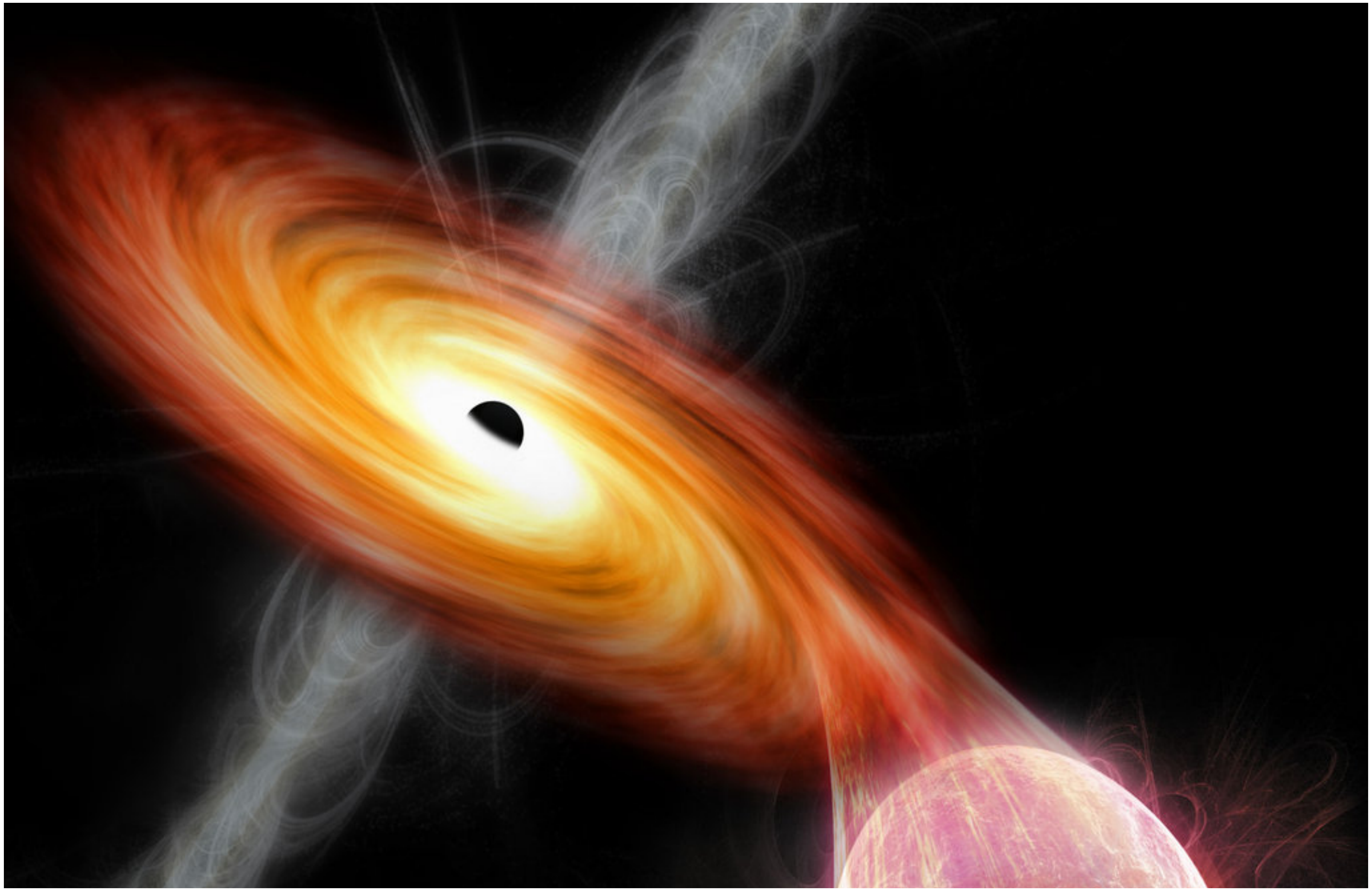
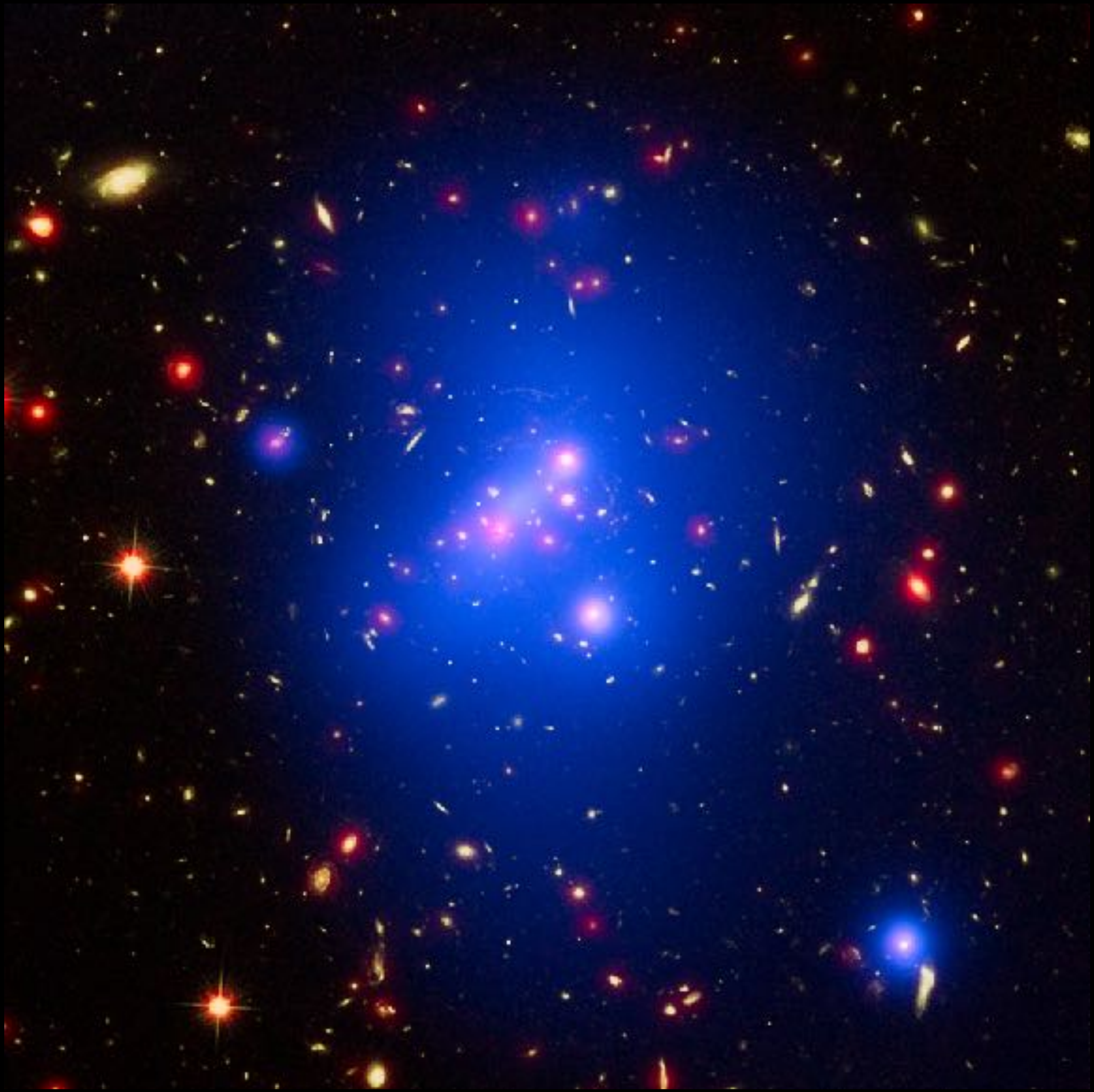
# High Energy Astrophysics

