable circular orbits in Newtonian gravity.
- In GR, close to a black hole gravity overcomes the centrifugal term: no stable orbits.

Schwarszchild black hole

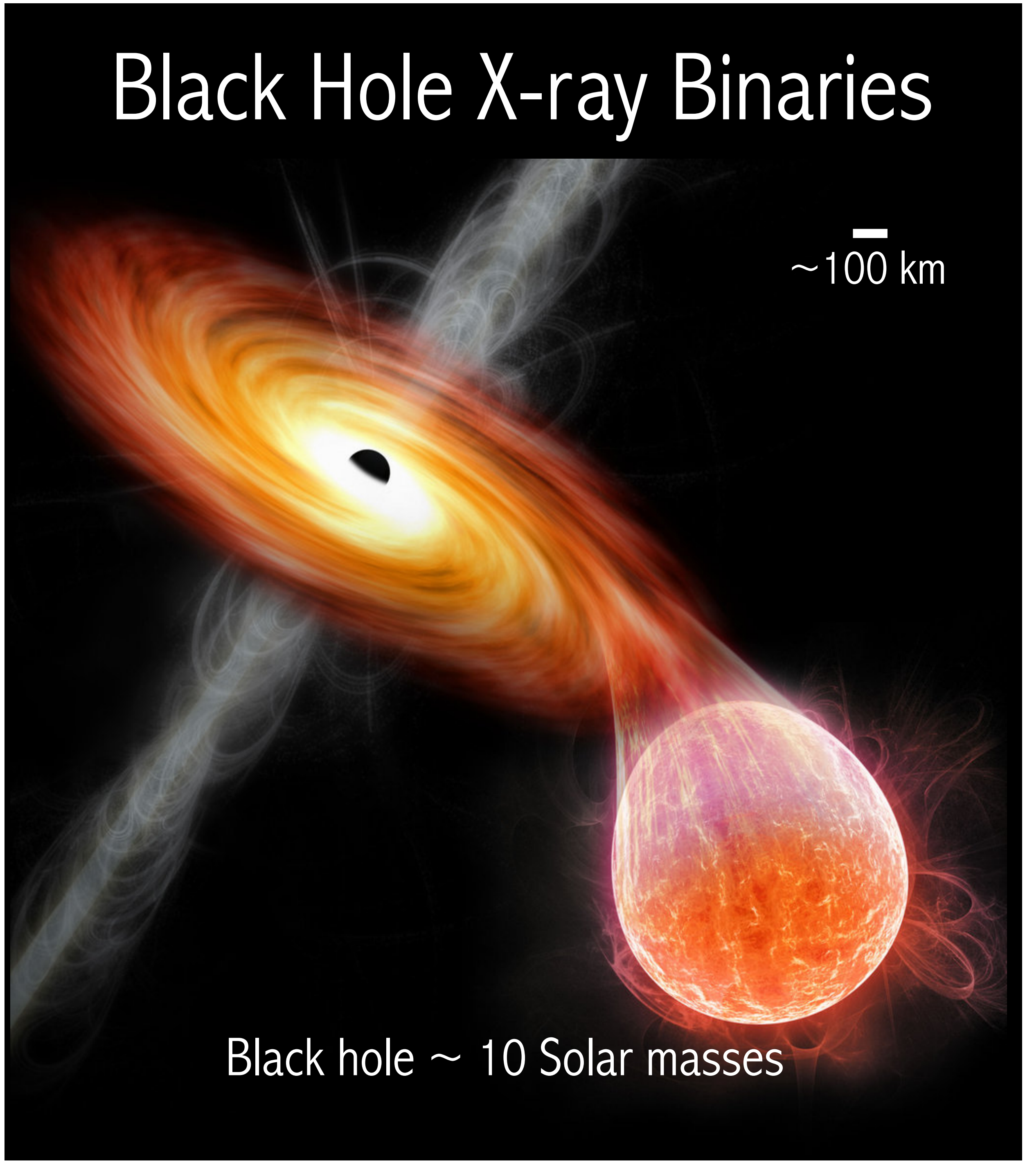


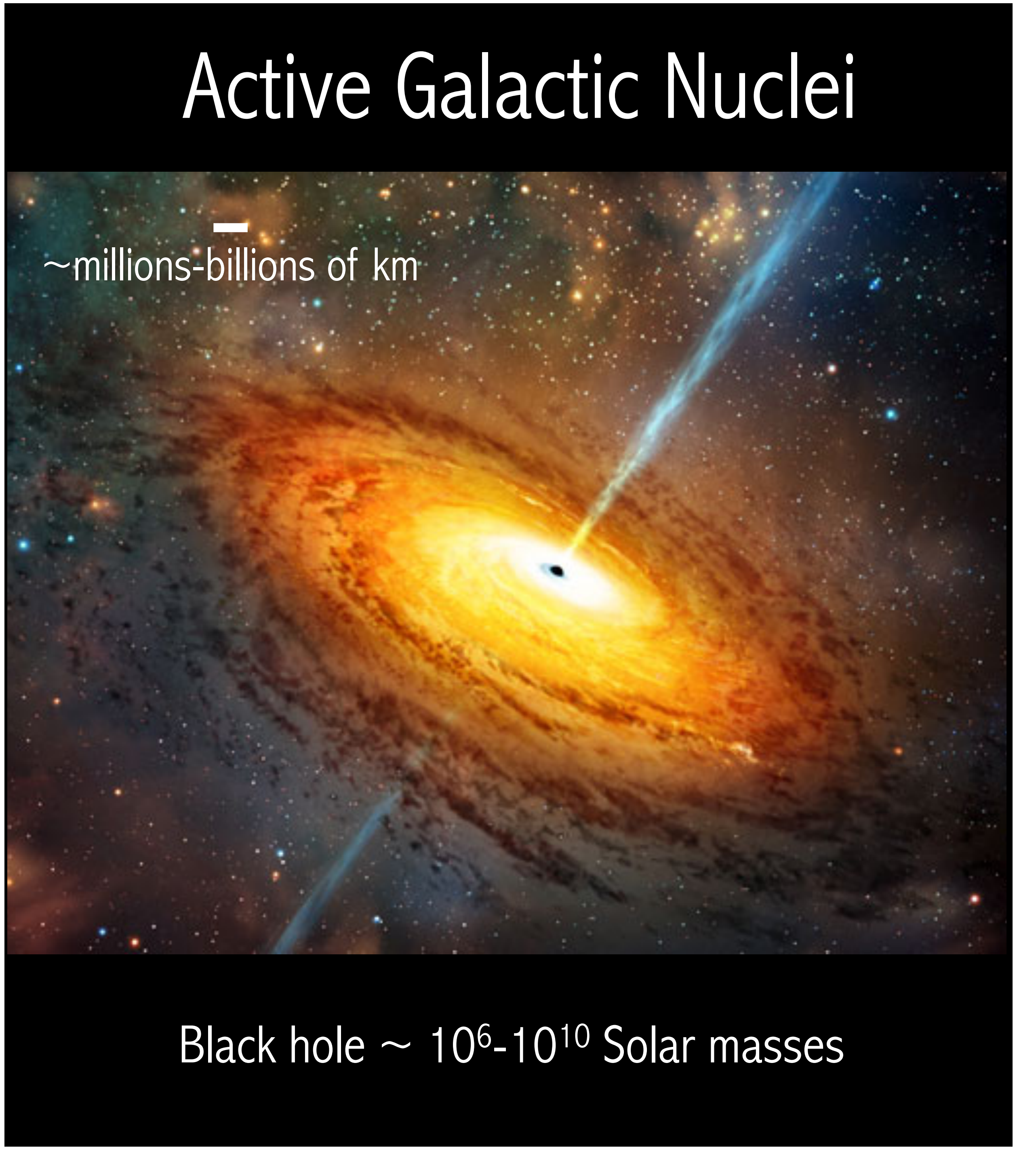
Innermost Stable Circular Orbit (ISCO)

ISCO depends on black hole spin: again, because spacetime is rotating (frame dragging effect), an orbiting test mass effectively has extra angular momentum, so black hole spin increases centrifugal barrier and enables stable orbits further in.



Accreting Black Holes





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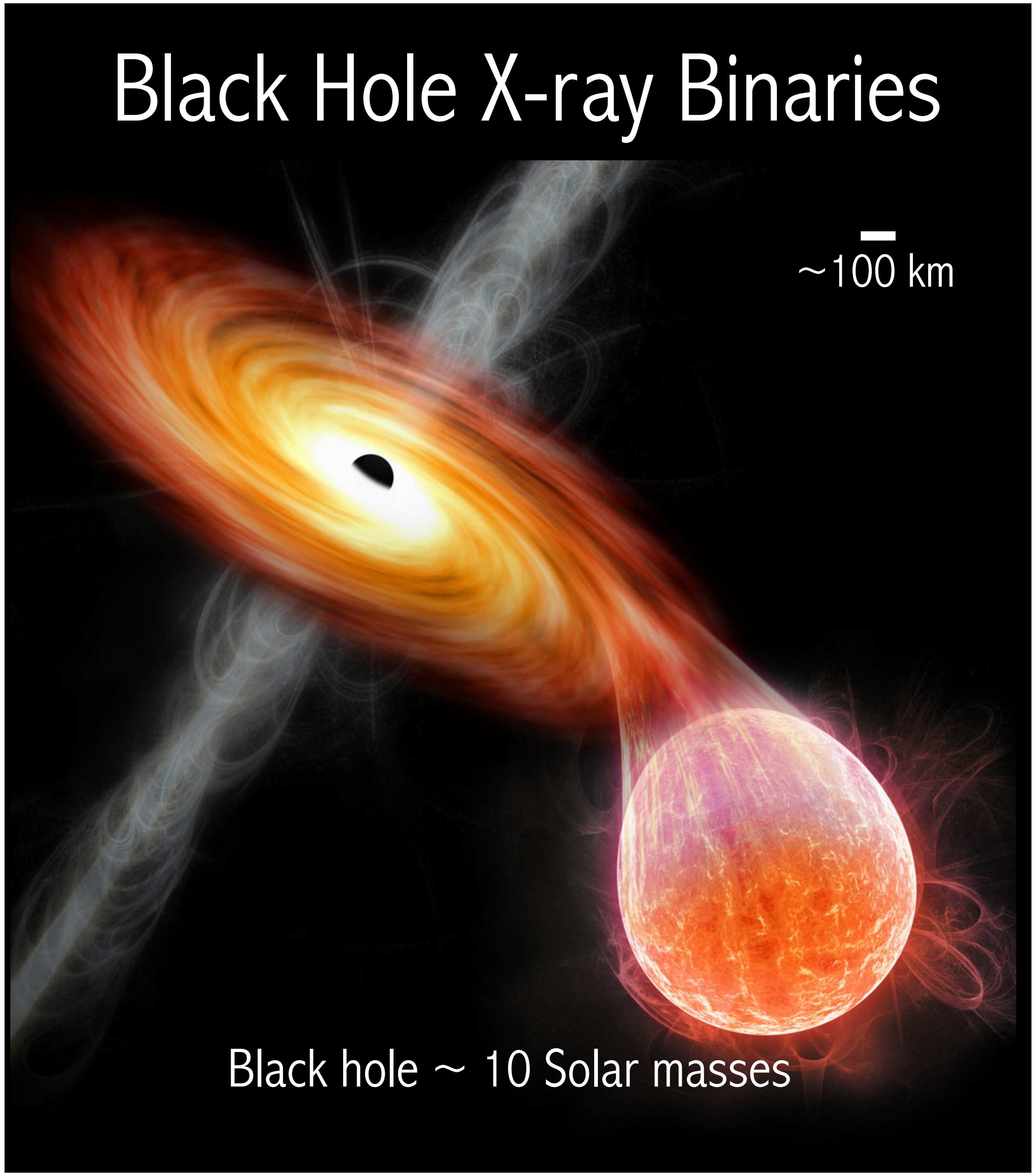

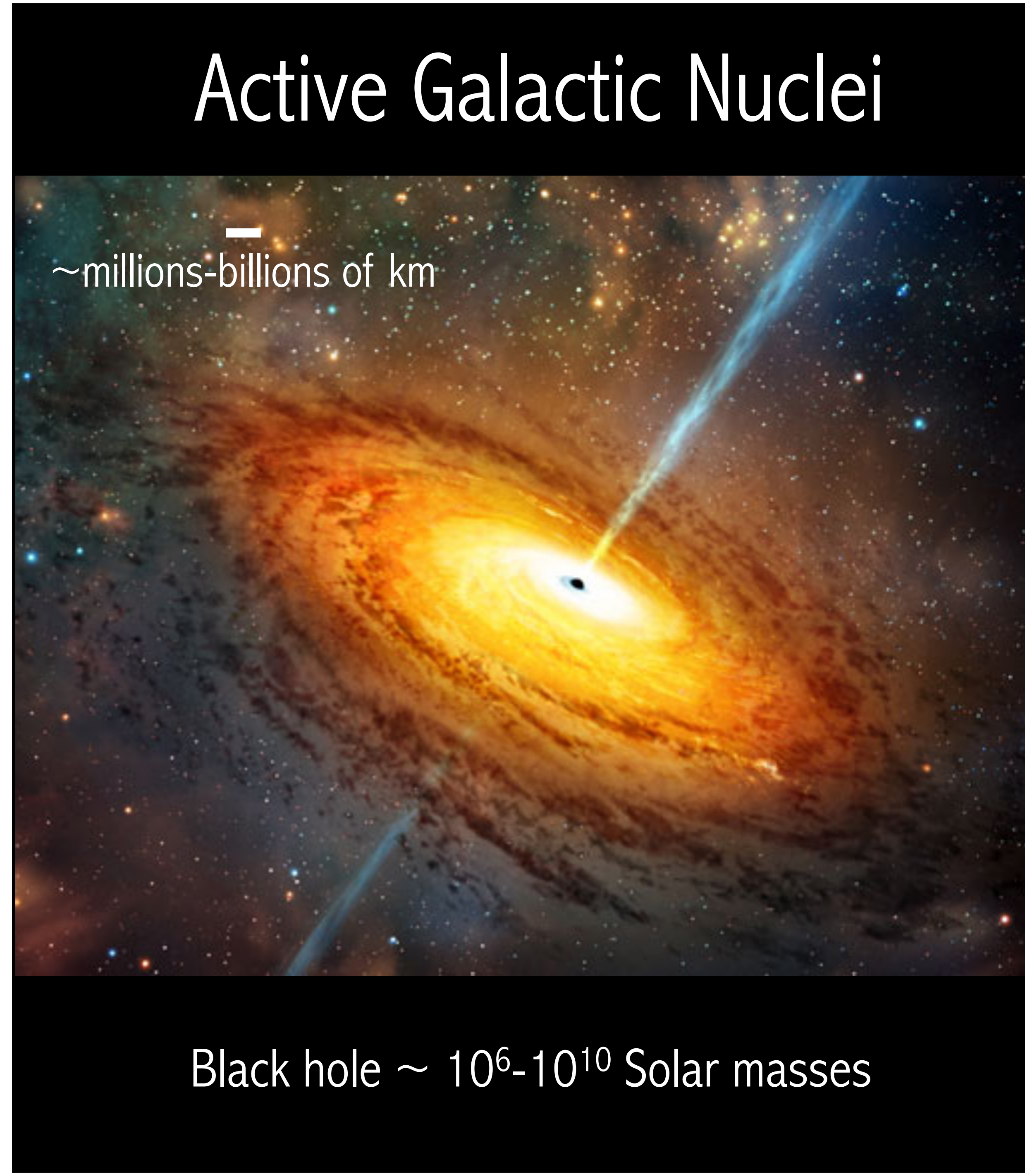

Power supply: gravitational potential energy of accreting material.

Therefore luminosity is the rest mass energy of accreted material multiplied by some efficiency factor: $L = \epsilon \dot{M} c^2$

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$$L_{\rm Edd} = \frac{4\pi G M c m_p}{\sigma_T}$$

$m_p =$ Proton mass

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Assumptions: photon-electron scattering opacity, spherical accretion

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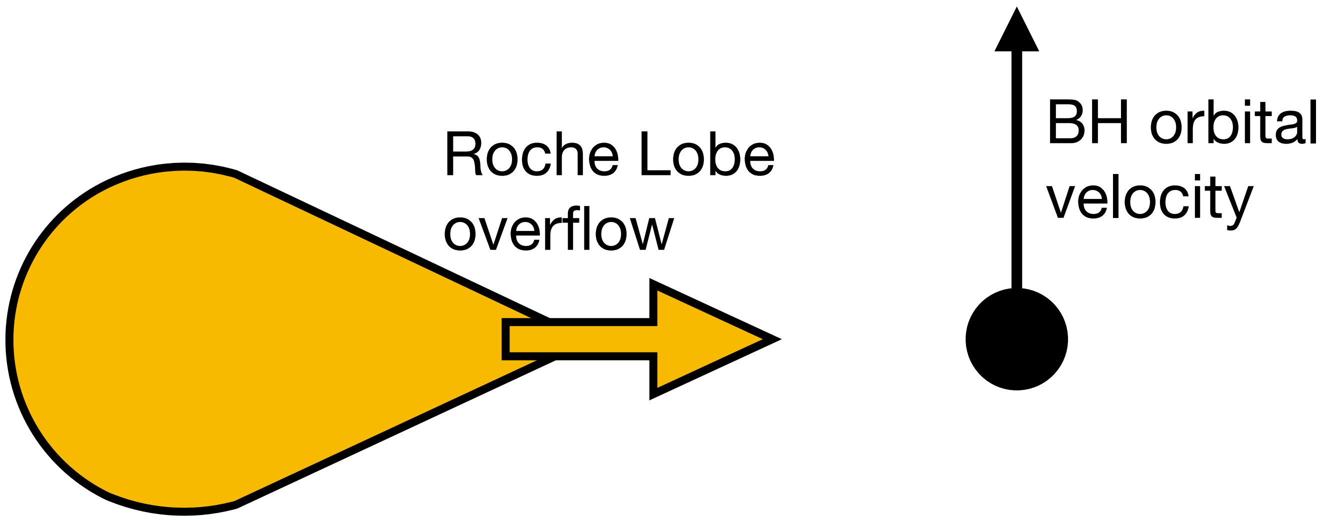
- But what is the efficiency?