Dr. Adam Ingram

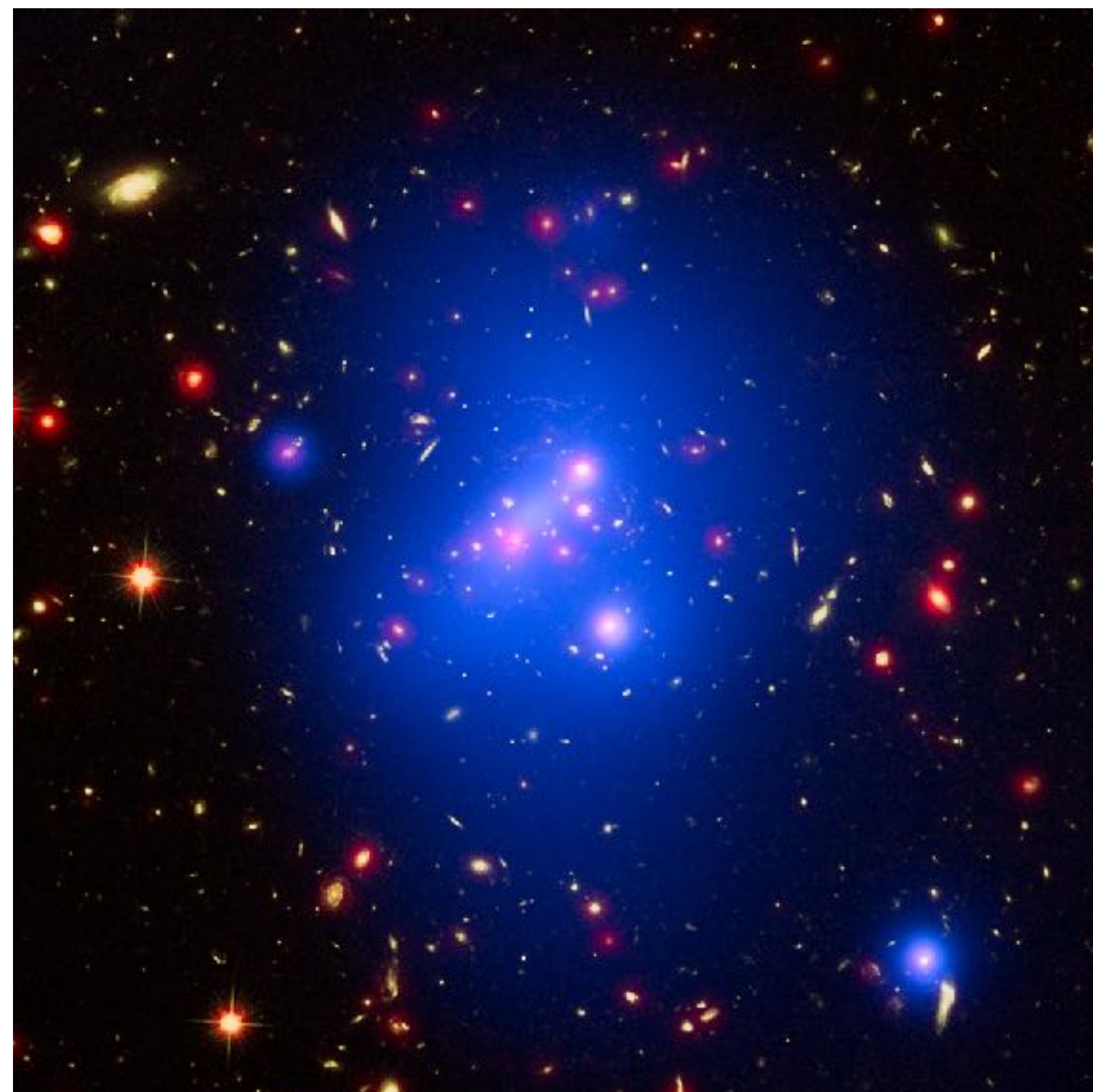


# Lecture 8
Galaxy Clusters



# Galaxy Clusters

- Largest gravitationally bound structures in the Universe.
- Intra Cluster Medium (ICM): hot gas in between the galaxies with characteristic temperature kT~1-15 keV [$kT=1$ keV => $T=1.16\times10^7$ K] — glows in X-rays (blue in picture).
- Dark matter mass ~10× ICM mass ~10× mass in stars.
- Typical total mass $\sim10^{14\text{-}15}\ M_\odot$.
- Typically ~100-1000 galaxies.
- Typical size ~2-10 Mpc across.

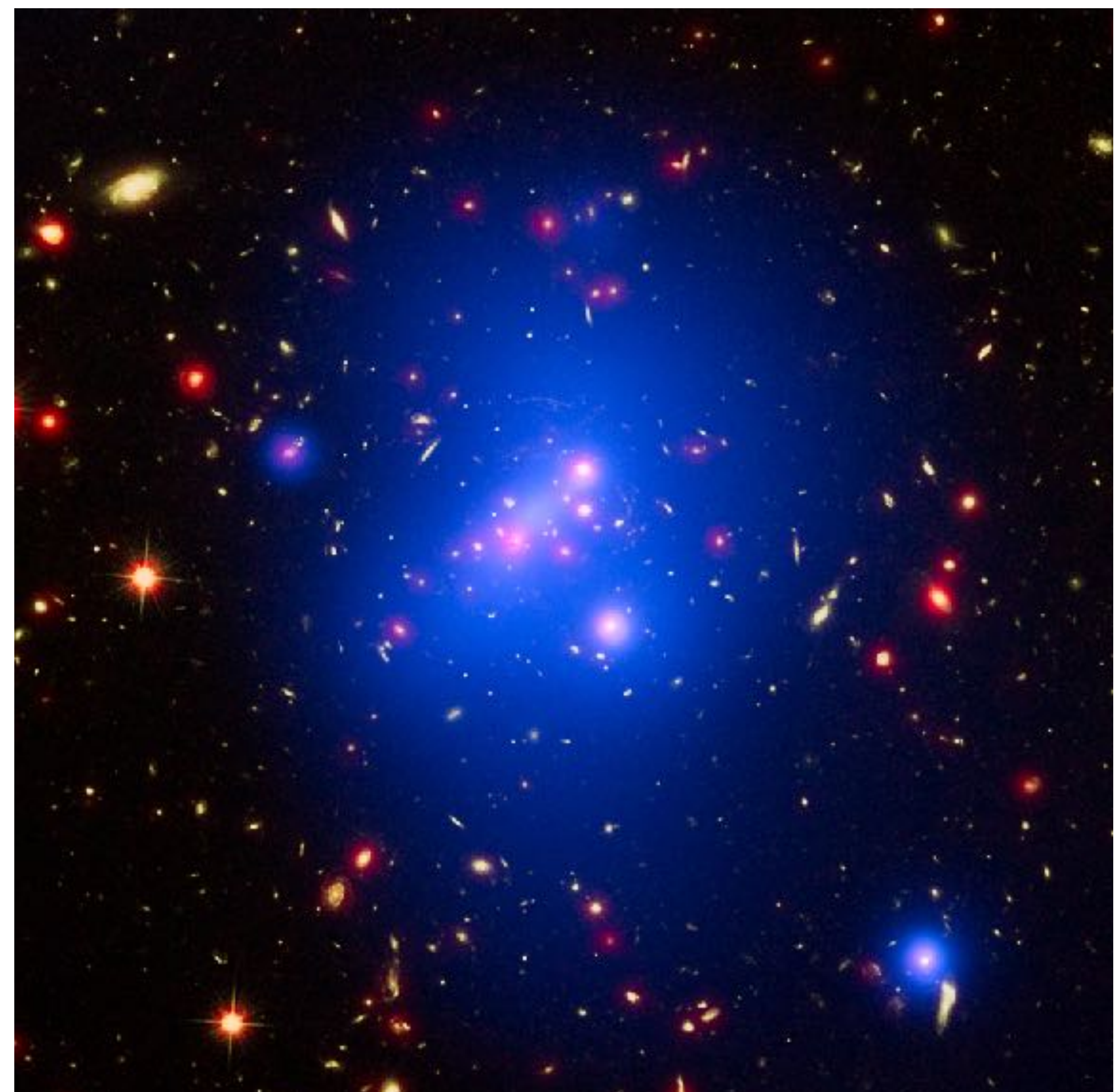


Pictured: Cluster IDCS J1426.5+3508 in X-rays (Chandra, blue), optical (HST, green) and IR (Spitzer, red). Mass $\sim5\times10^{14}\ M_\odot$

# Galaxy Clusters

- Fritz Zwicky was the first to suggest “invisible” mass in clusters in the 1930s.
- He found that the orbital speeds of galaxies were too fast for the structures to stay bound without more invisible (in optical at least) gravitating mass.
- We now see the ICM in X-rays, but only see the influence of dark matter.

Fritz Zwicky



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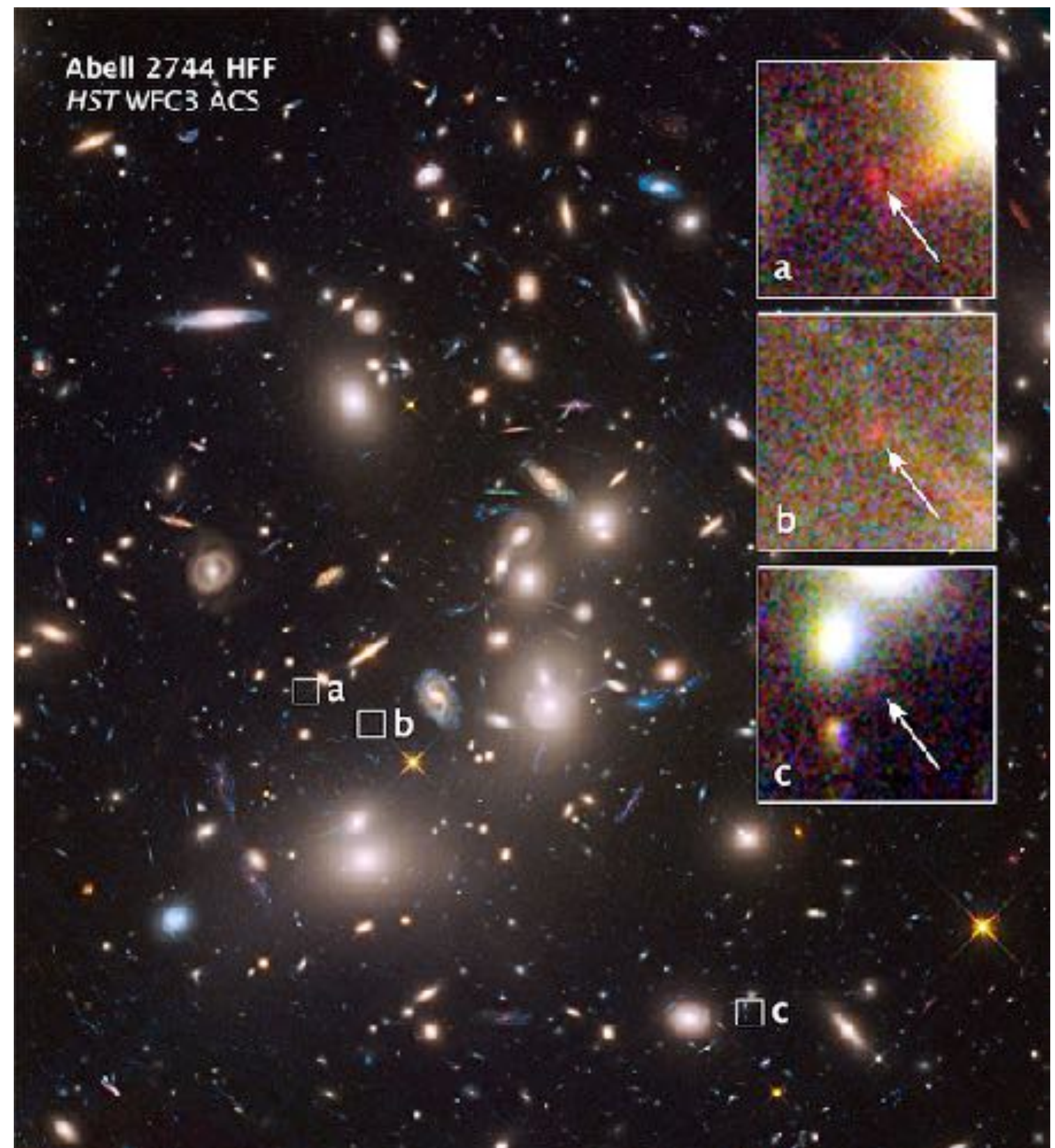
- We now see the ICM in X-rays, but only see the influence of dark matter.
- We also infer DM from gravitational lensing of background galaxies (Zwicky also suggested this)

Fritz Zwicky



# The Intra Cluster Medium

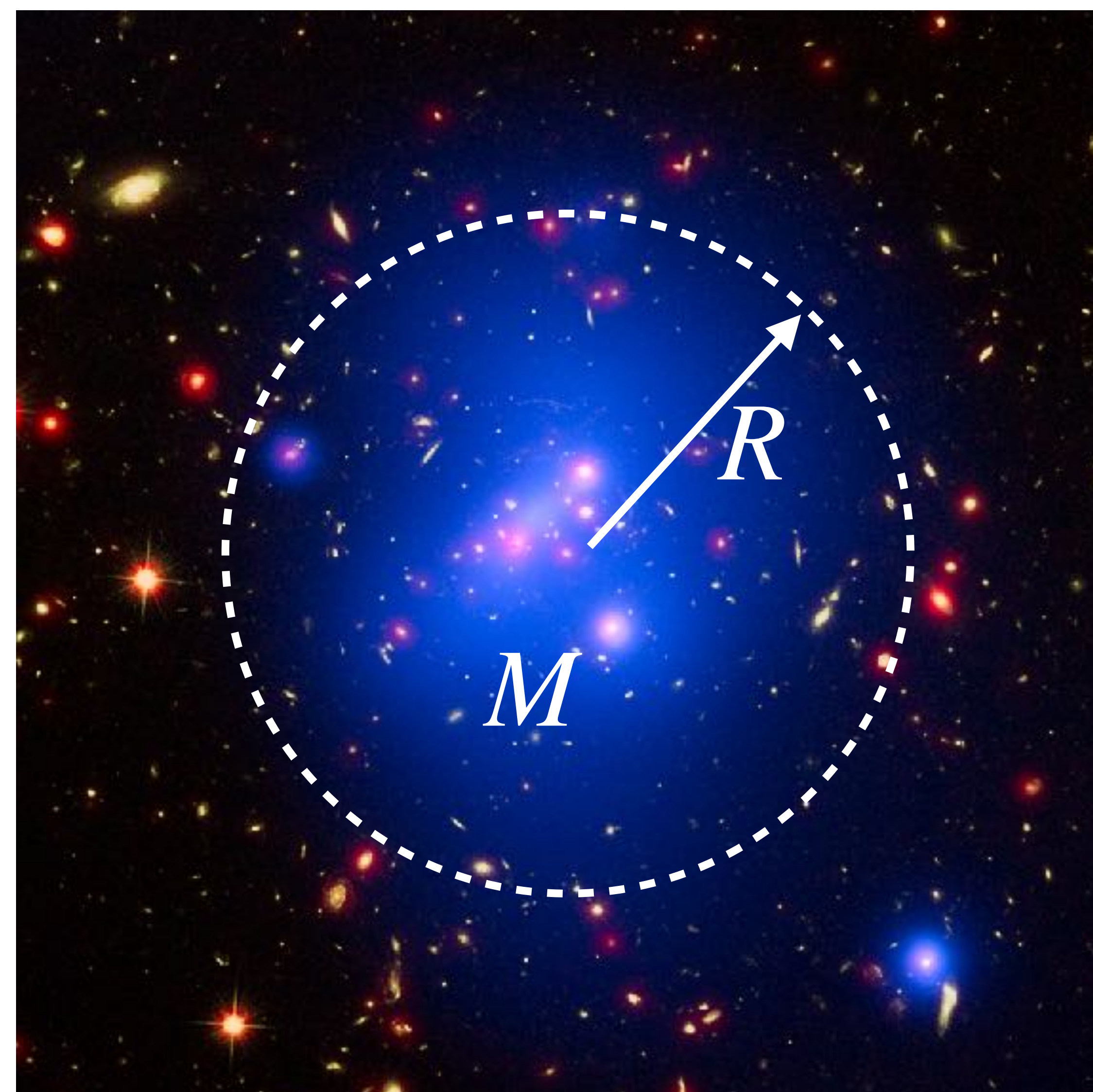
- Plasma particles in the ICM have large random thermal motions, preventing gravitational collapse in the cluster's potential well.