Disc formation

Companion star rest frame:



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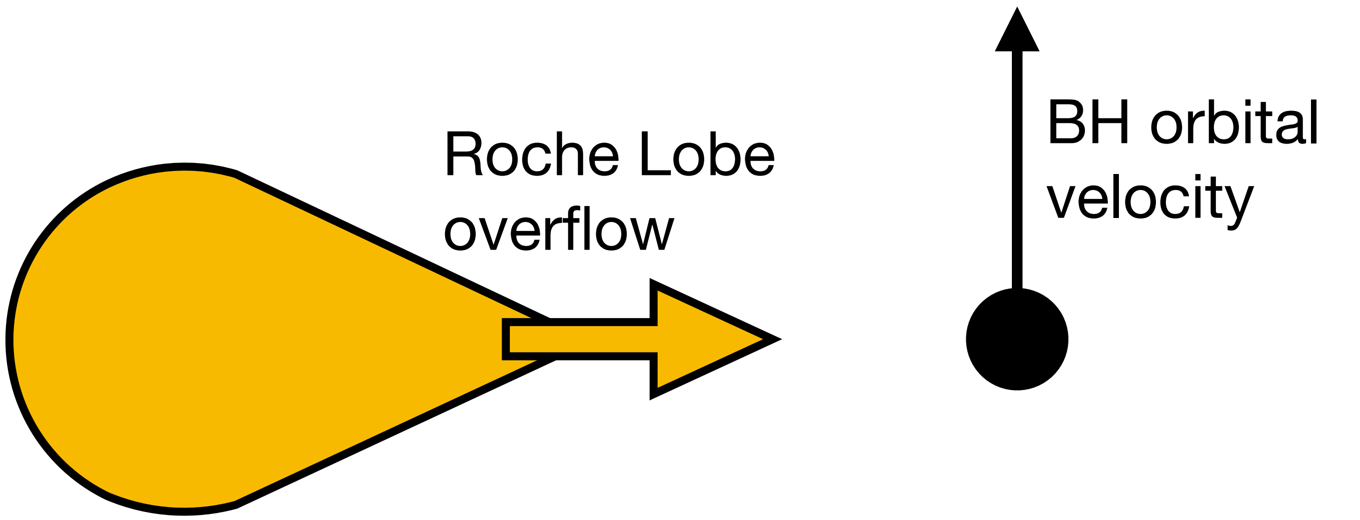

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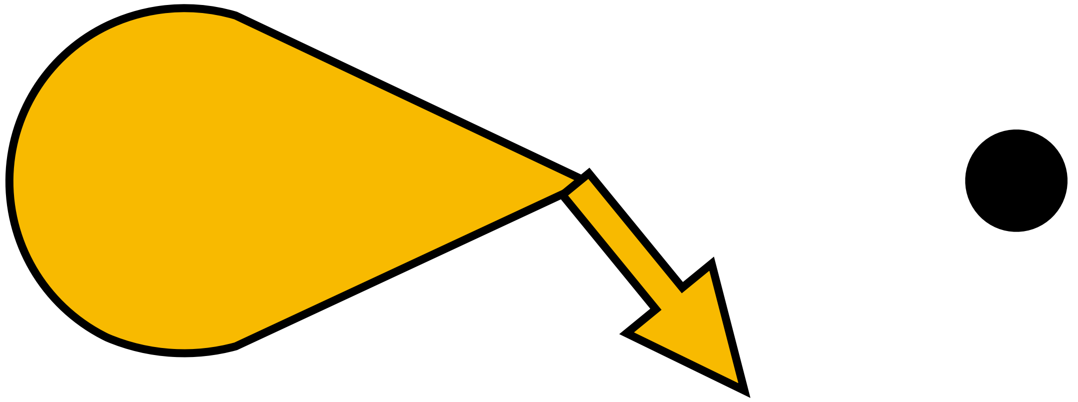

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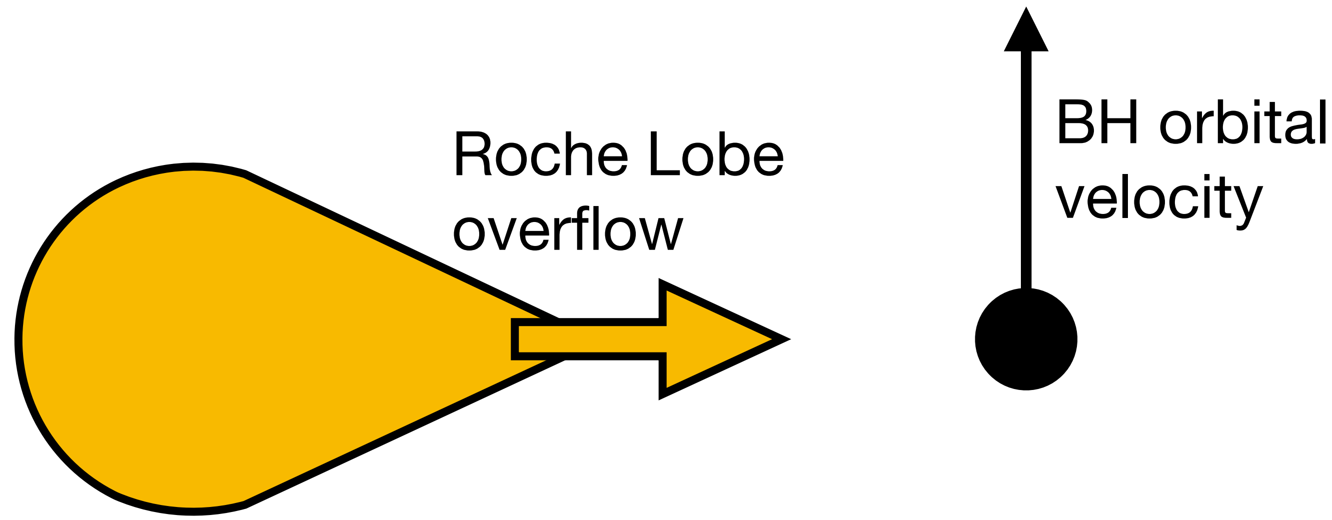

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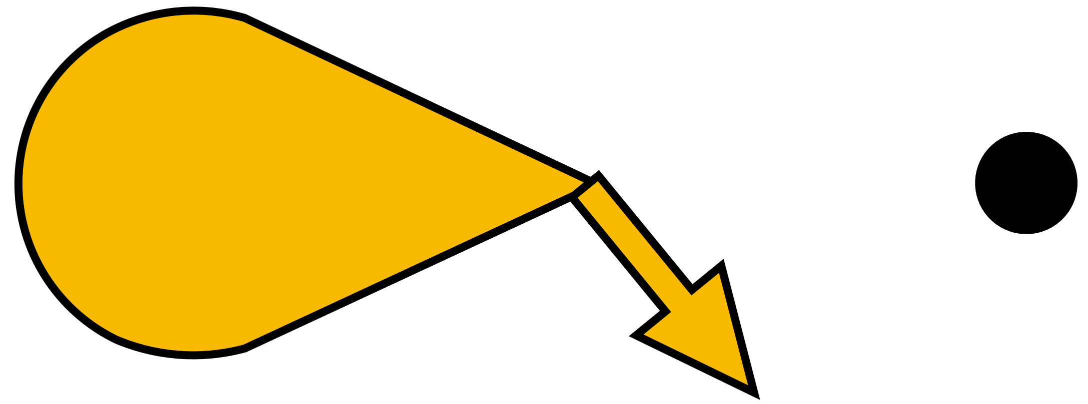
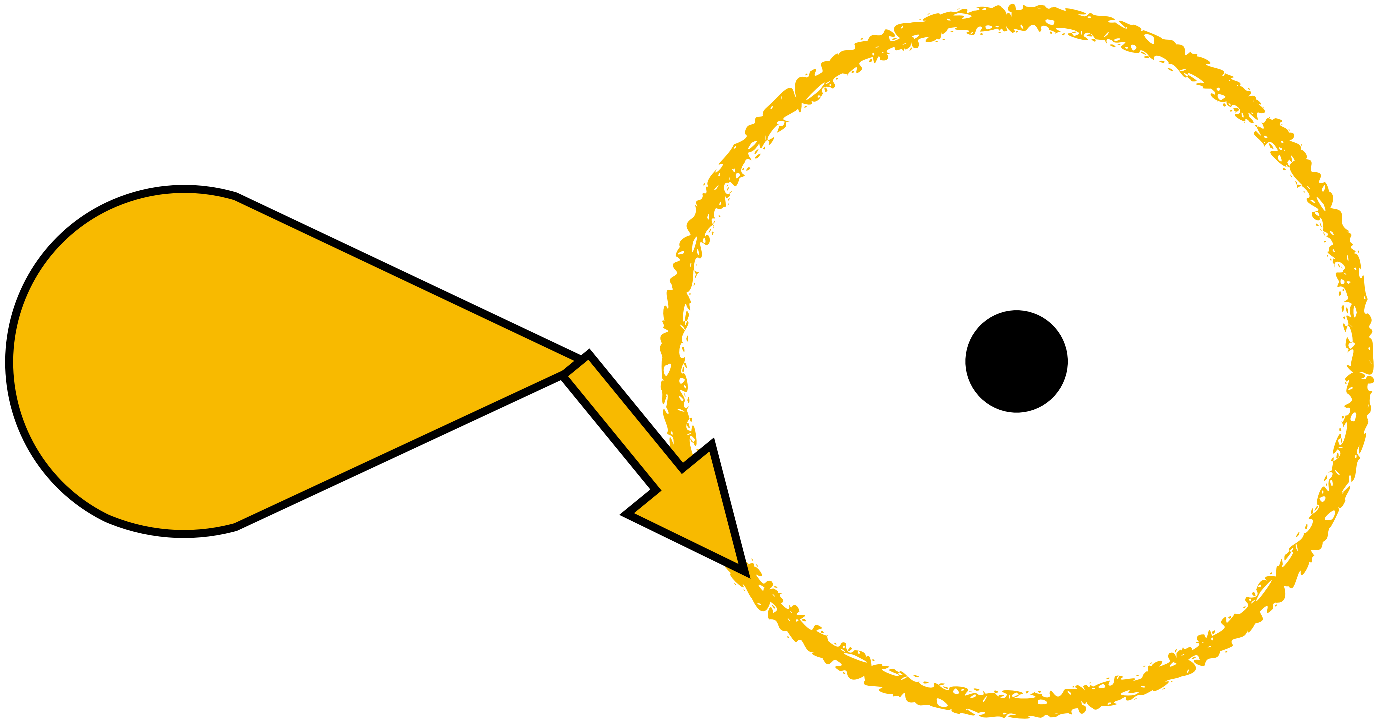


Therefore material has *angular momentum* about the black hole (similar arguments hold for AGN).

Disc formation

- Orbit quickly circularises via collisions (circular orbit is minimum energy configuration).



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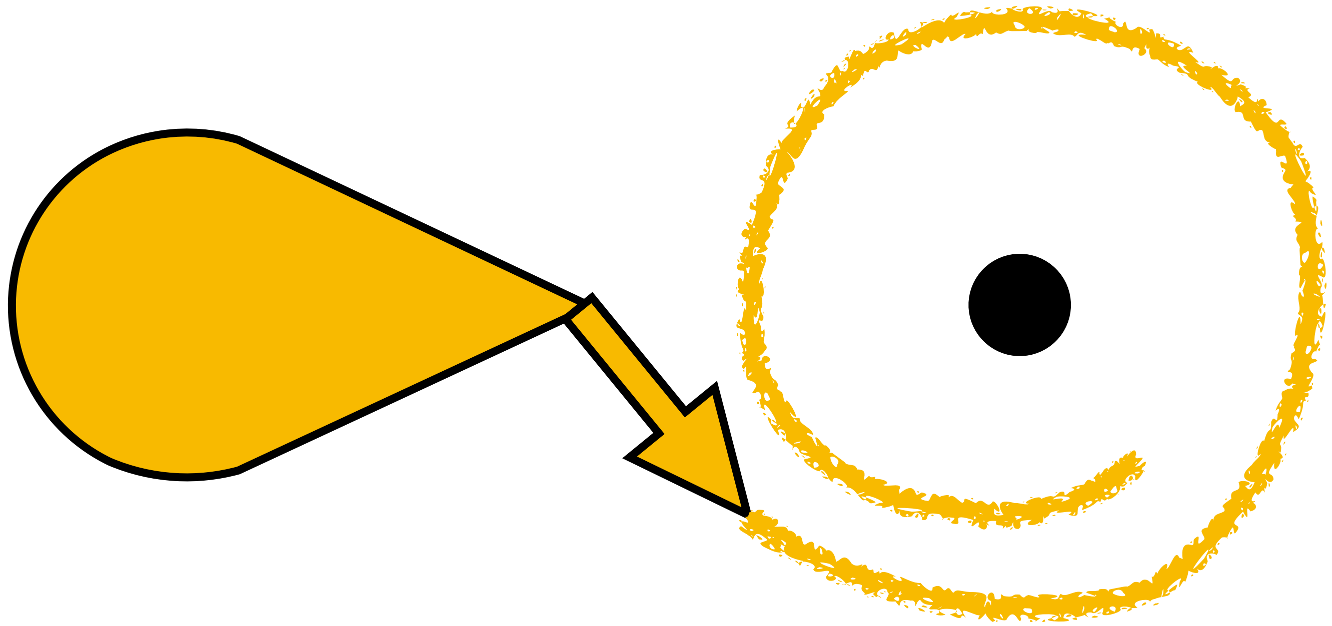
- Conservation of angular momentum: ~flat disc forms



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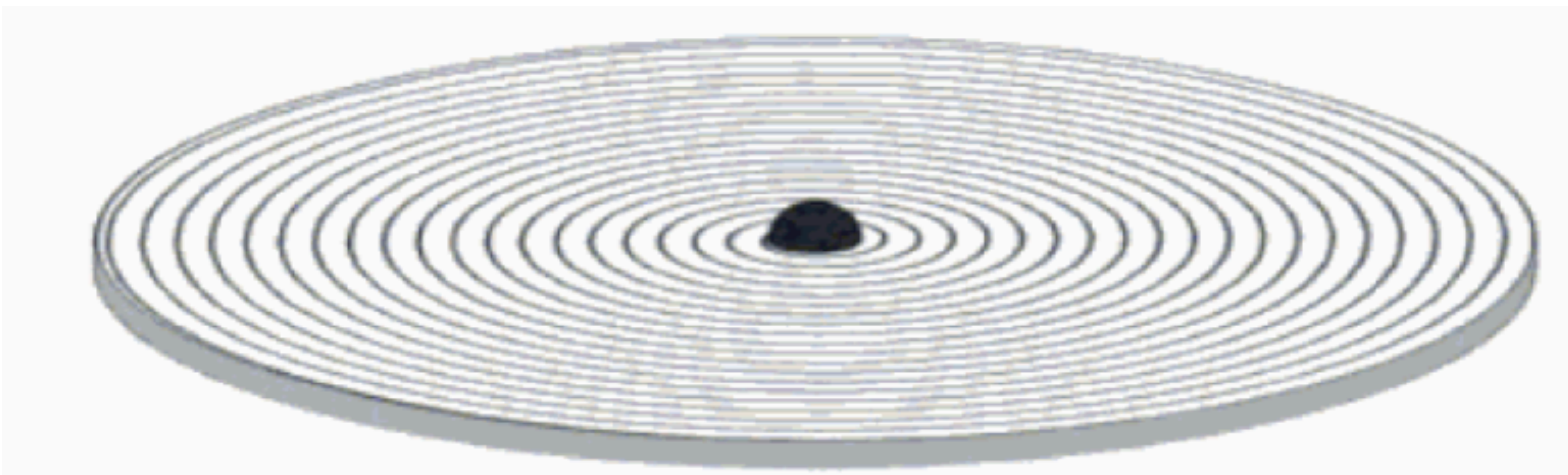
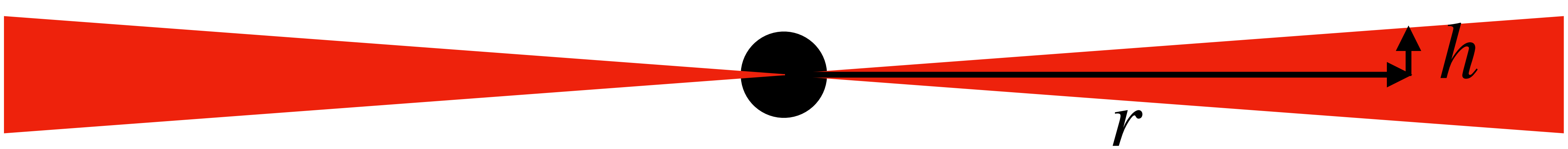