- In equilibrium, we can estimate the temperature from the viral theorem:
  Thermal energy = (1/2) gravitational potential energy

- Ignoring some pre-factors of order unity, gives:

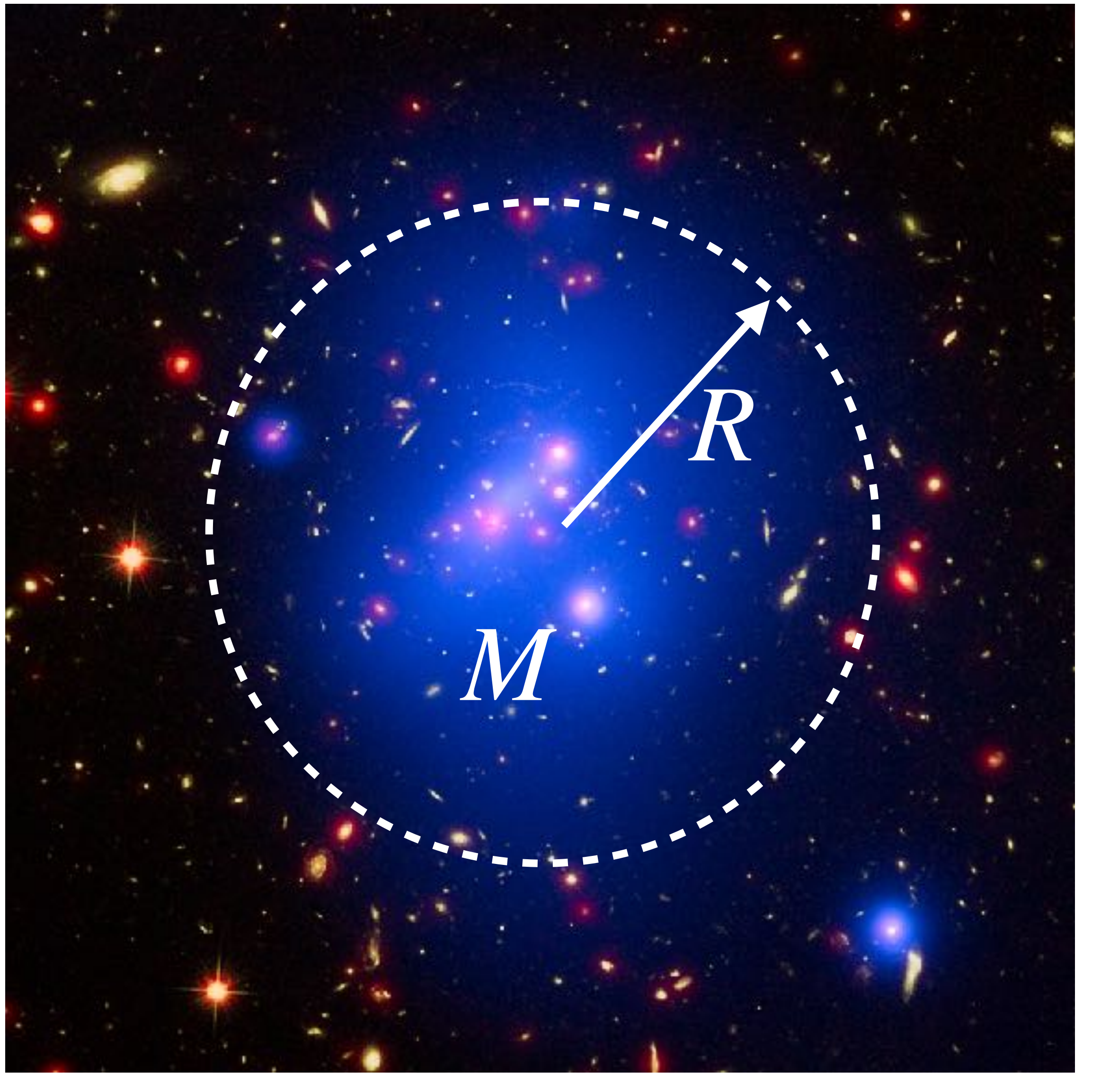
$$k_B T_e = k_B T_i \sim \frac{G M m_H}{R}$$

M = total cluster mass
R = radius of cluster
$T_e$ = electron temp
$T_i$ = ion temp
$m_H$ = mass of hydrogen atom



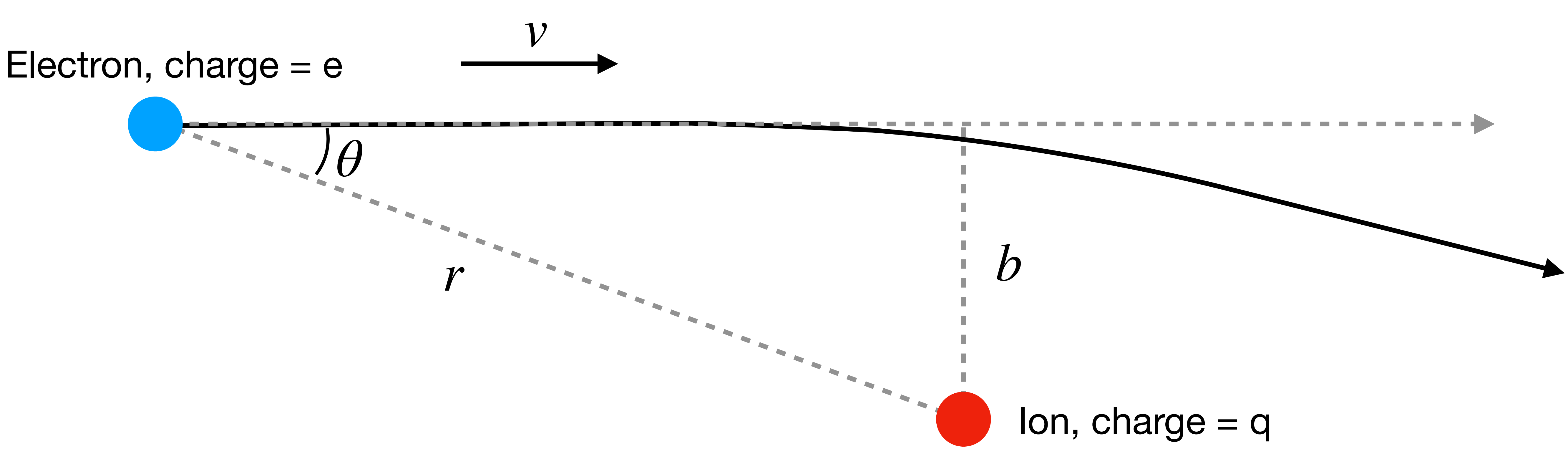
# The Intra Cluster Medium

- ICM is thermalised; i.e. electrons have Maxwellian velocity distribution.
- BUT plasma density is too low for blackbody radiation (i.e. photons are not in thermal equilibrium with electrons).
- Dominant emission mechanism: thermal bremsstrahlung radiation.



# Bremsstrahlung

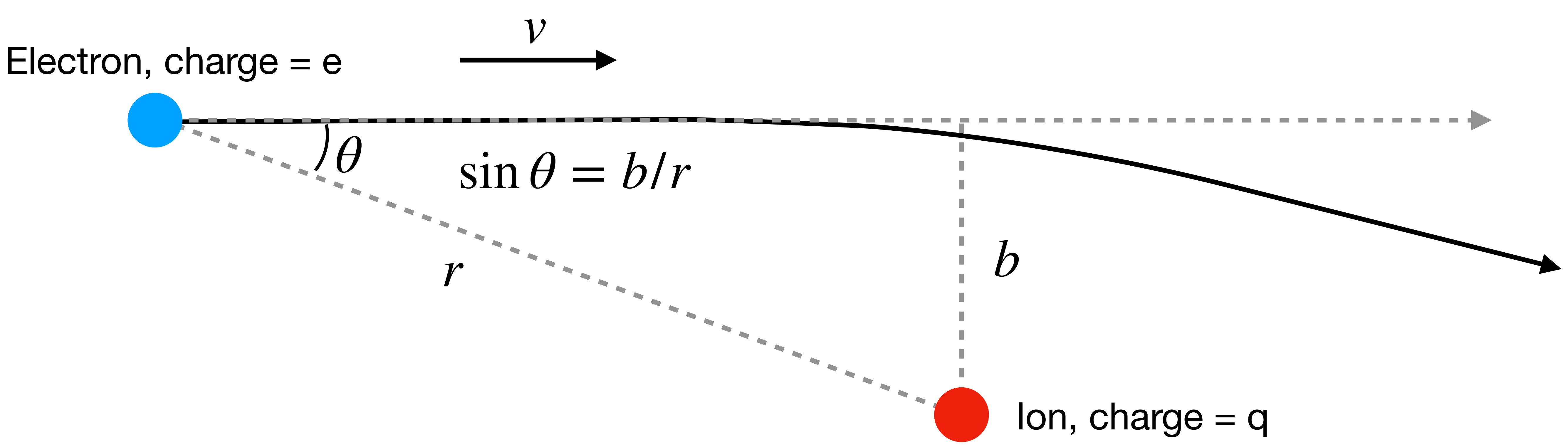
- Bremsstralung radiation = “Breaking radiation”
- Electron moving at velocity v on a trajectory to miss an ion by a distance b.
- b = the impact parameter.
- Coulomb force will accelerate the electron towards the ion.
- Power of EM radiation given by Larmor formula.
- i.e. like synchrotron, except an ion deflects the electron instead of a magnetic field.