- Disc scale height: $h/r \sim \text{constant}$



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$$E_{\text{kinetic}} = \frac{1}{2} m v^2 = \frac{1}{2} m r^2 \omega^2 = \frac{1}{2} \frac{GMm}{r} = \frac{1}{2} E_{\text{potential}}$$

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- Therefore *half* of the GPE liberated can be radiated away (virial theorem). Luminosity of annulus is:

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- This is *enormous*! Nuclear fusion has efficiency $\epsilon \approx 0.007$

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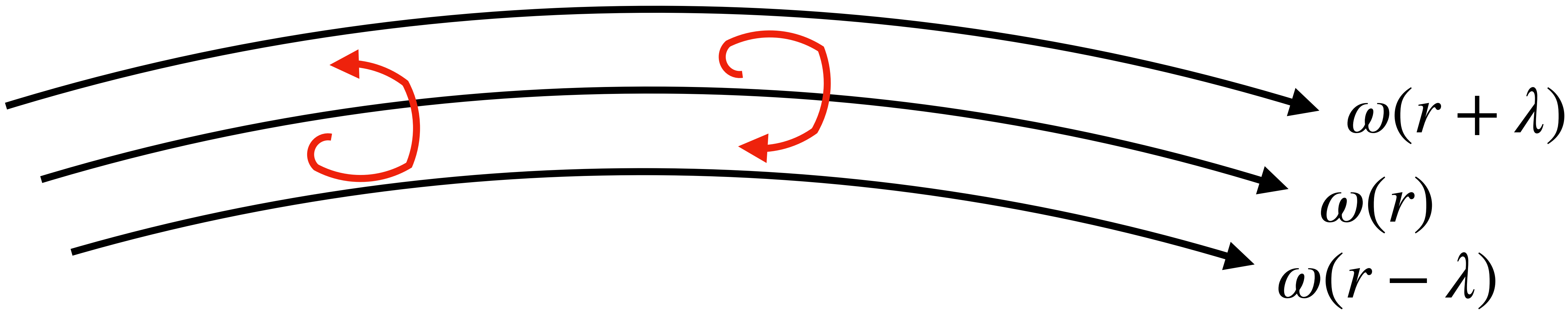

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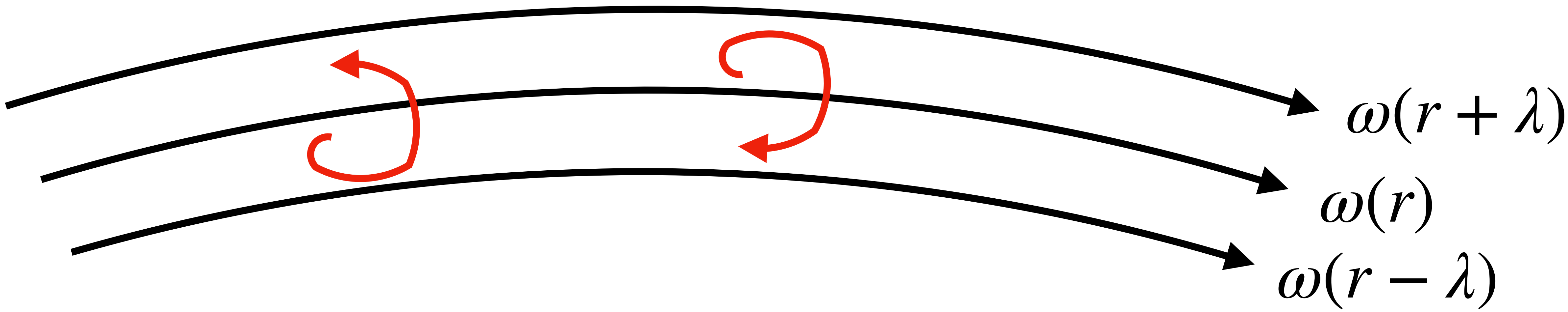

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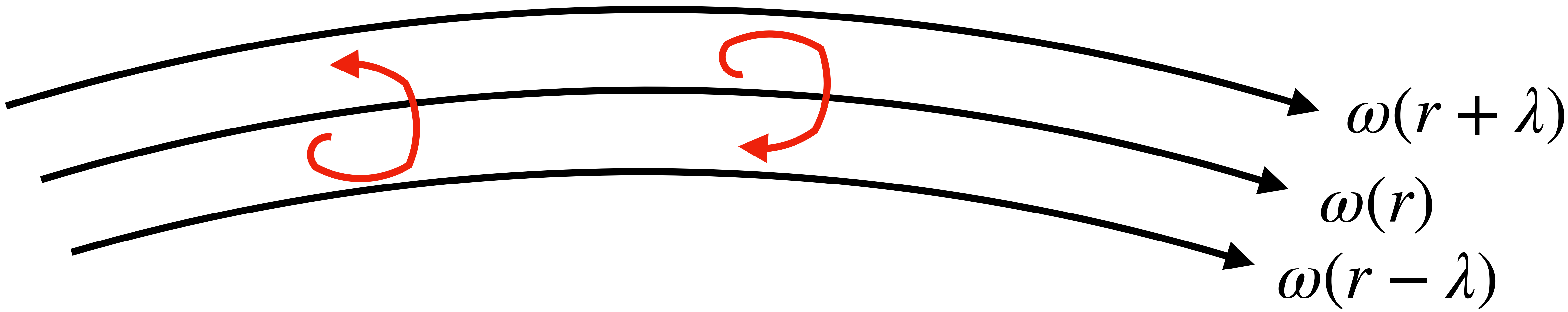

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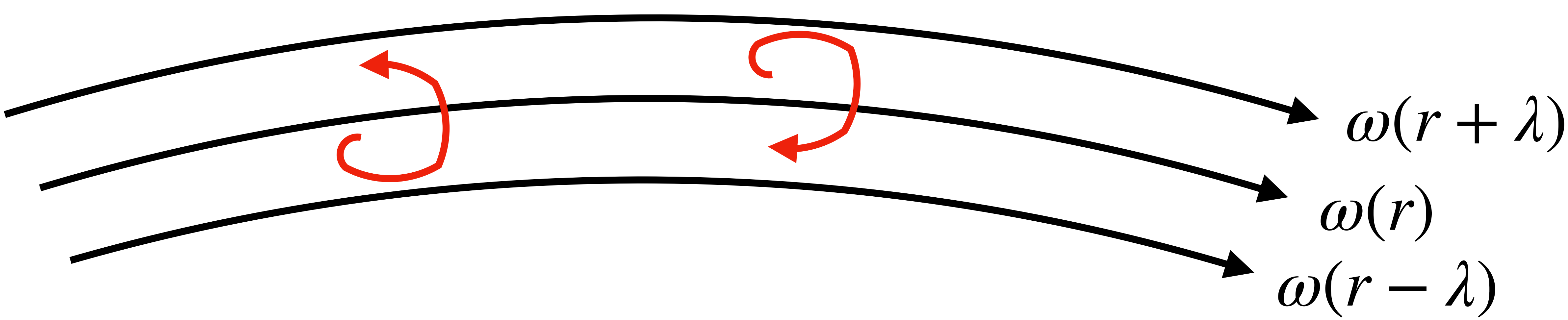
- No net mass transfer: mass in = mass out.
- Therefore angular momentum transfer, since particles from the inner ring have larger angular velocity around BH than those from outer ring.
- In other words: viscosity (~friction) *slows down* the inner ring and *speeds up* the outer ring
- i.e. angular momentum transported outwards, material can spiral towards the BH.

Angular Momentum Transport

So what is causing the turbulent motions (~viscosity)?



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  $\bar{v} \sim c_s$ = sound speed $\lambda \sim$ mean free path of particles

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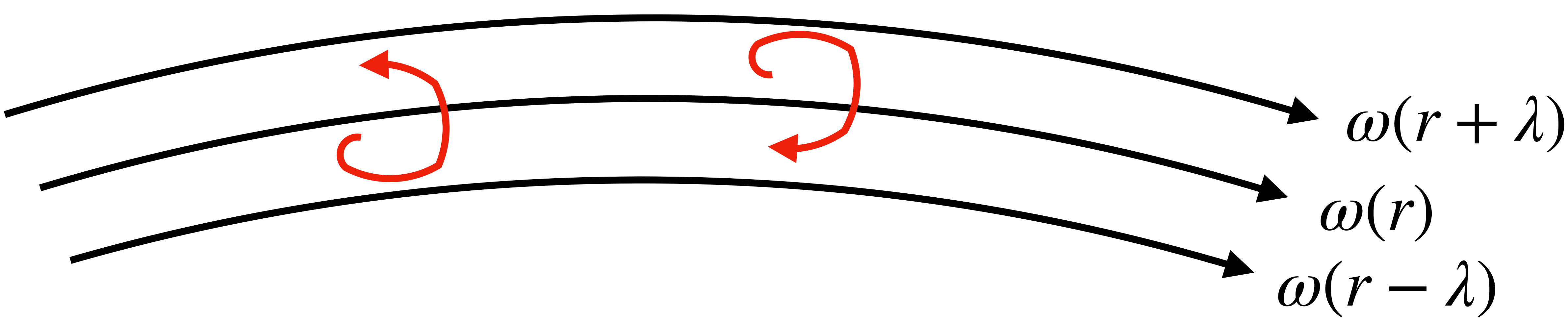

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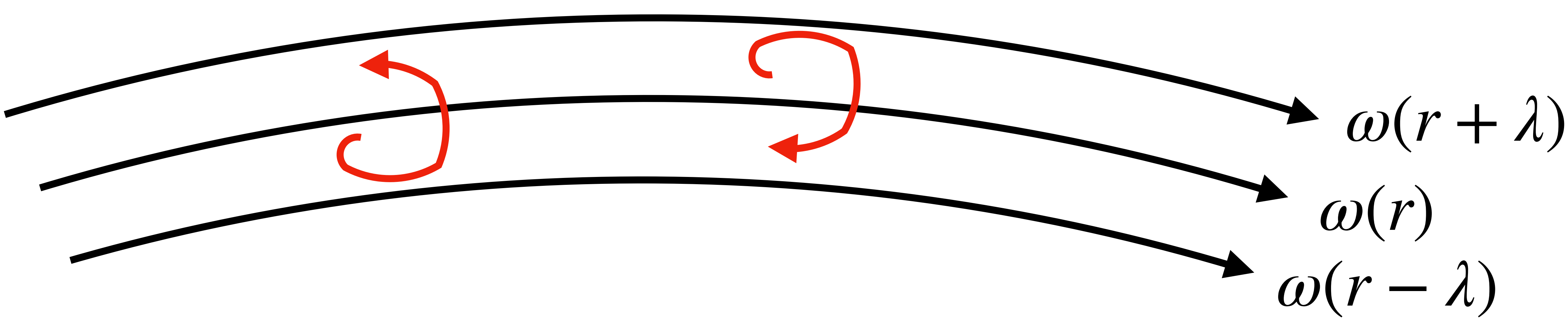

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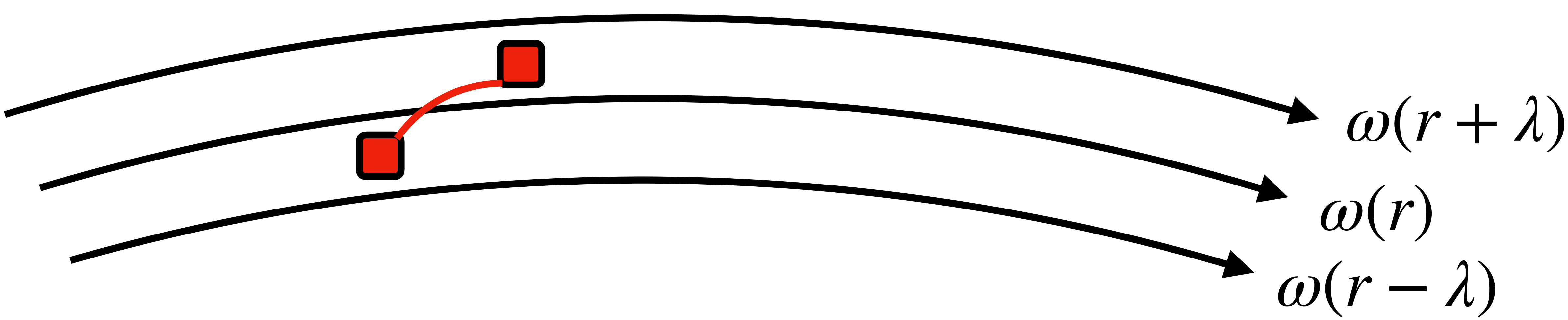

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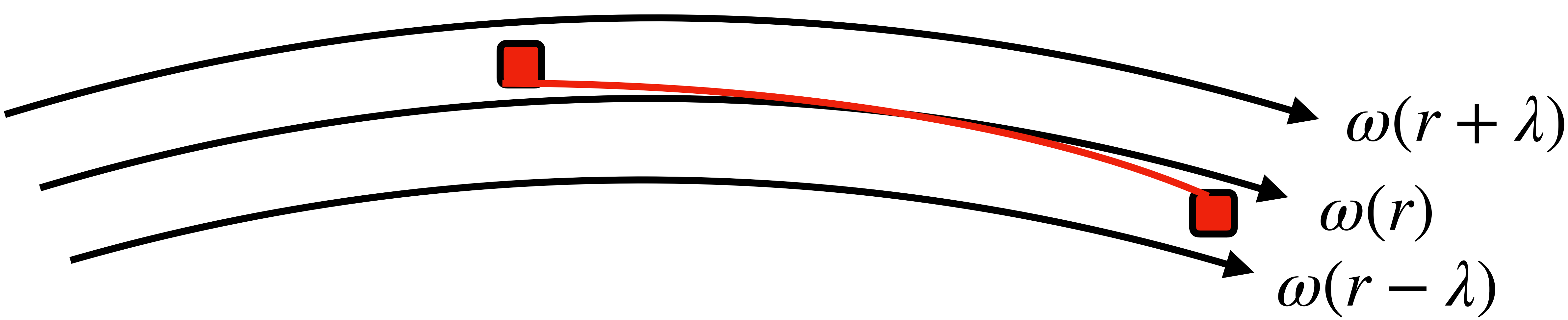

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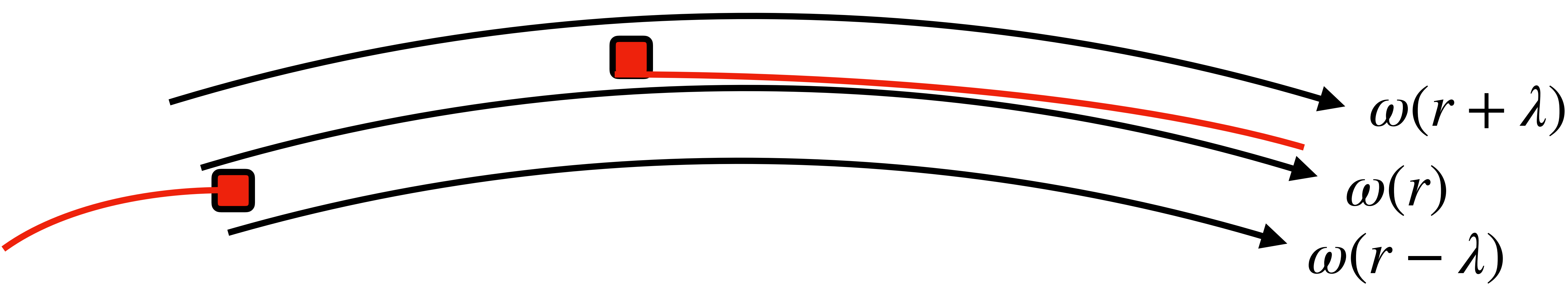
- Effective viscosity must be provided by <u>*hydrodynamic turbulence*</u>.
- The magneto-rotational instability provides the required viscosity: B-field lines connect parcels of gas, differential rotation stretches distance between these parcels, causes field lines to become tangled and generate turbulence. Like winding an elastic band round and round the disc.



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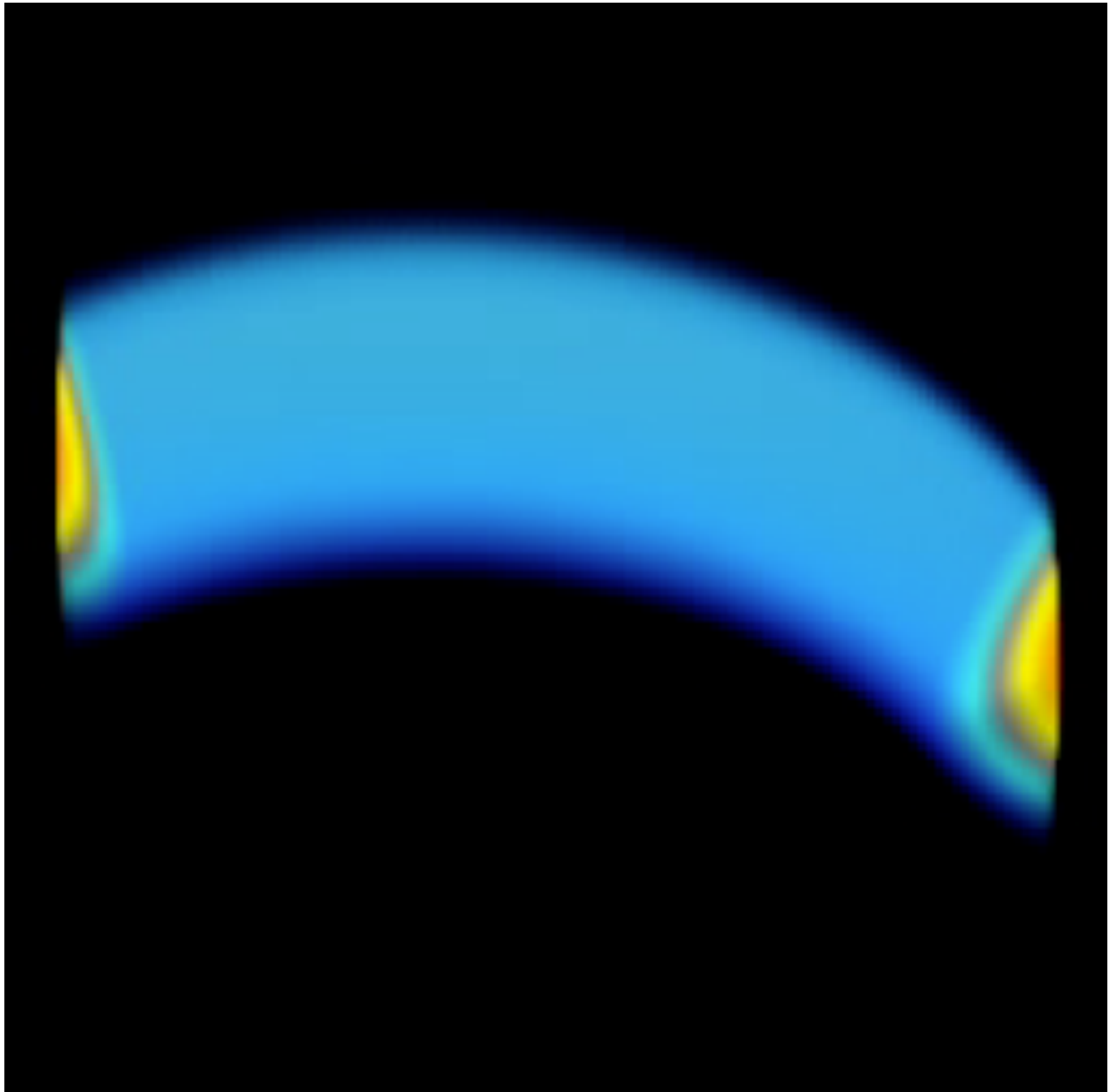

Krolik, de Villiers & Hawley

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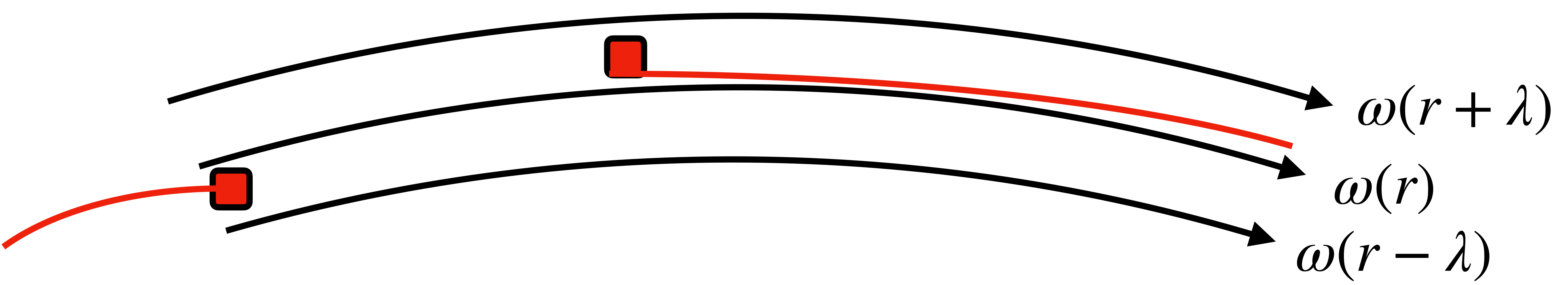
So what is causing the turbulent motions (~viscosity)?

- Shakura & Sunyaev (1973) famously assumed for the *kinematic viscosity*:

$$\nu = \alpha c_s h$$

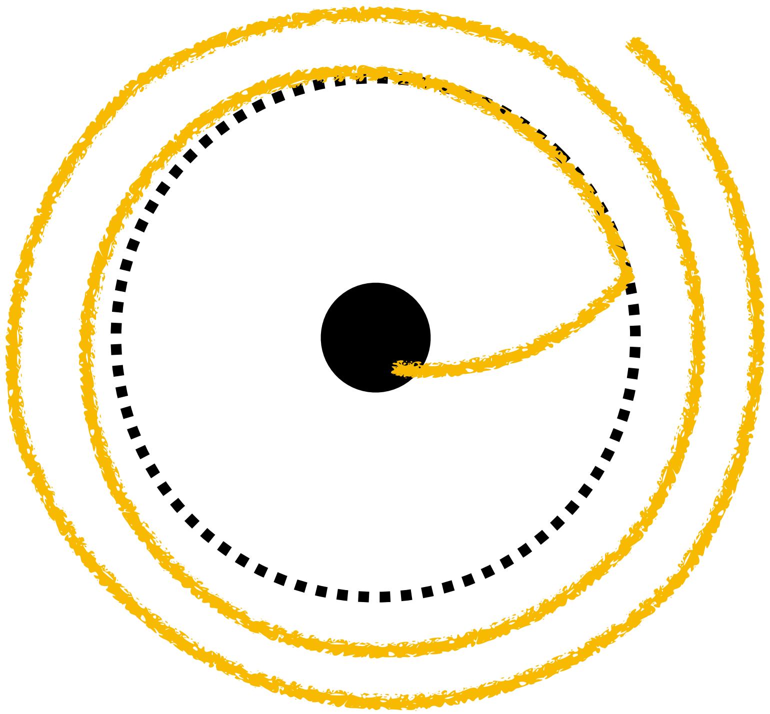
- α is a dimensionless constant: speed of eddies $\lesssim c_s$; length scale of eddies $\lesssim h$, therefore $\alpha \lesssim 1$.

- The "alpha-disc" model has been remarkably successful.



What happens at r_{isco}?

- Inside of ISCO, no stable orbits so material is in free fall.



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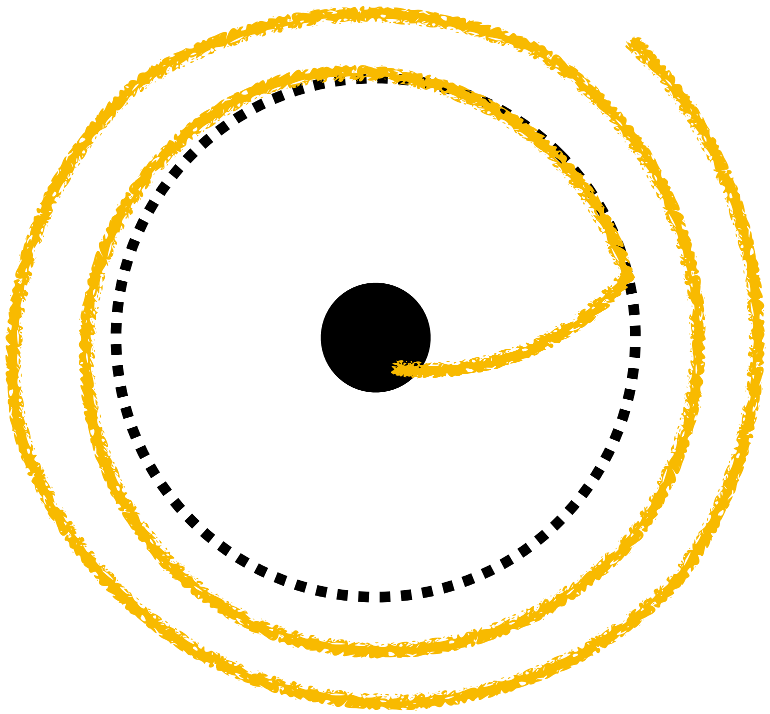

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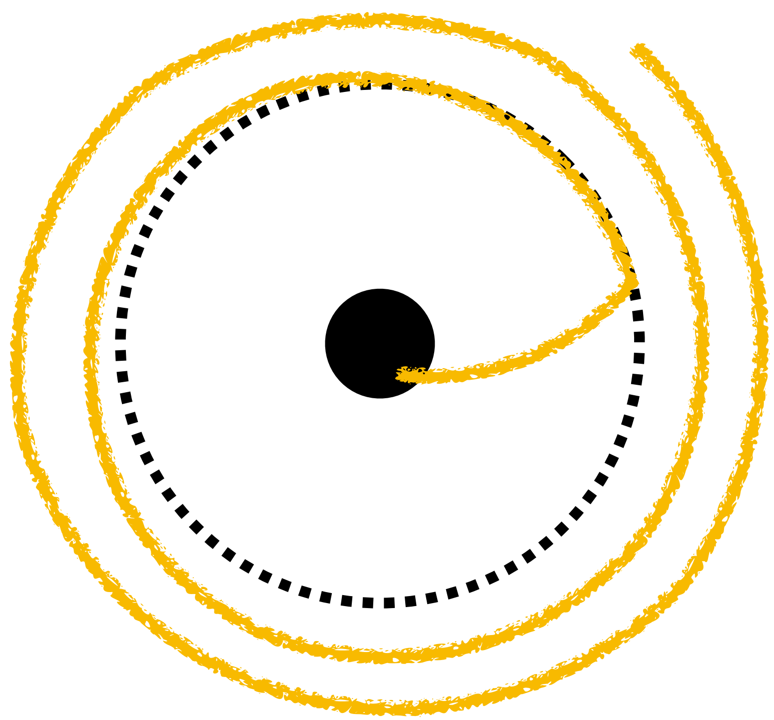

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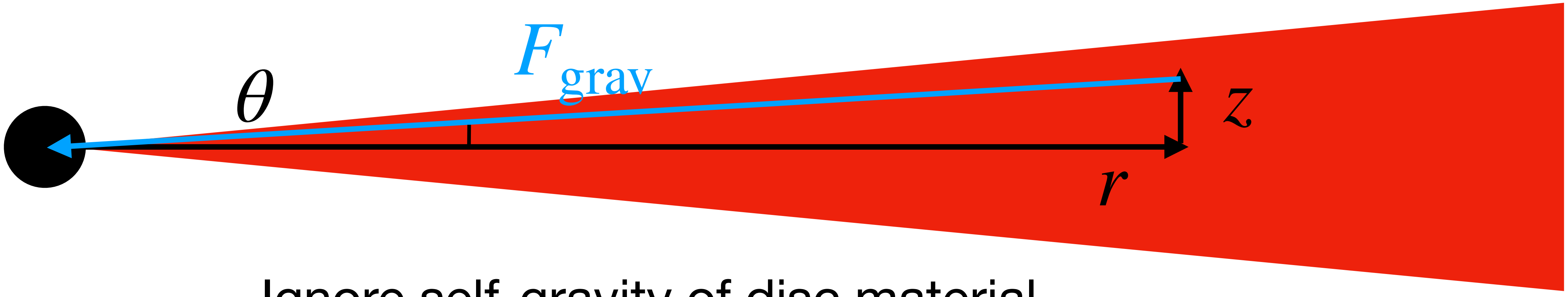
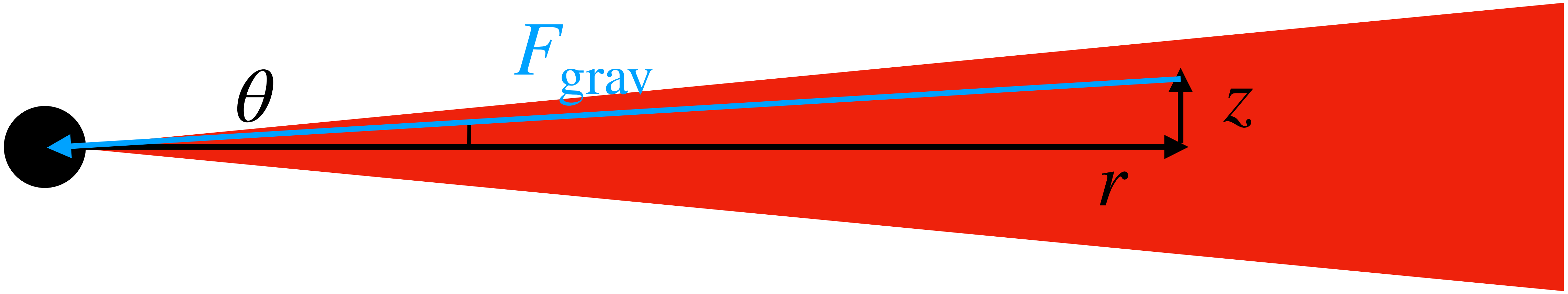
- Therefore lower spin supermassive black holes can presumably grow faster than higher spin ones.

Disc vertical structure



Ignore self-gravity of disc material

Disc vertical structure



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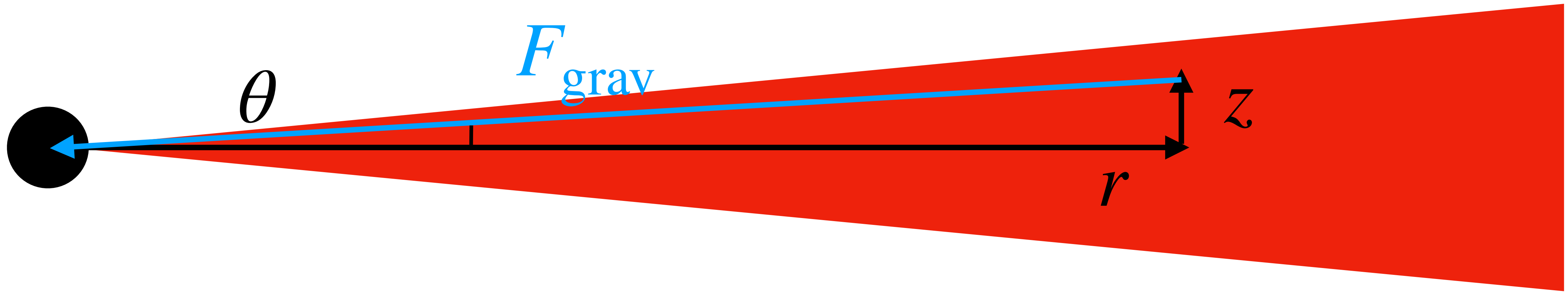

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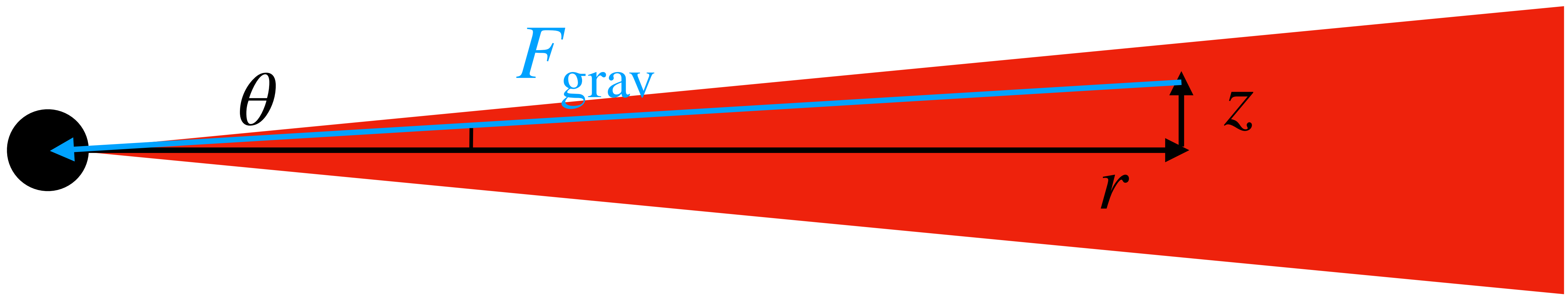

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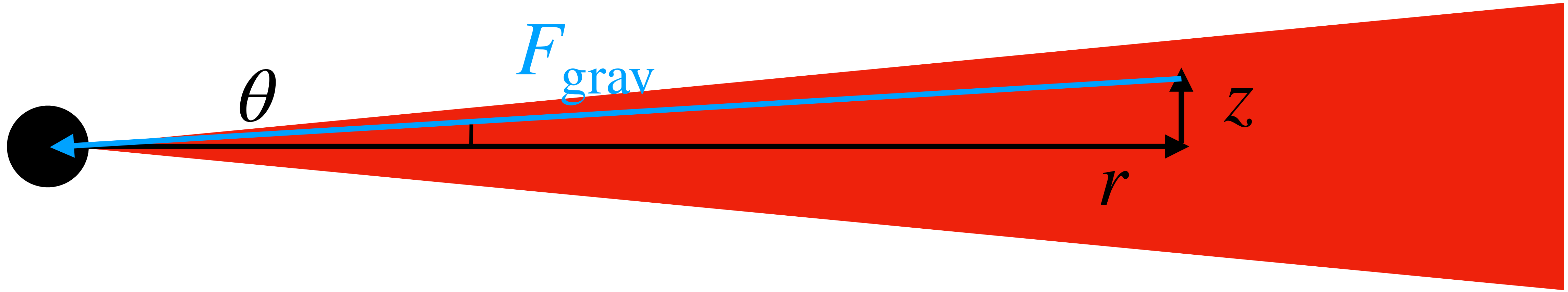