# Bremsstrahlung

- Vertical acceleration from Coulomb formula:

$$a_z = -\frac{qe}{4\pi m_e \epsilon_0 r^2}\sin\theta$$



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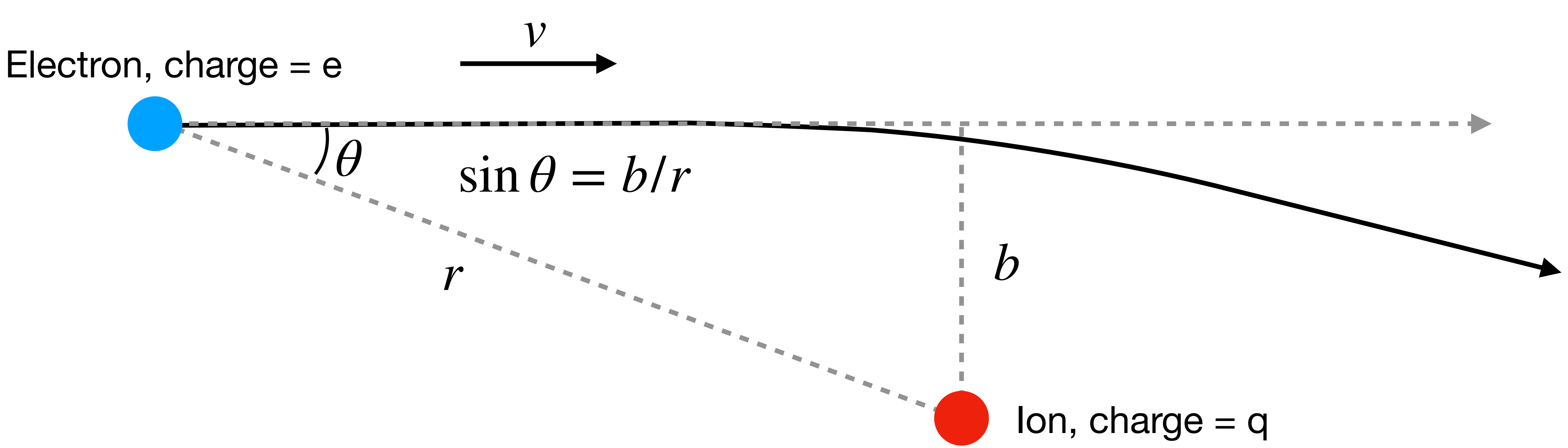
$$\implies a_z = -\frac{qe}{4\pi m_e \epsilon_0 b^2} \sin^3\theta$$



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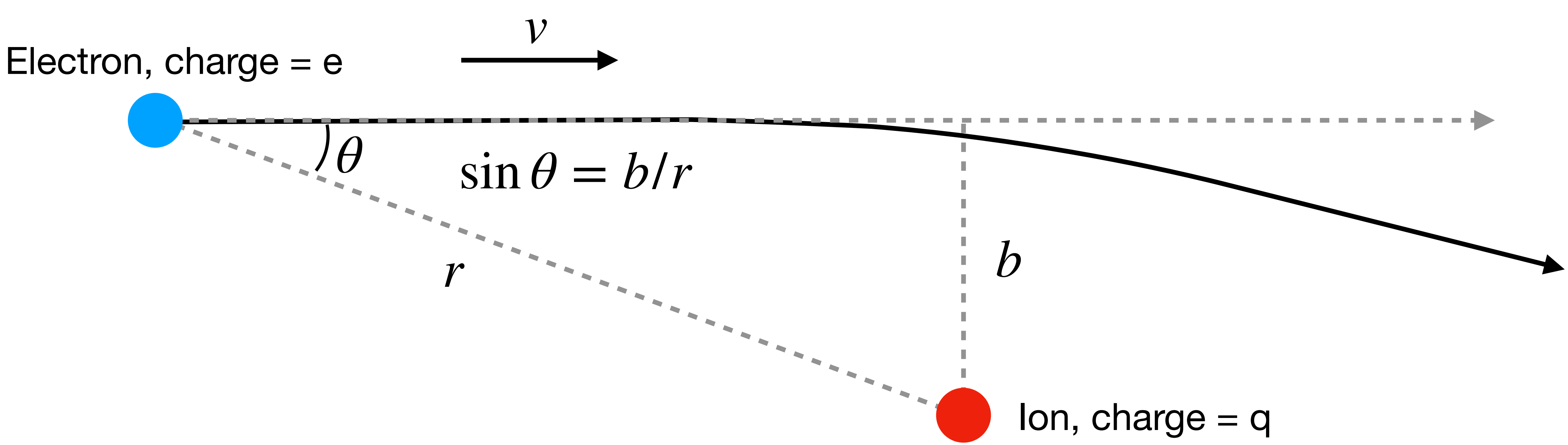
- Ignore horizontal acceleration (acceleration before collision, deceleration after)



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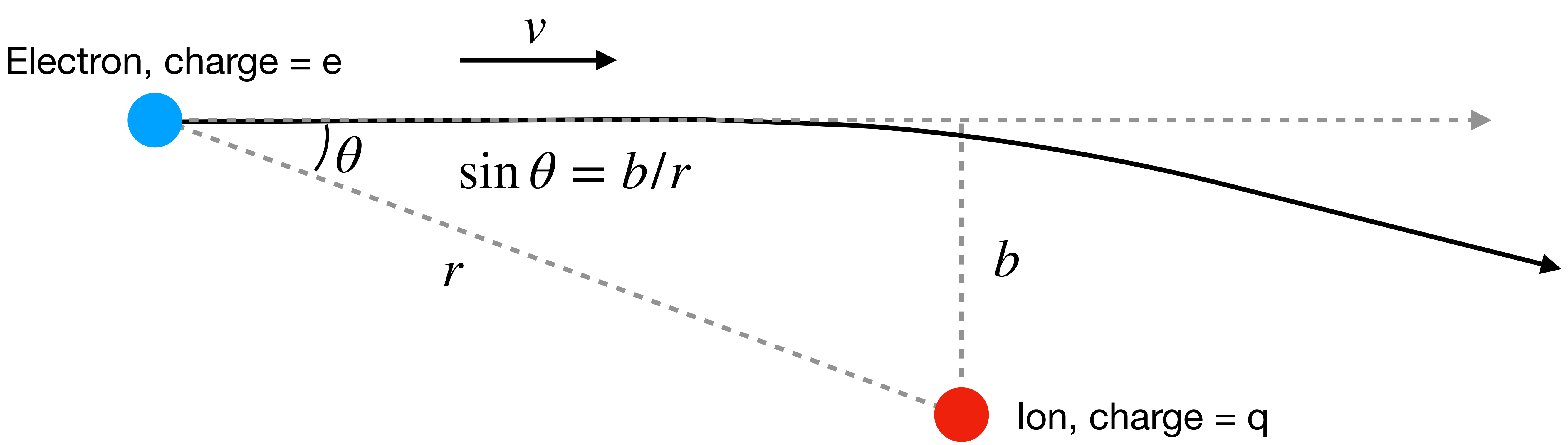

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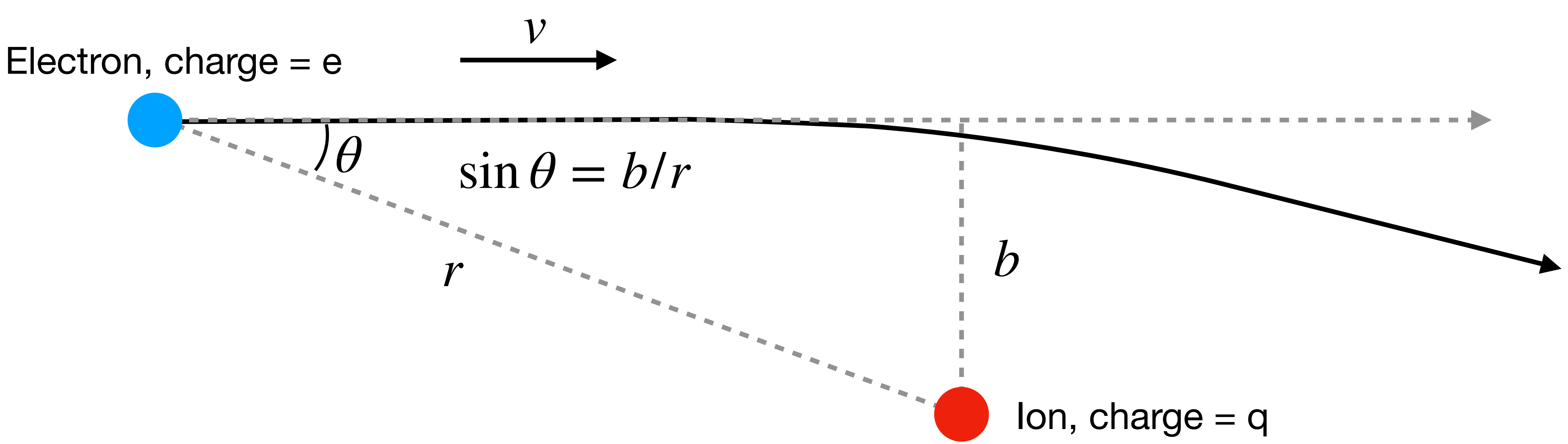
$$P = \frac{e^2 a^2}{6\pi\epsilon_0 c^3}$$



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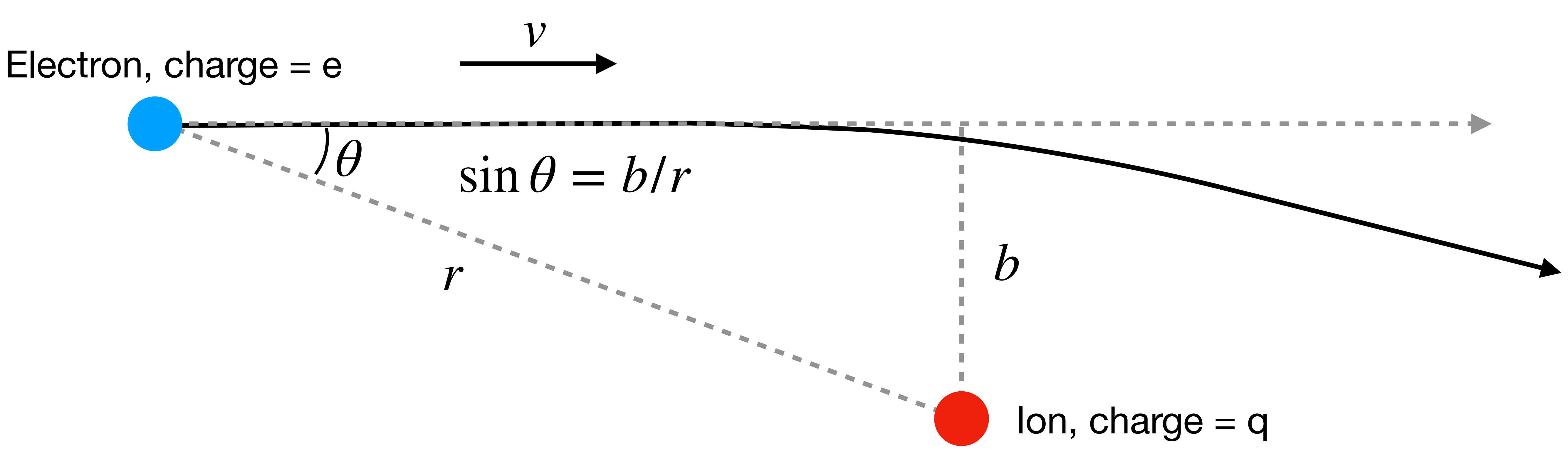

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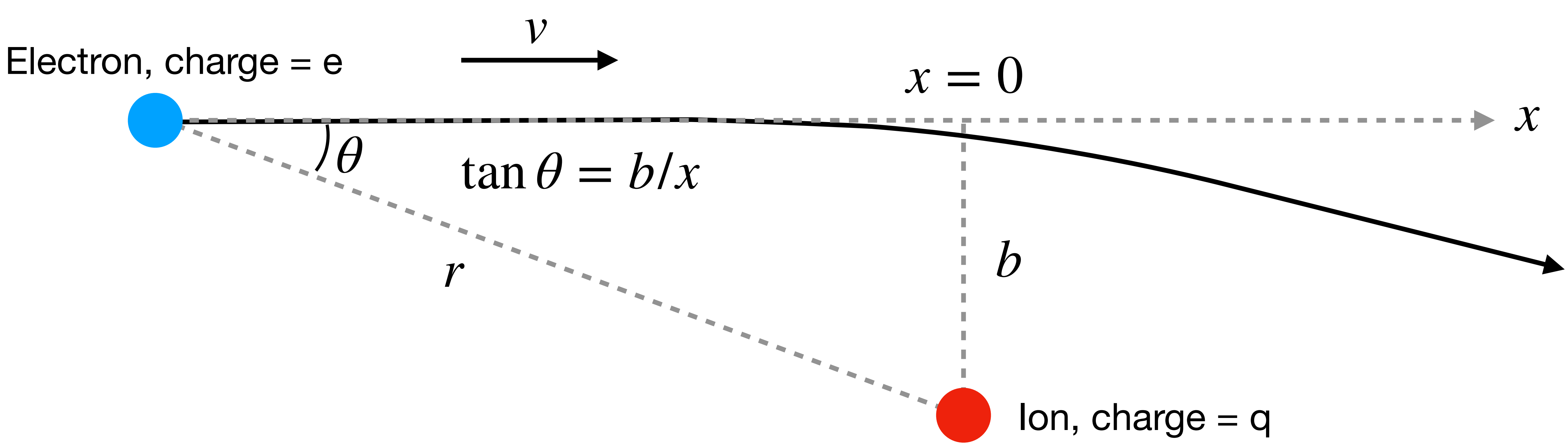
- Thomson cross-section:

$$\sigma_T = \frac{1}{6\pi}\left(\frac{e^2}{\epsilon_0 m_e c^2}\right)^2$$

$$\implies P = \frac{q^2 \sigma_T c}{16\pi^2 \epsilon_0 b^4} \sin^6\theta$$

# Bremsstrahlung

- Solve equation of motion to get P(t). Hard in general but get a feel by ignoring vertical motion of electron: $x(t) = vt$
- v is constant and x=0 when t=0.
- In this case: $\tan[\theta(t)] = b/x(t); \quad P(t) \propto \sin^6[\theta(t)]$