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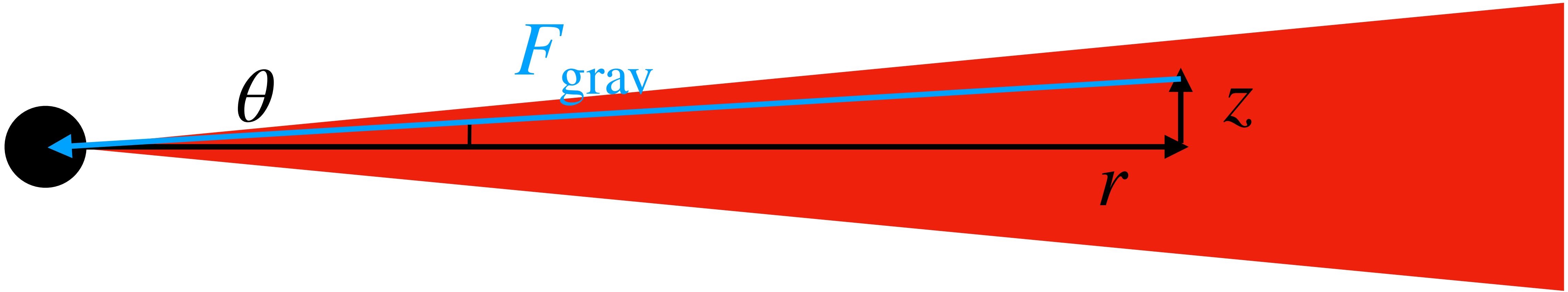

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$\frac{h}{r} \ll 1 \implies$ <u>*disc rotation is highly supersonic.*</u>

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- From Stefan-Boltzmann law, the temperature is:

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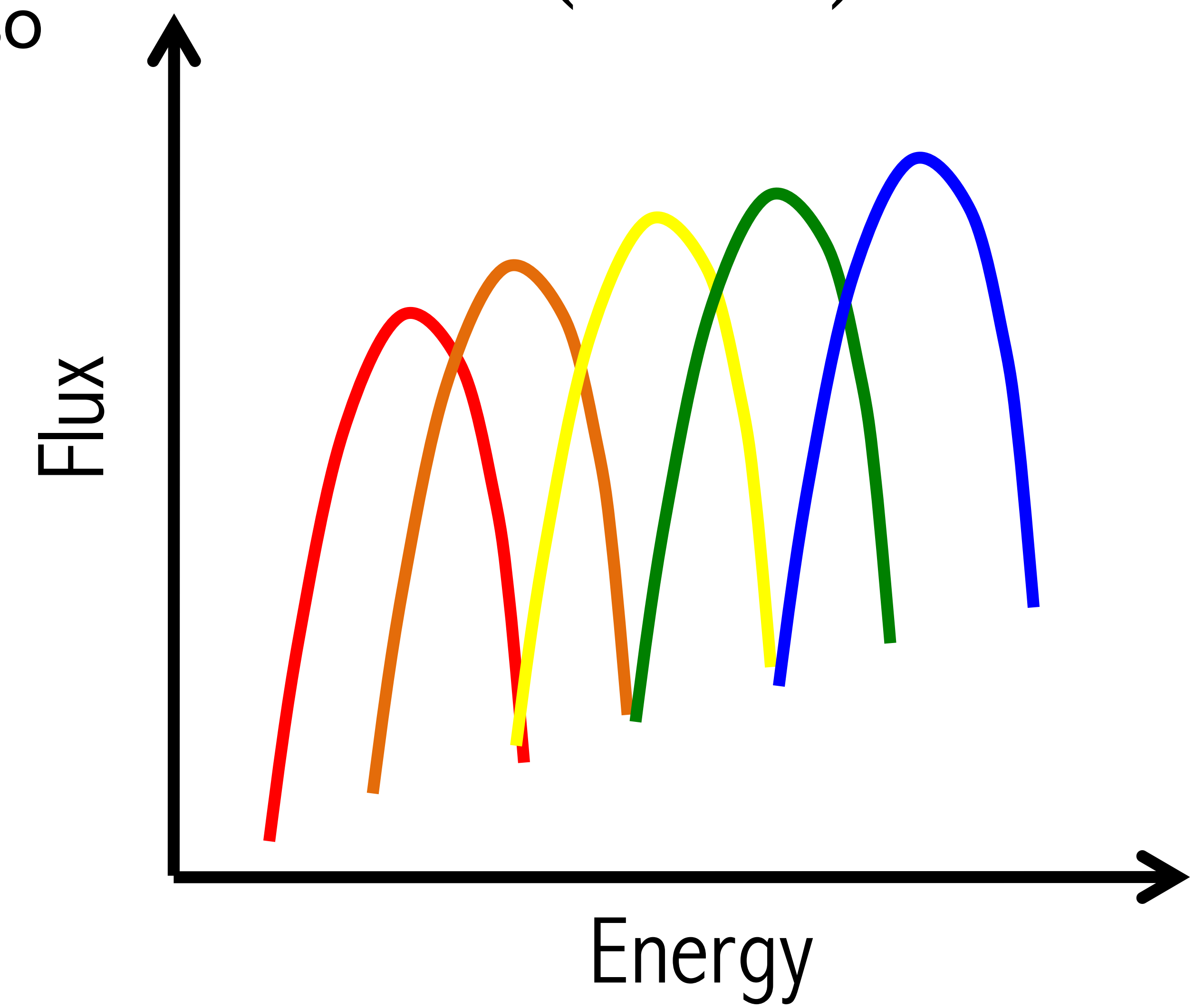

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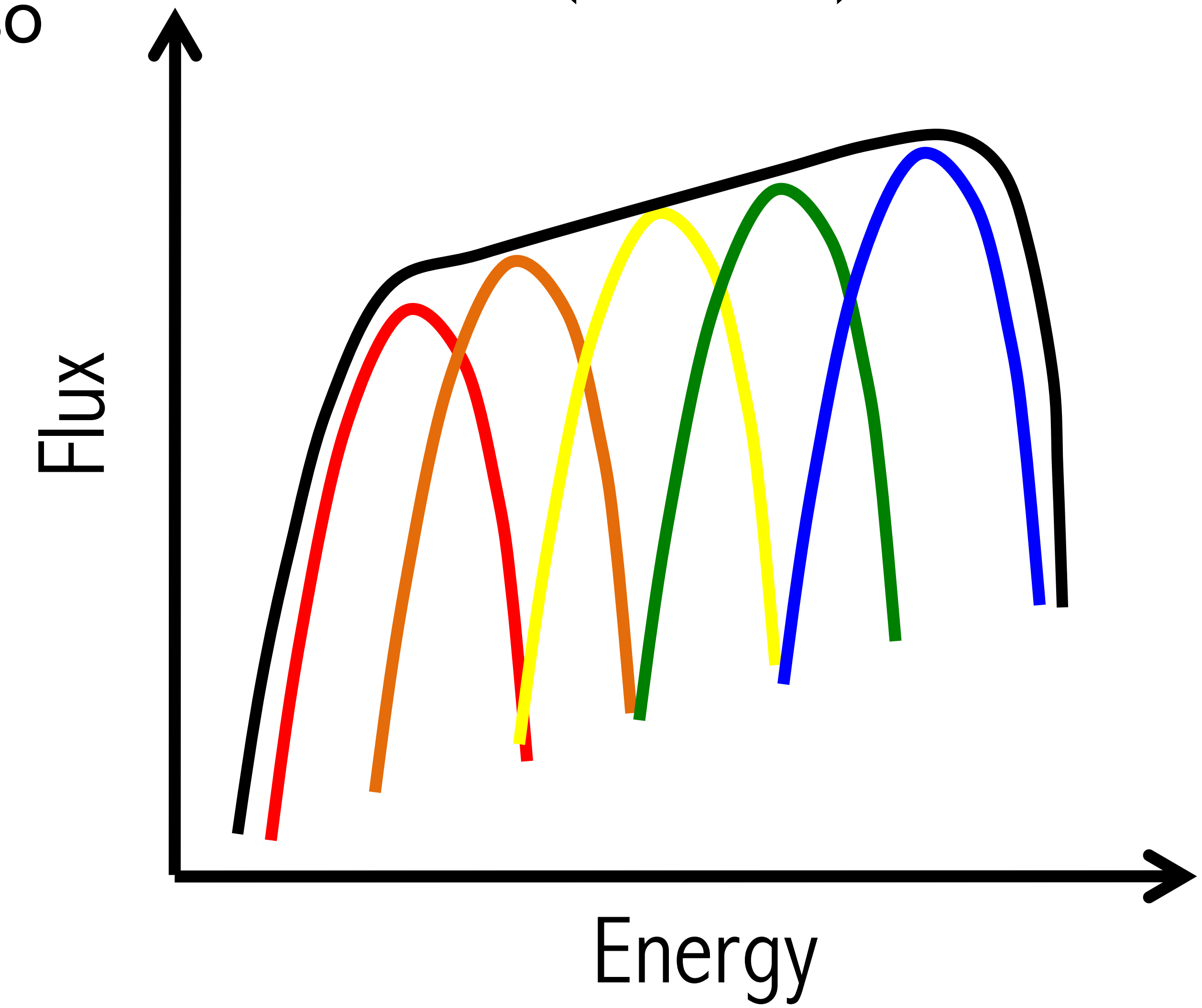

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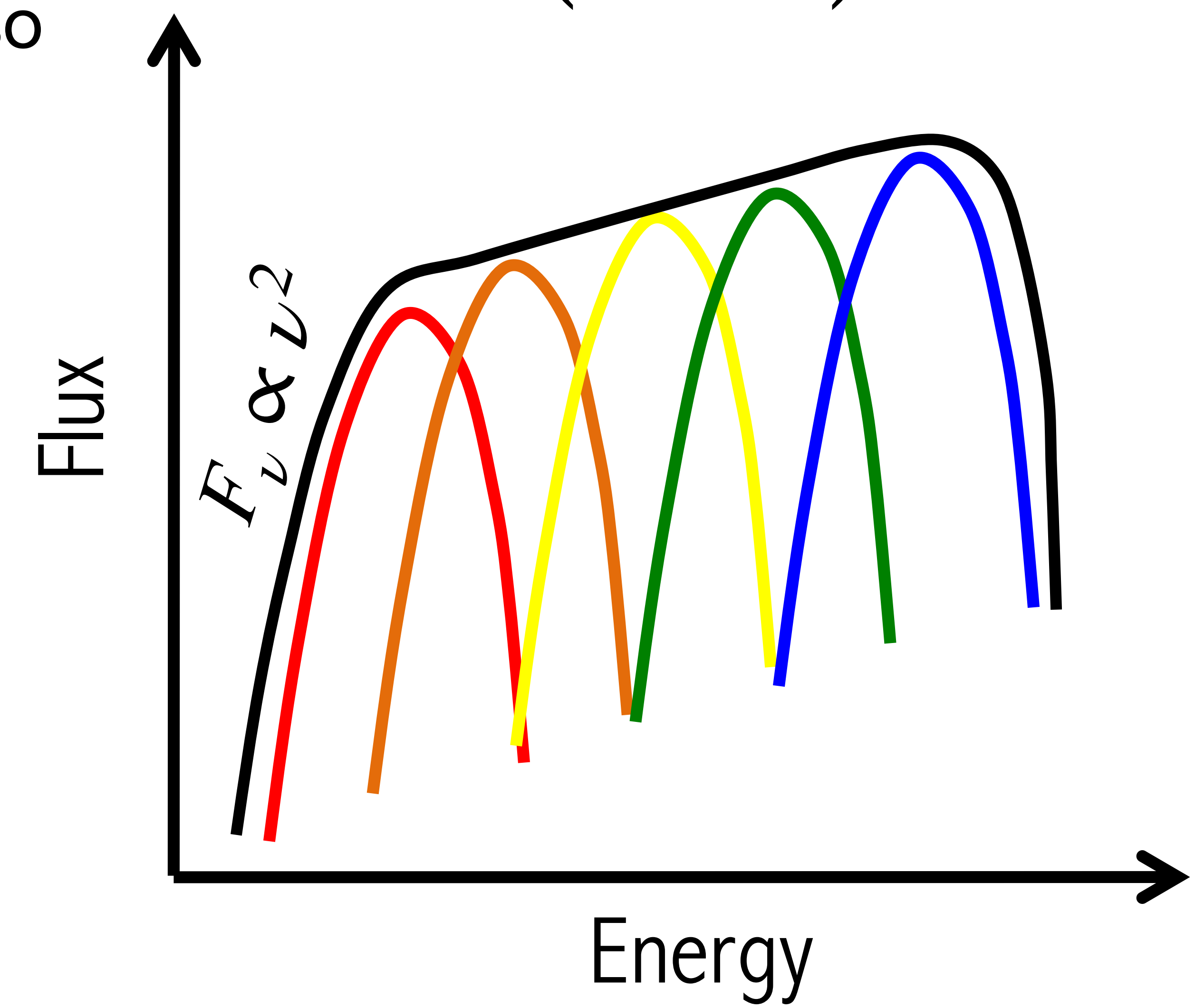