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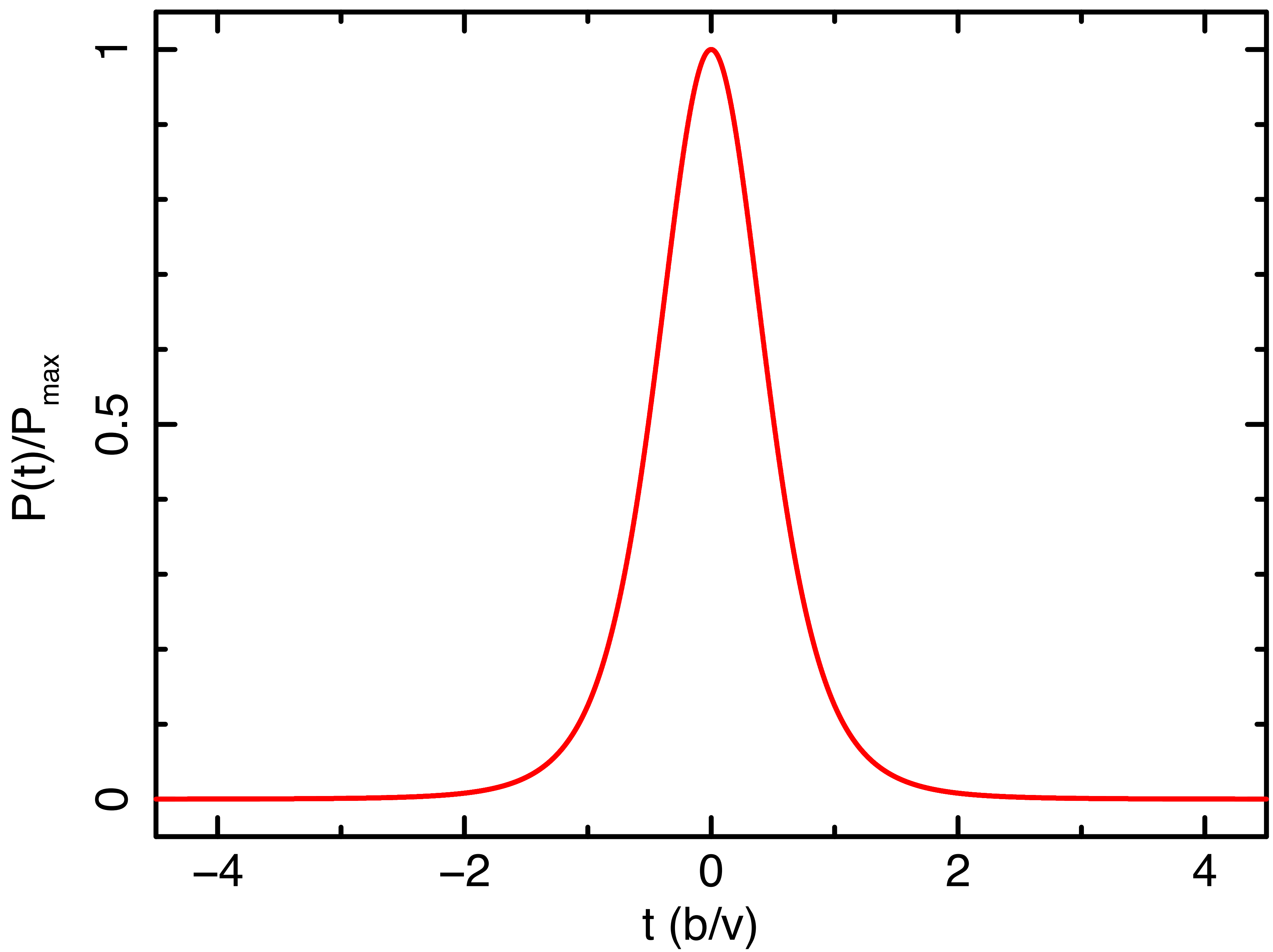
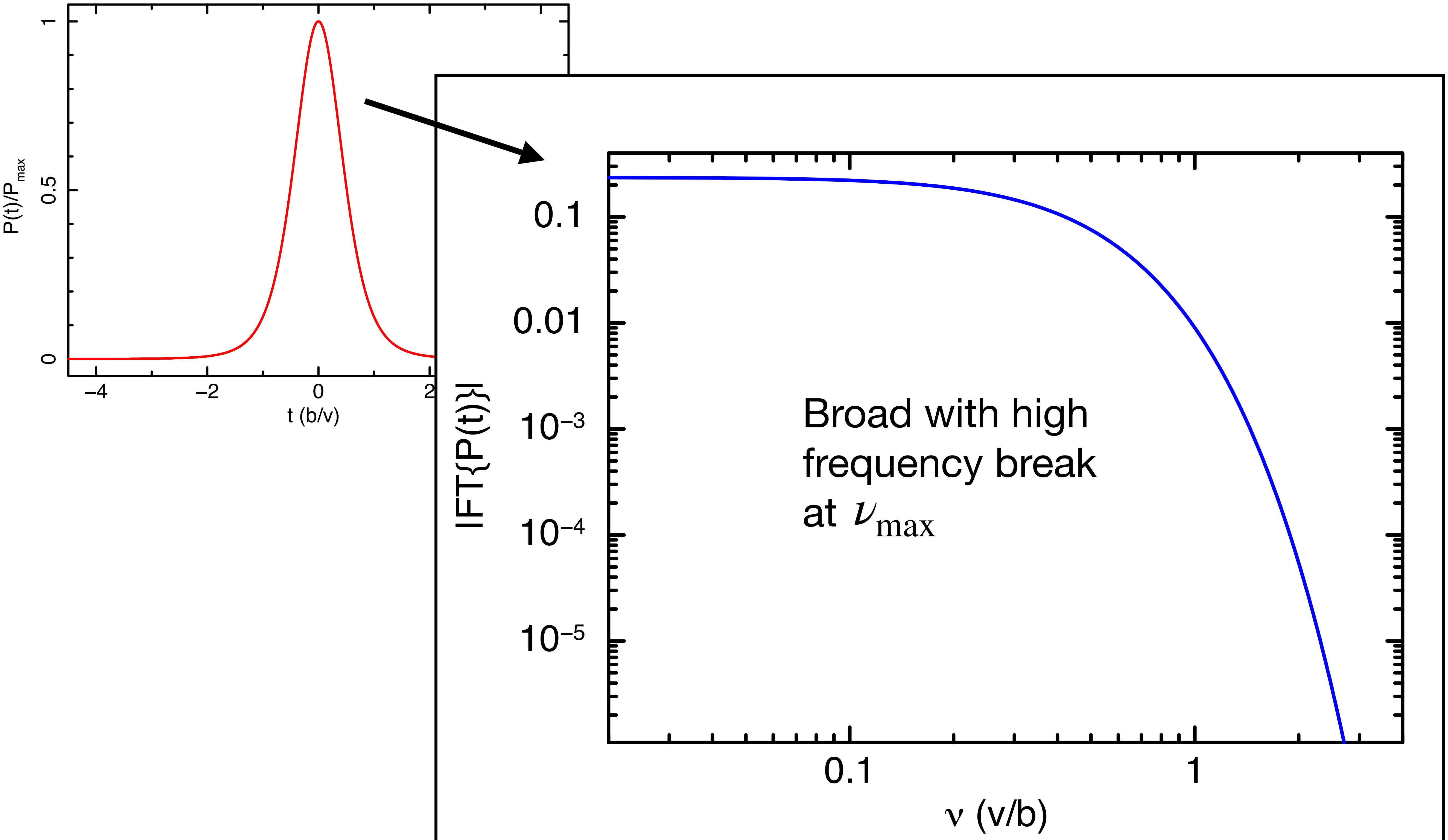


# Bremsstrahlung

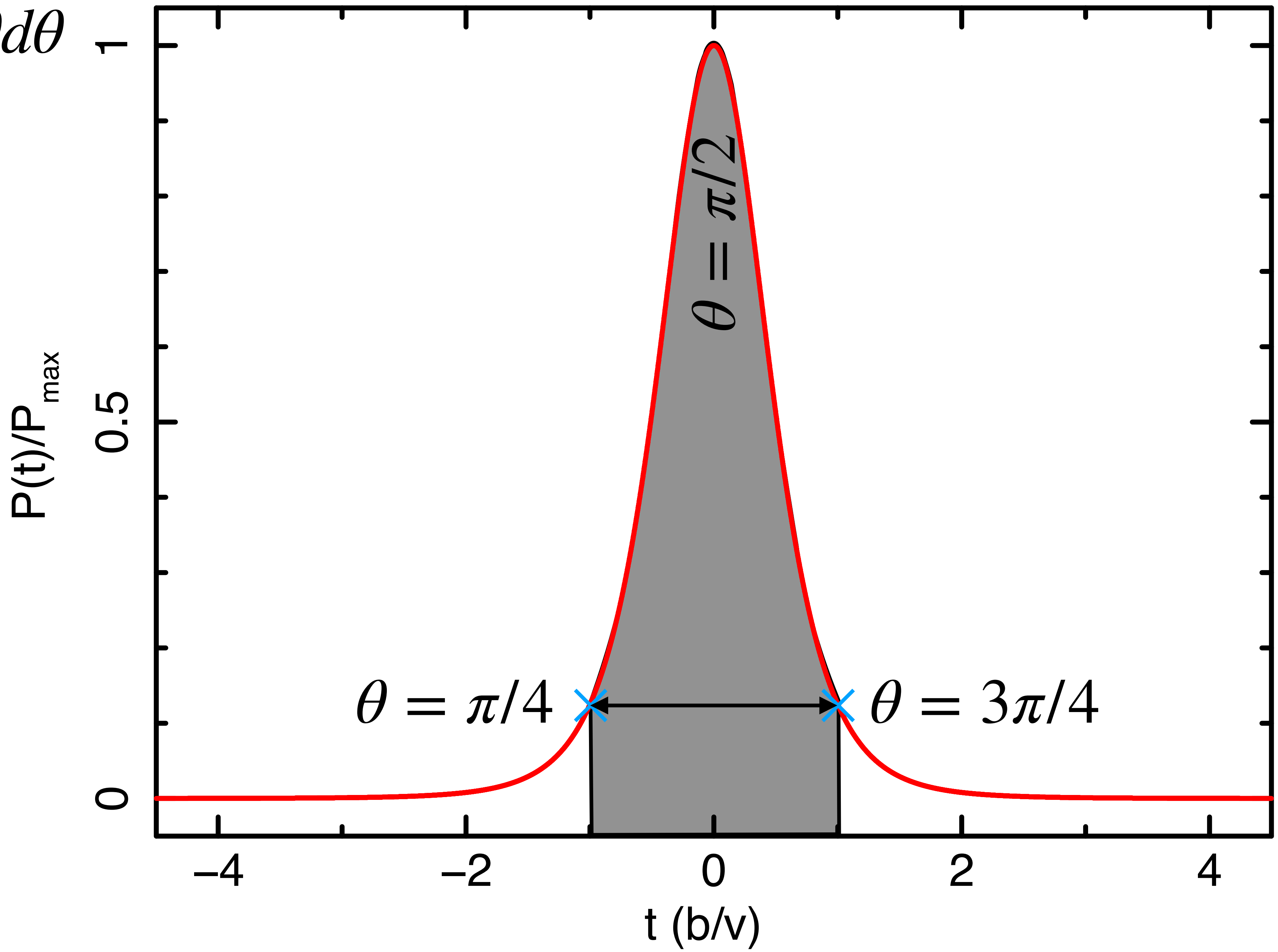
- To get the spectrum, we need the same trick we used for synchrotron radiation: take the Fourier transform