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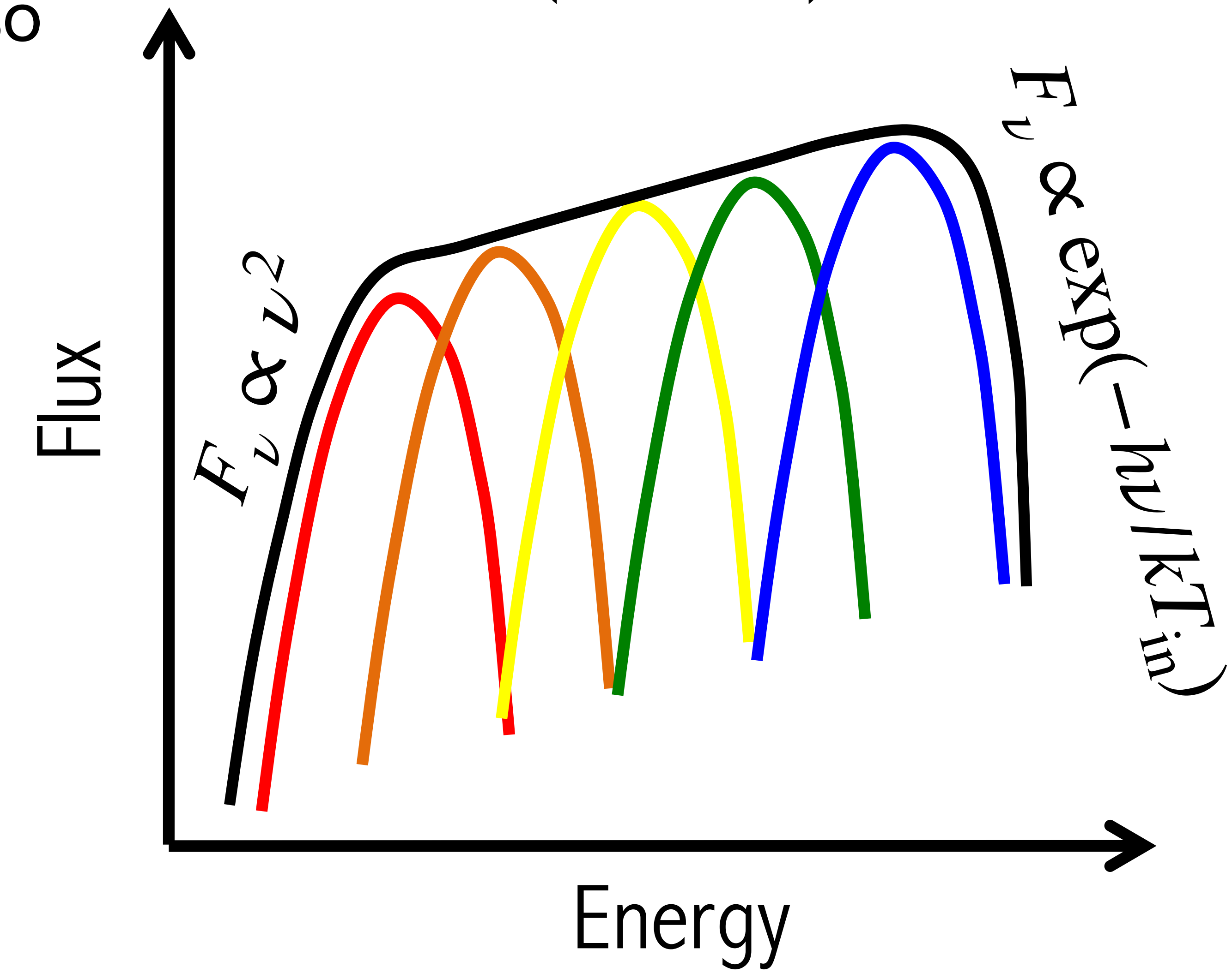
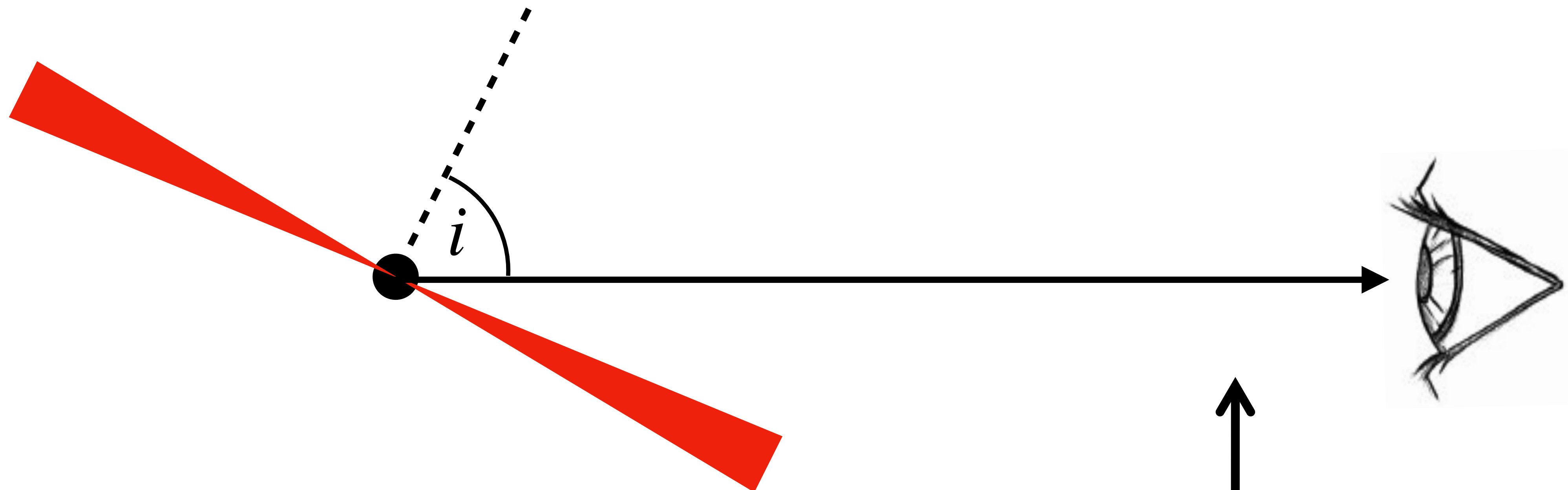
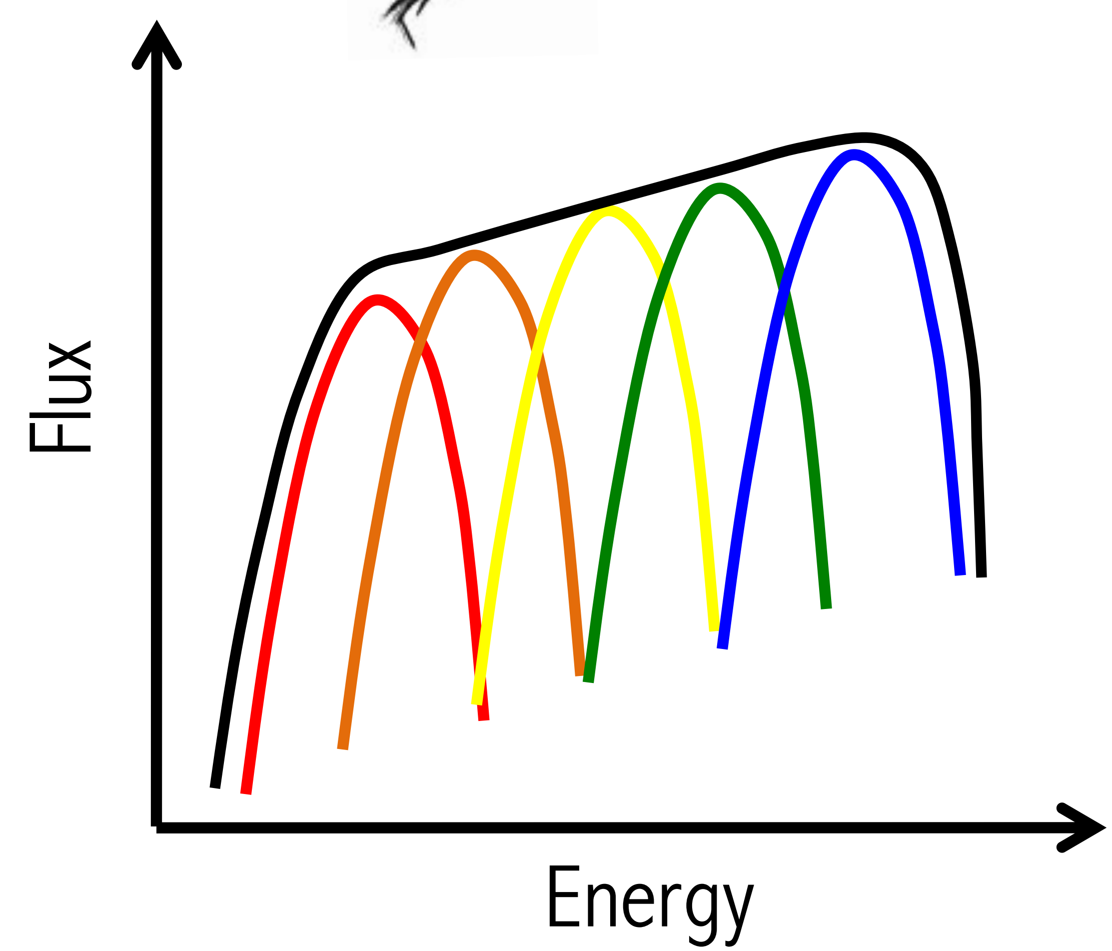


Disc Spectrum

Intermediate frequencies:

$$F_\nu = \int_{r_{\rm in}}^{r_{\rm out}} B_\nu[T(r)]d\Omega(r) \approx \int_{r_{\rm in}}^{r_{\rm out}} B_\nu[T(r)]\frac{2\pi r \cos i \ dr}{D^2}$$





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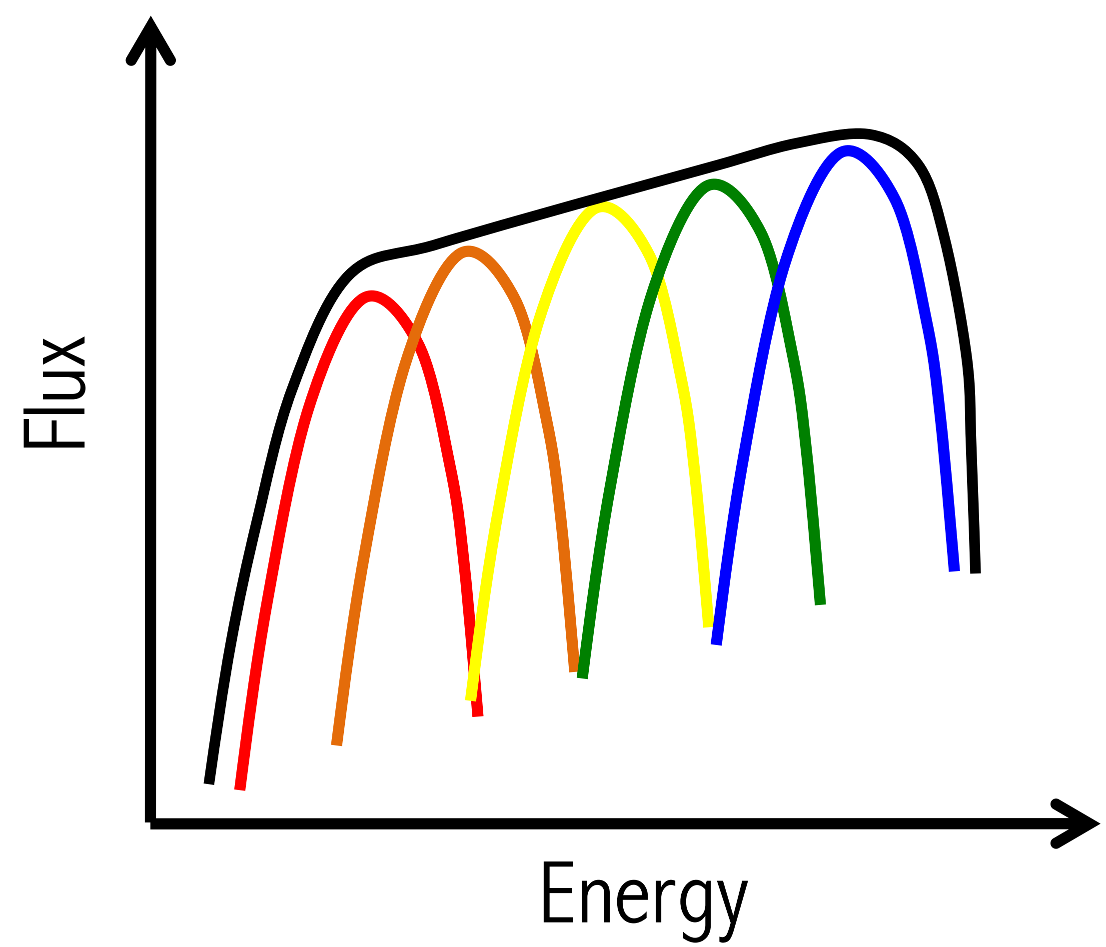

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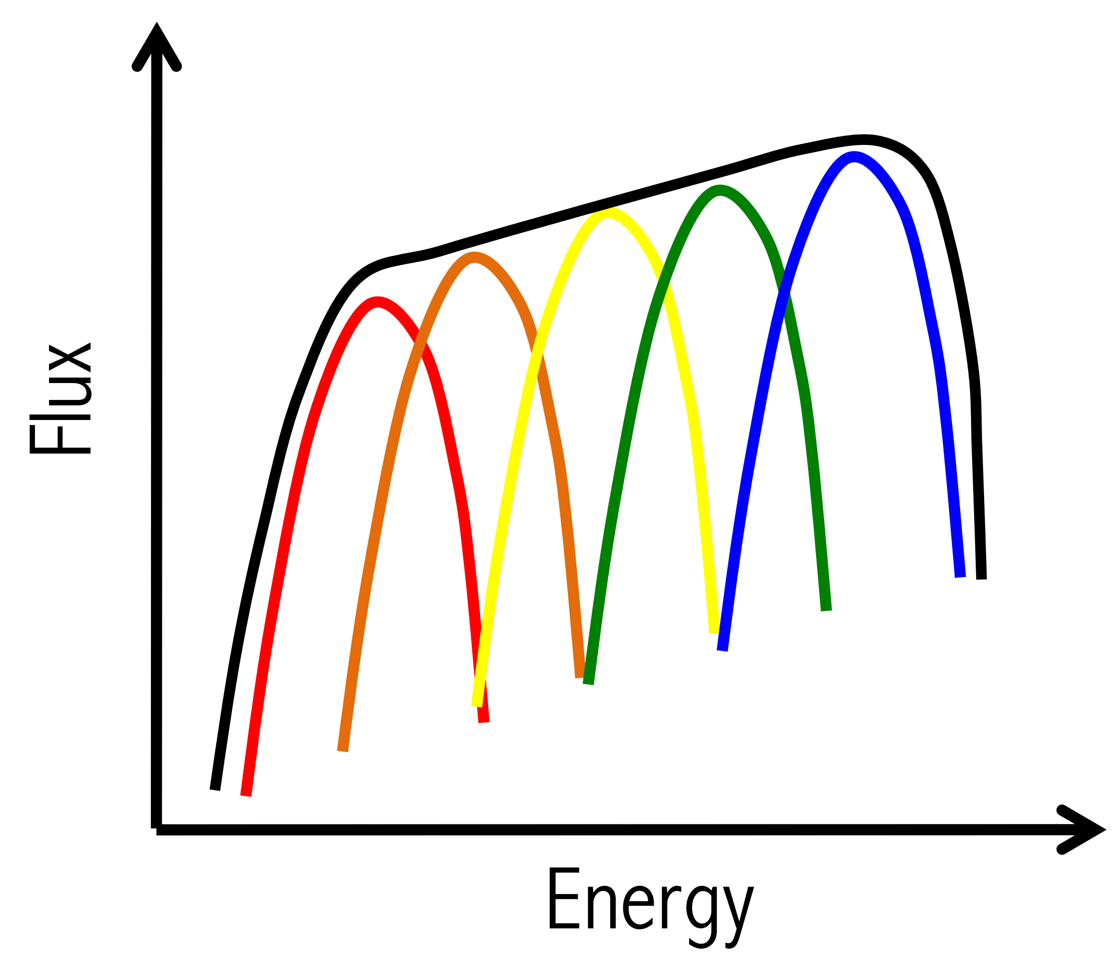

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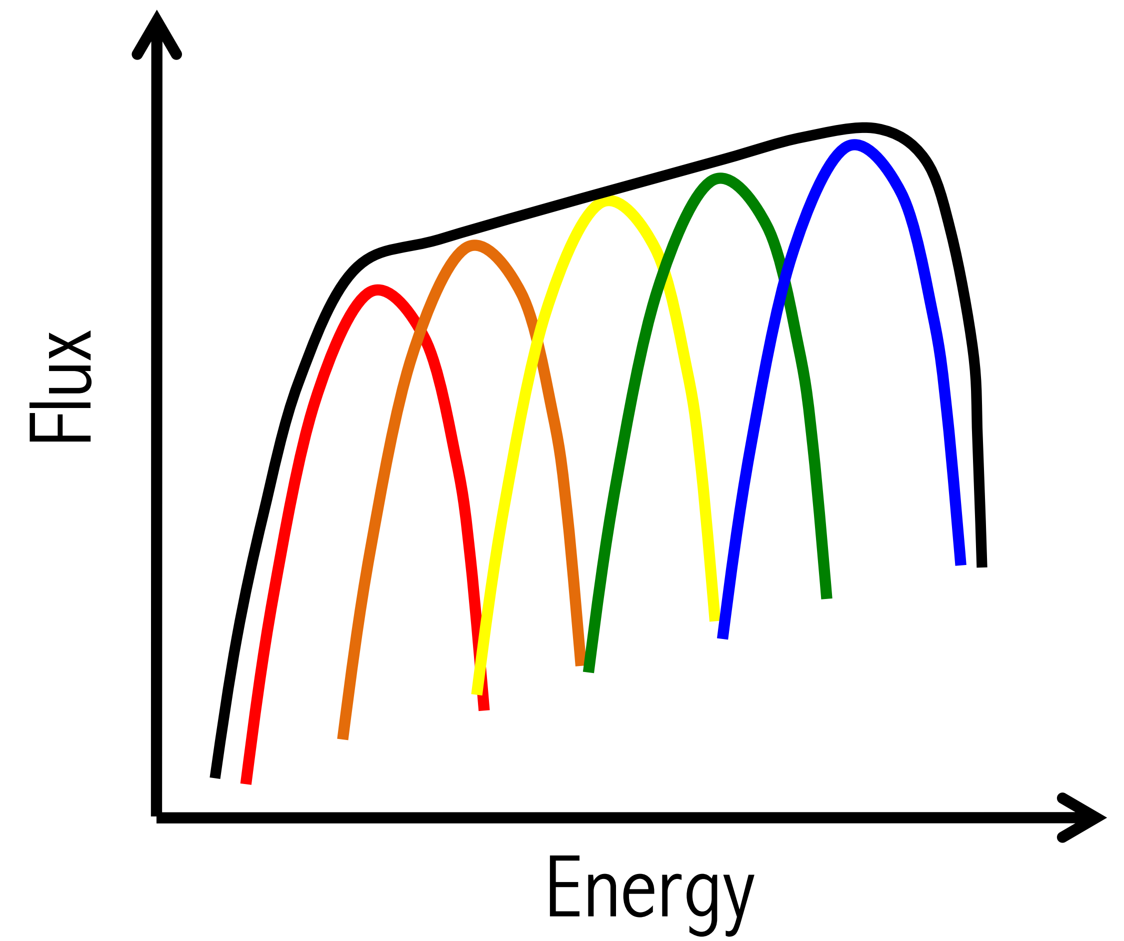

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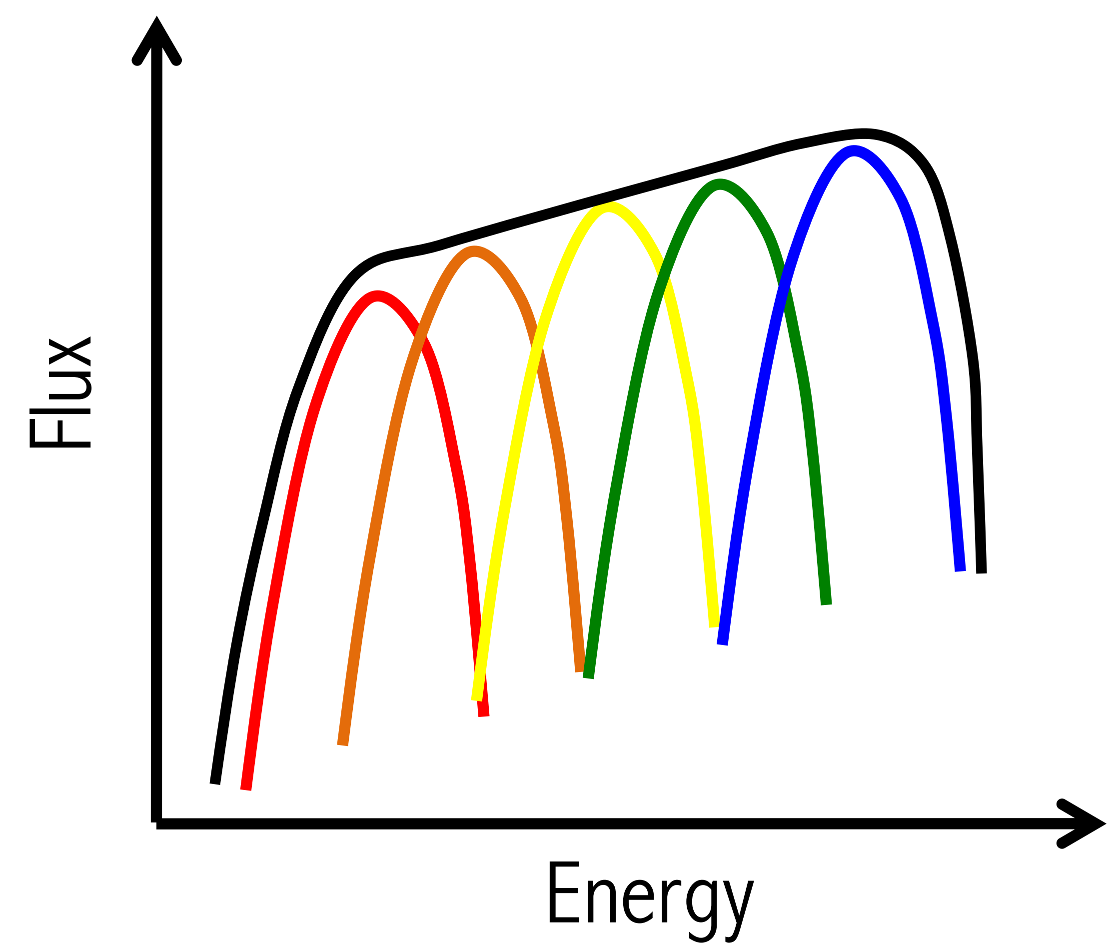

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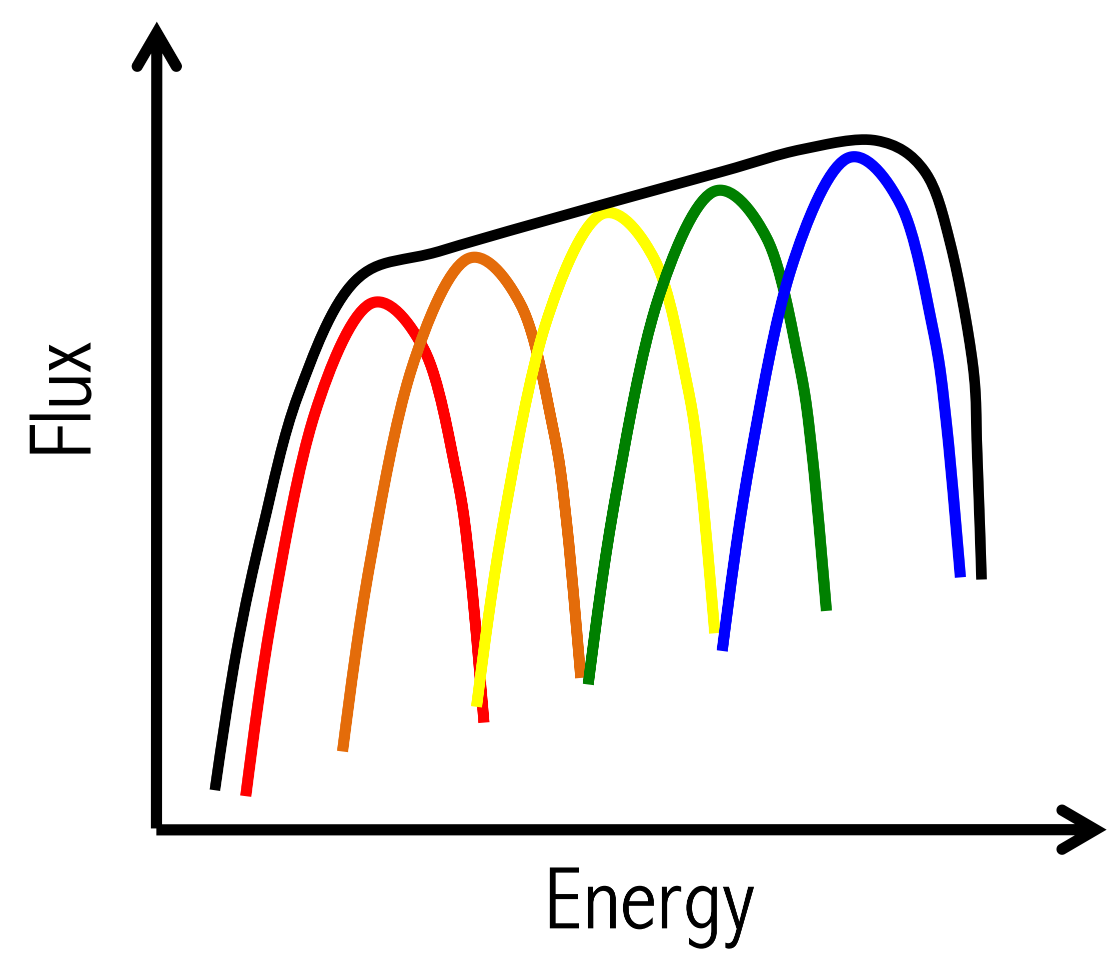

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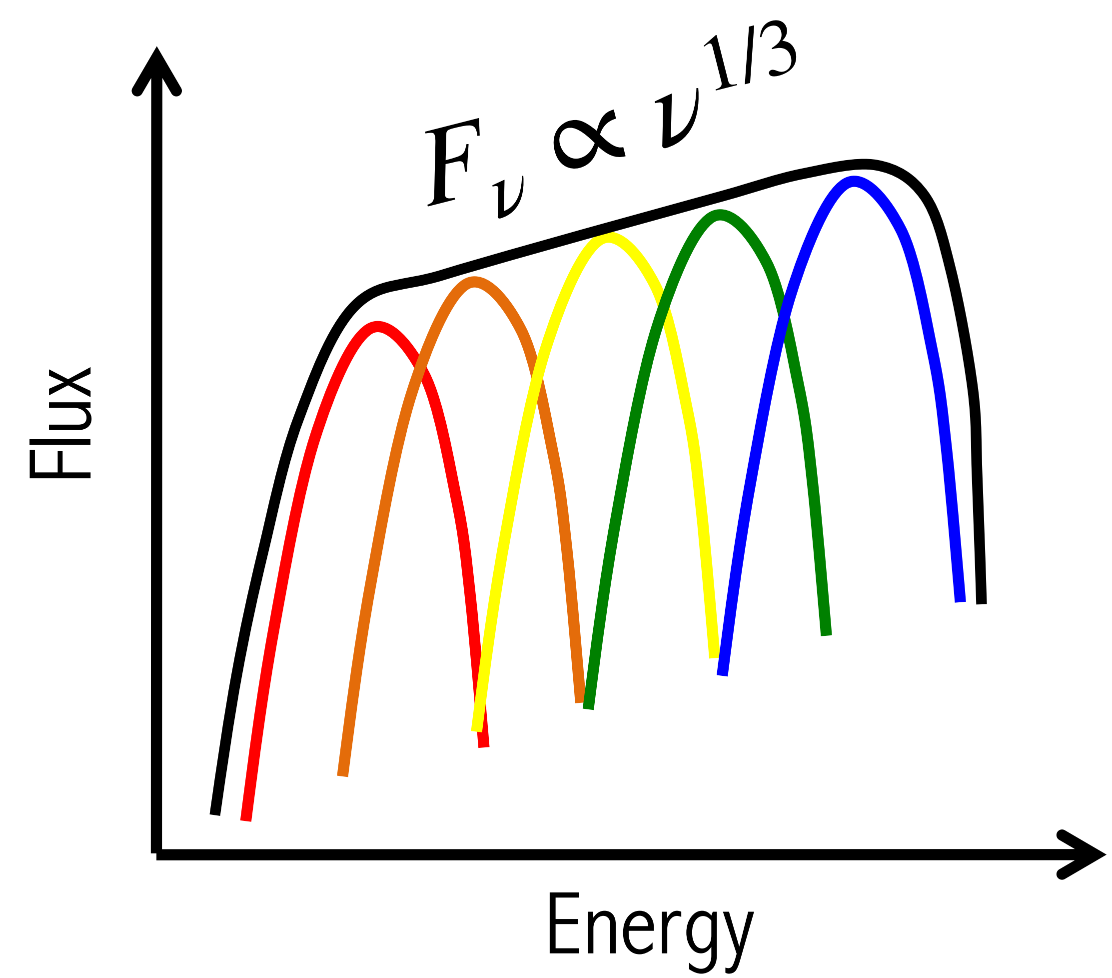

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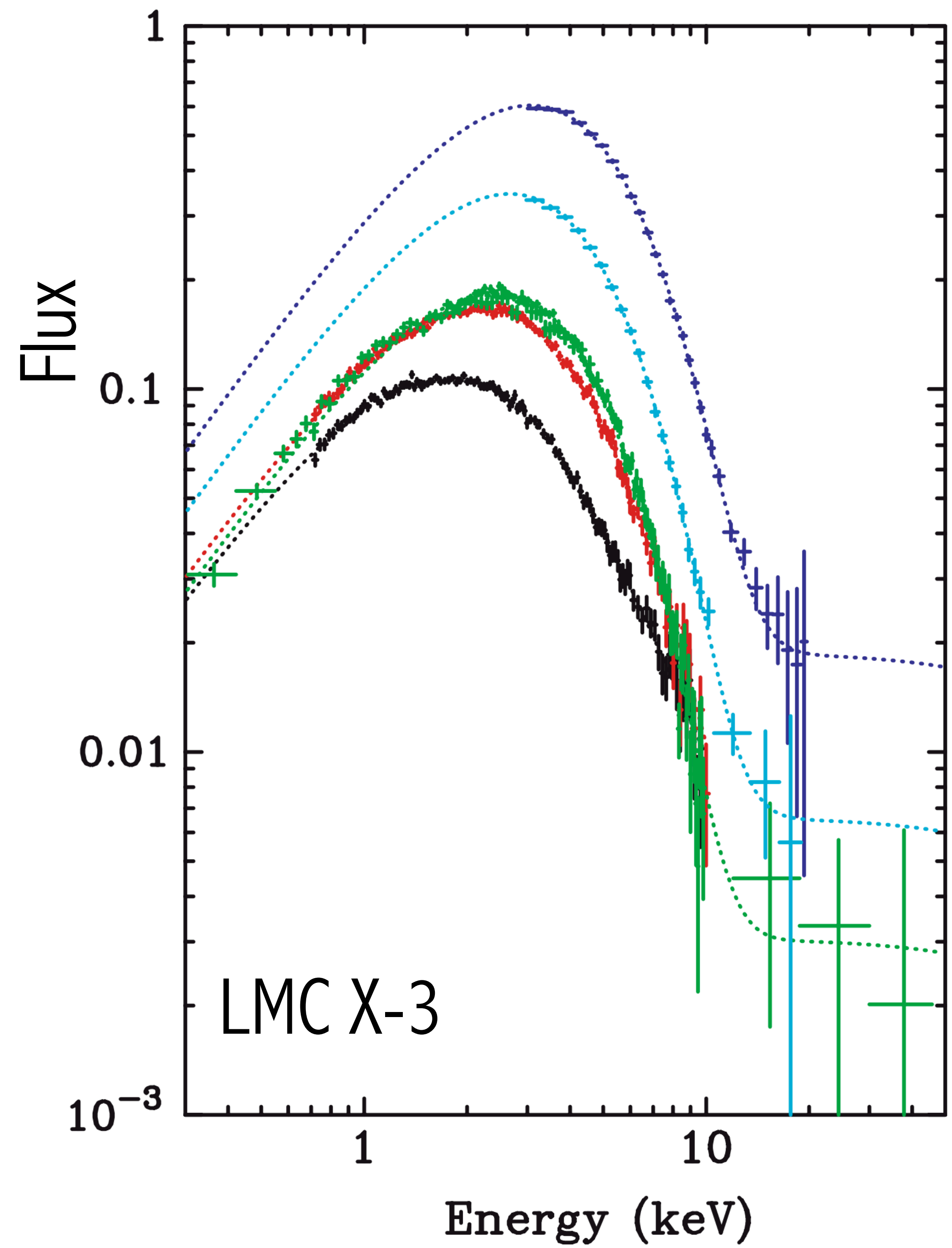

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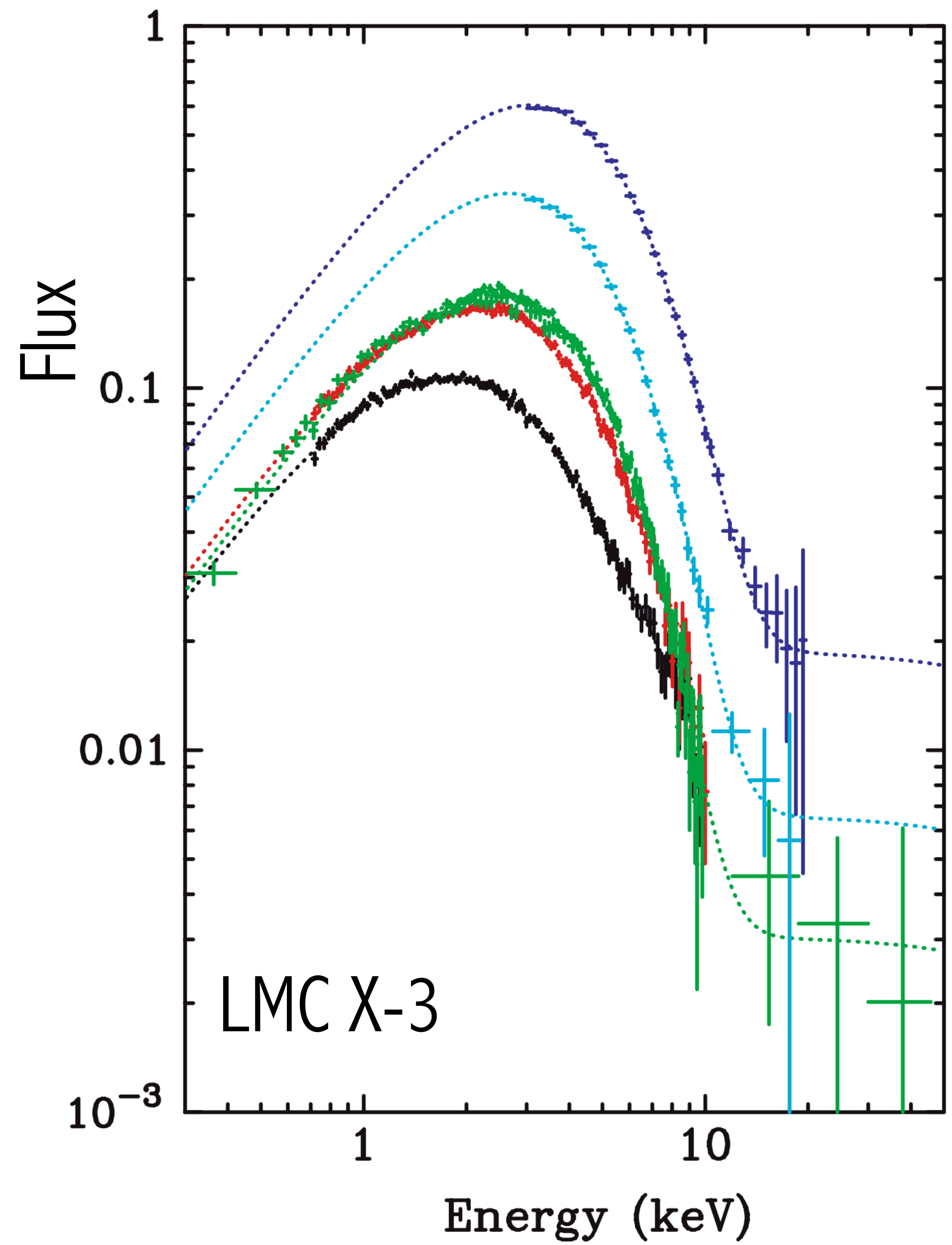

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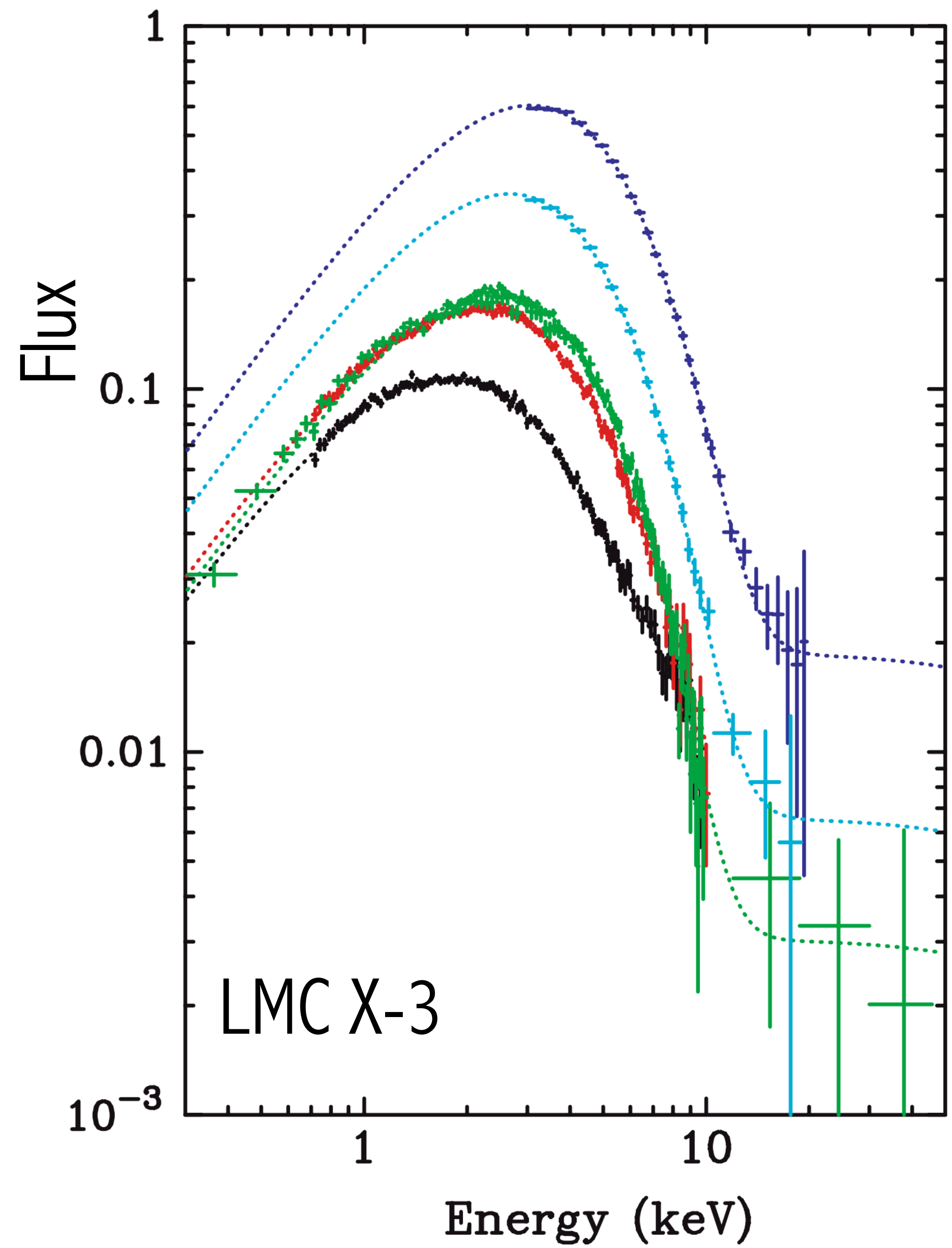

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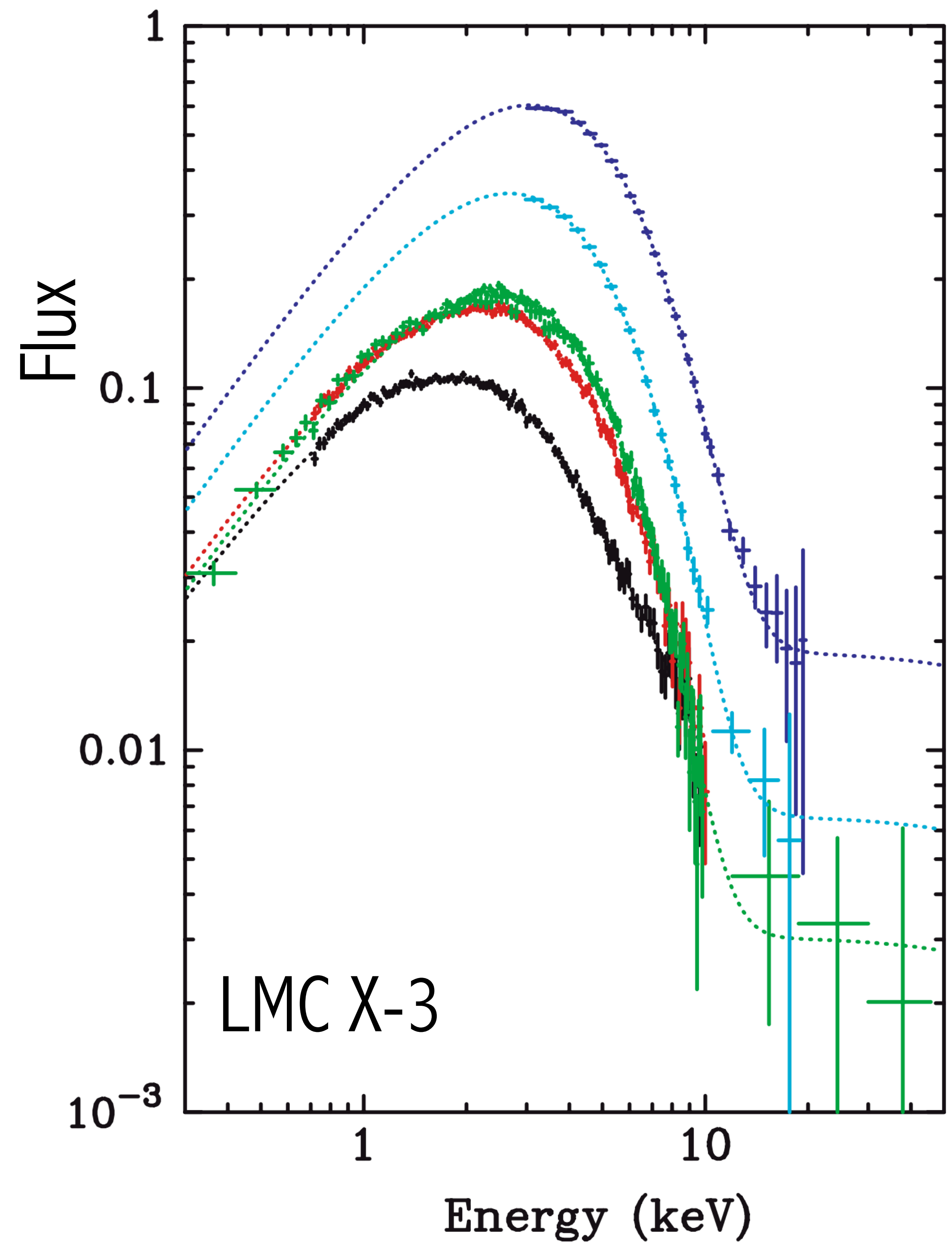

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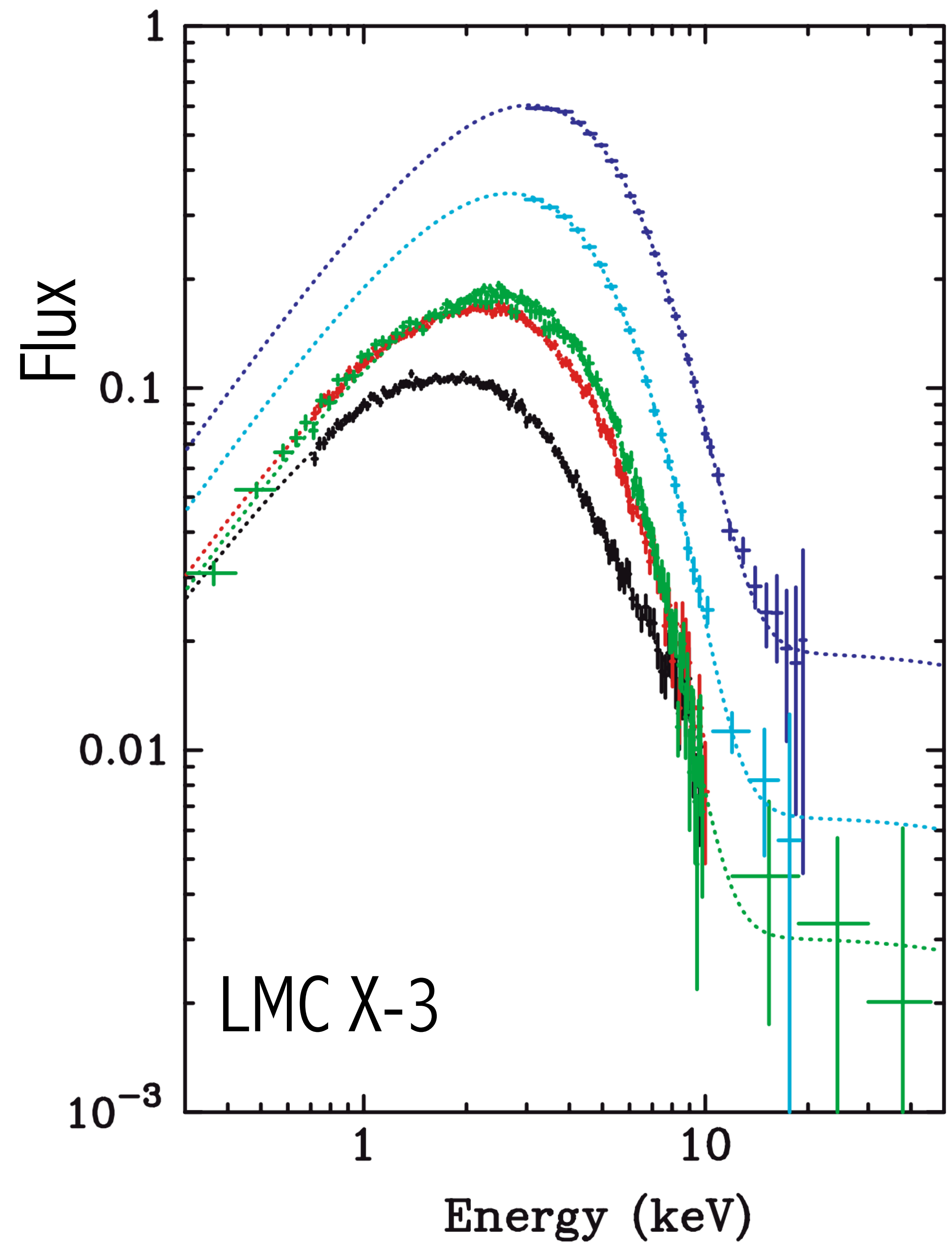

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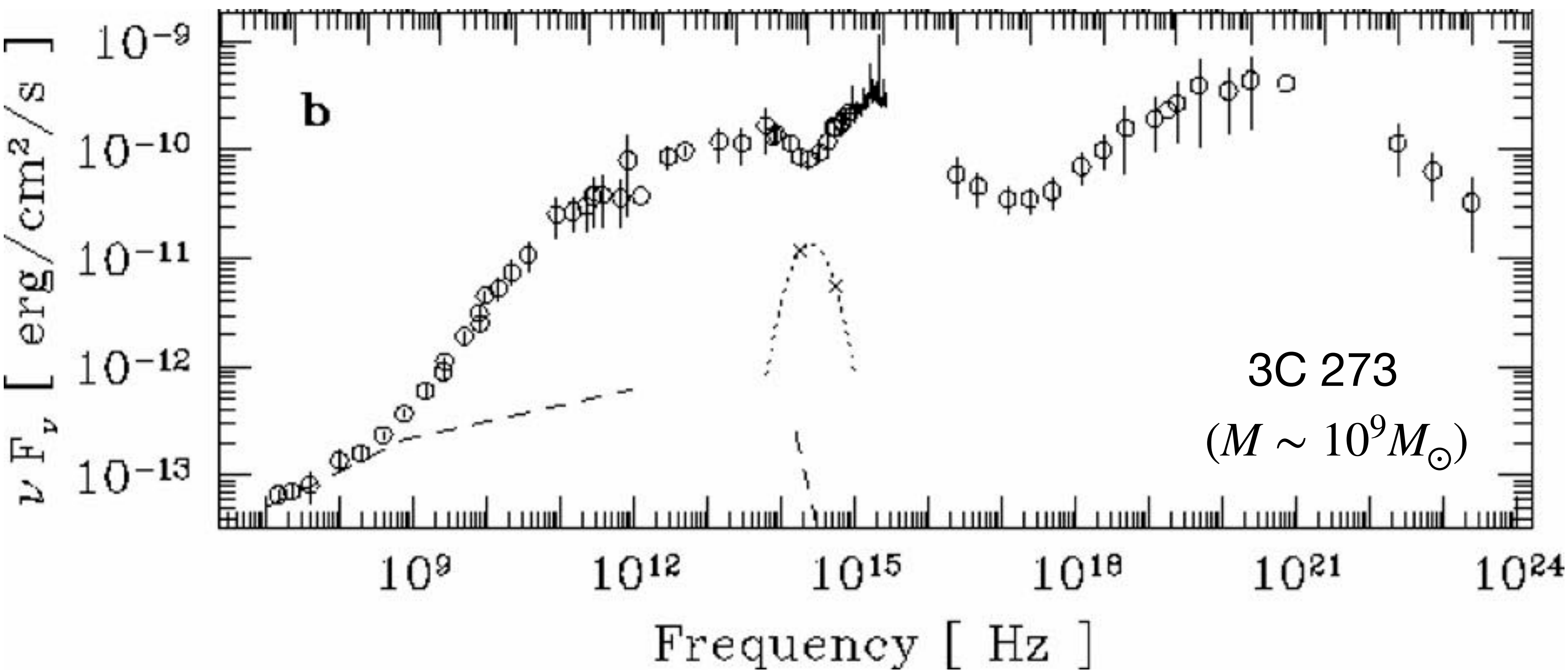

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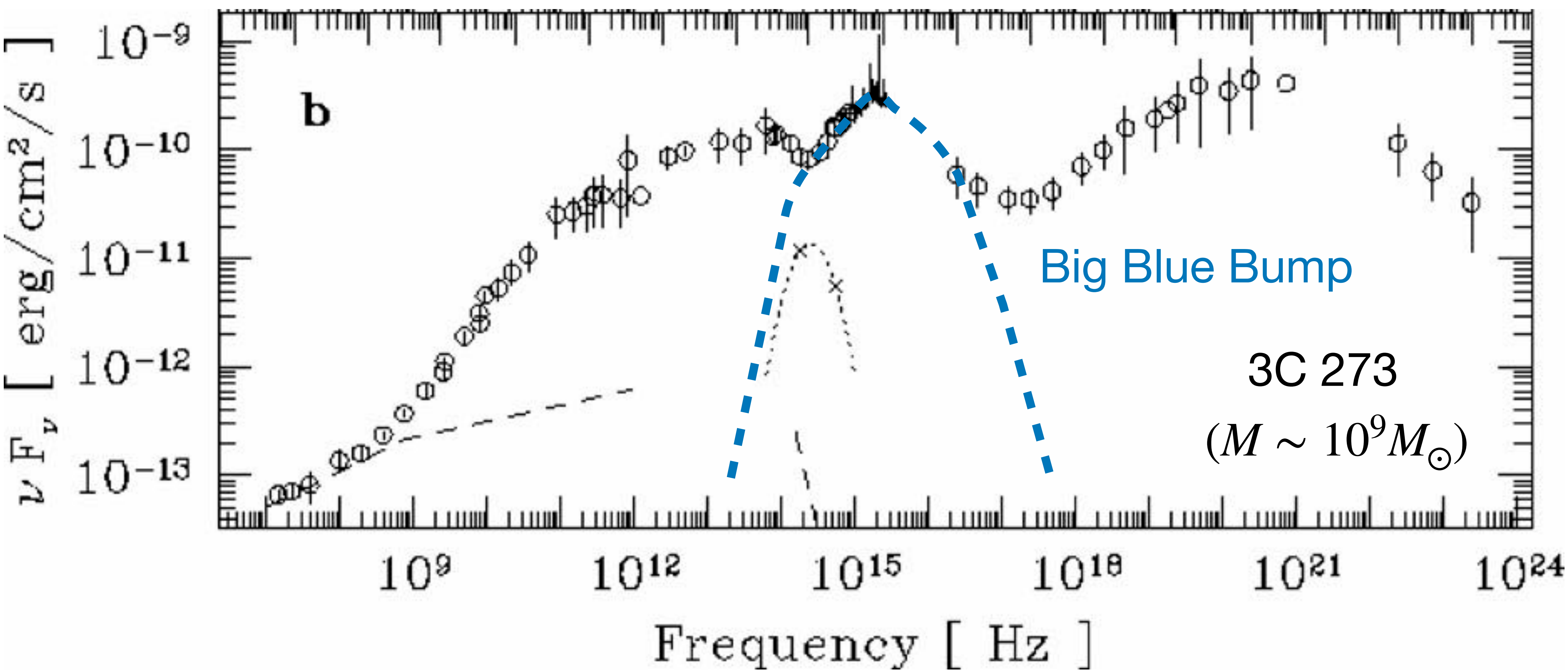

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- But see lots of other stuff, some of which we will study over the following two lectures.