# Bremsstrahlung

- Can estimate the break frequency by appreciating that most of the energy in the pulse is irradiated between $\theta = \pi/4$ and $\theta = 3\pi/4$

$$\frac{\int_{\pi/4}^{\pi/2} \sin^6\theta d\theta}{\int_0^{\pi/2} \sin^6\theta d\theta} > 0.96$$



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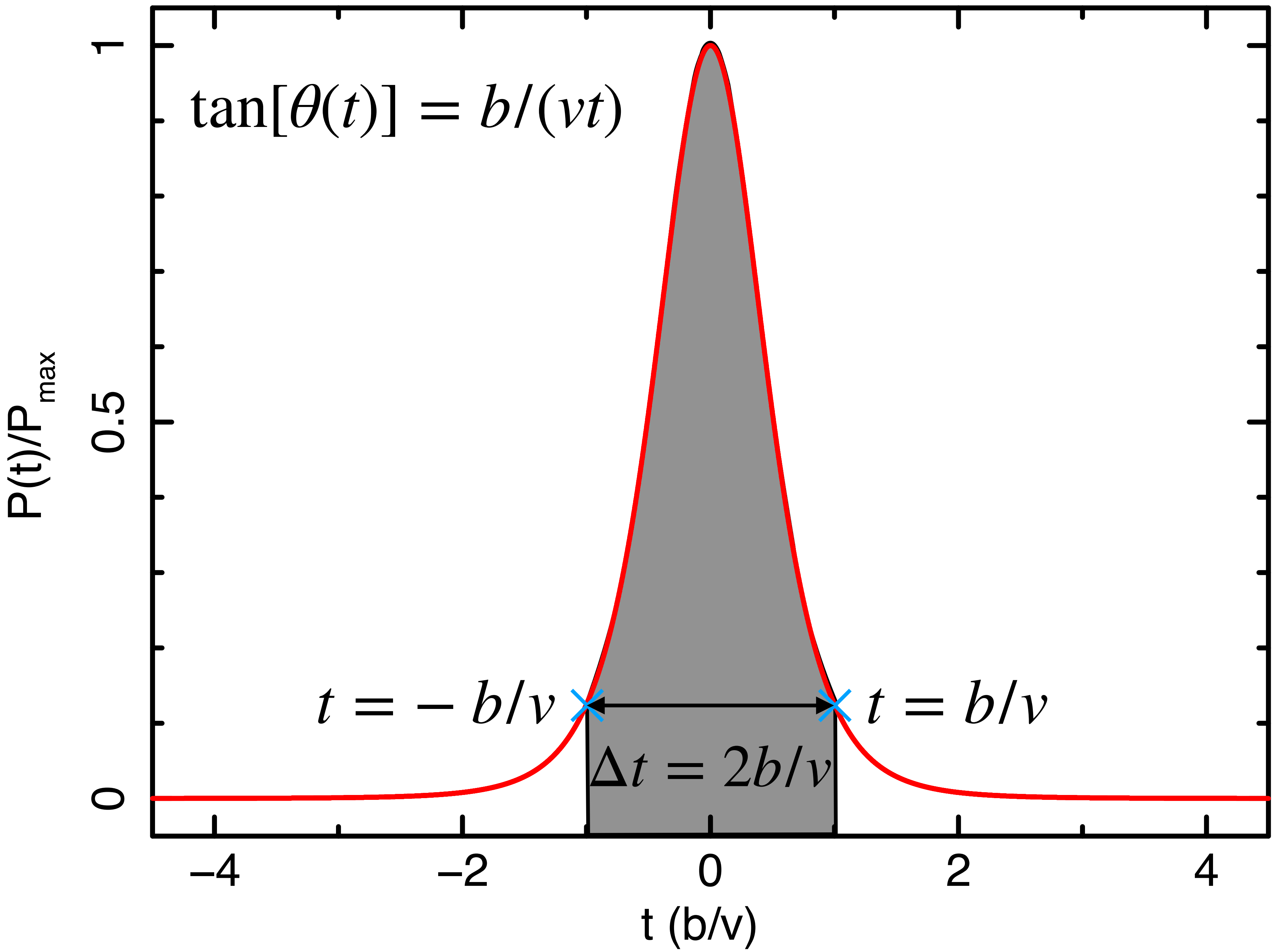
- Corresponds to $\tan\theta = \pm 1$
- Therefore pulse is ~a top hat with duration $\Delta t = 2b/v$



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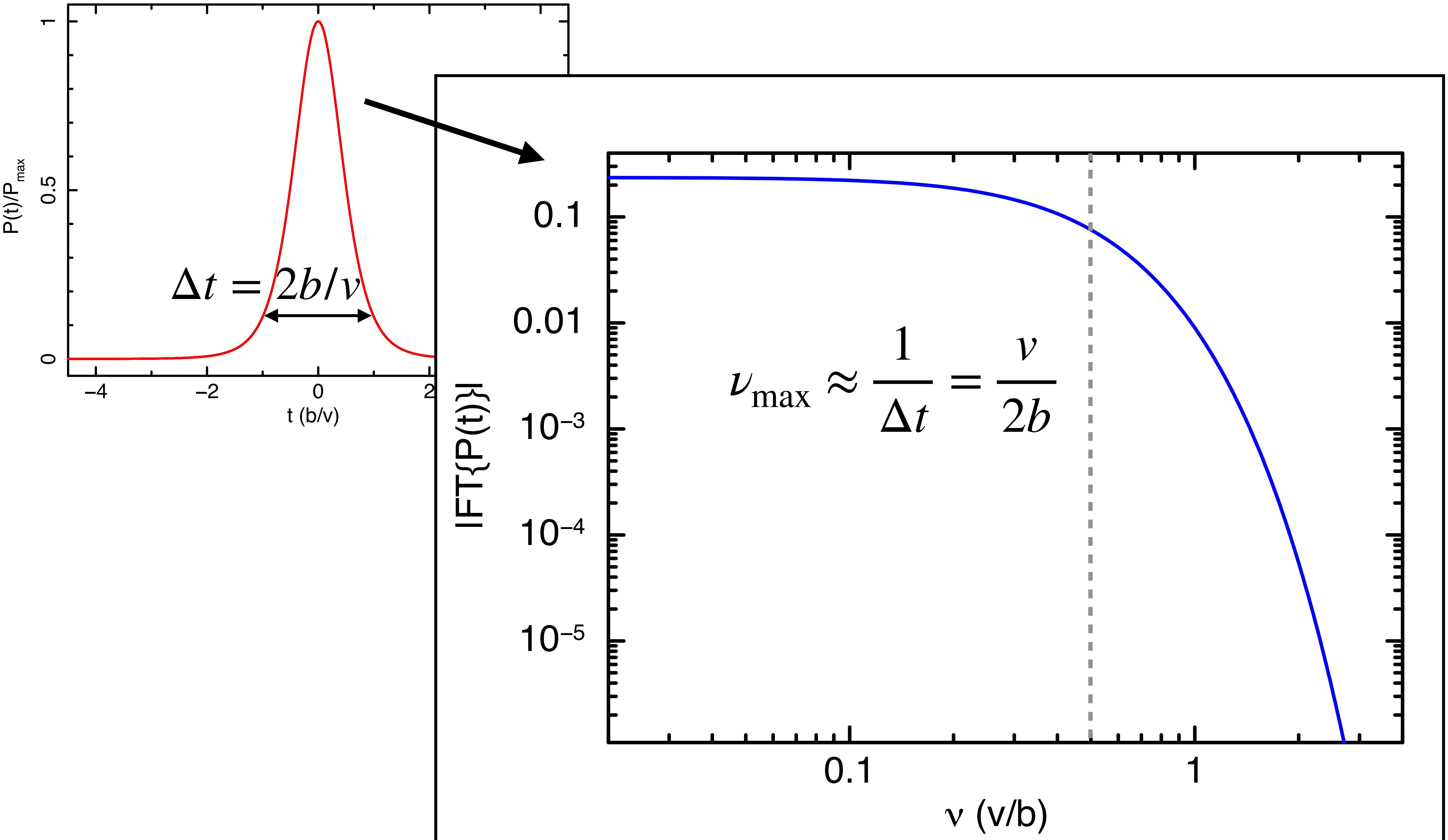

# Bremsstrahlung

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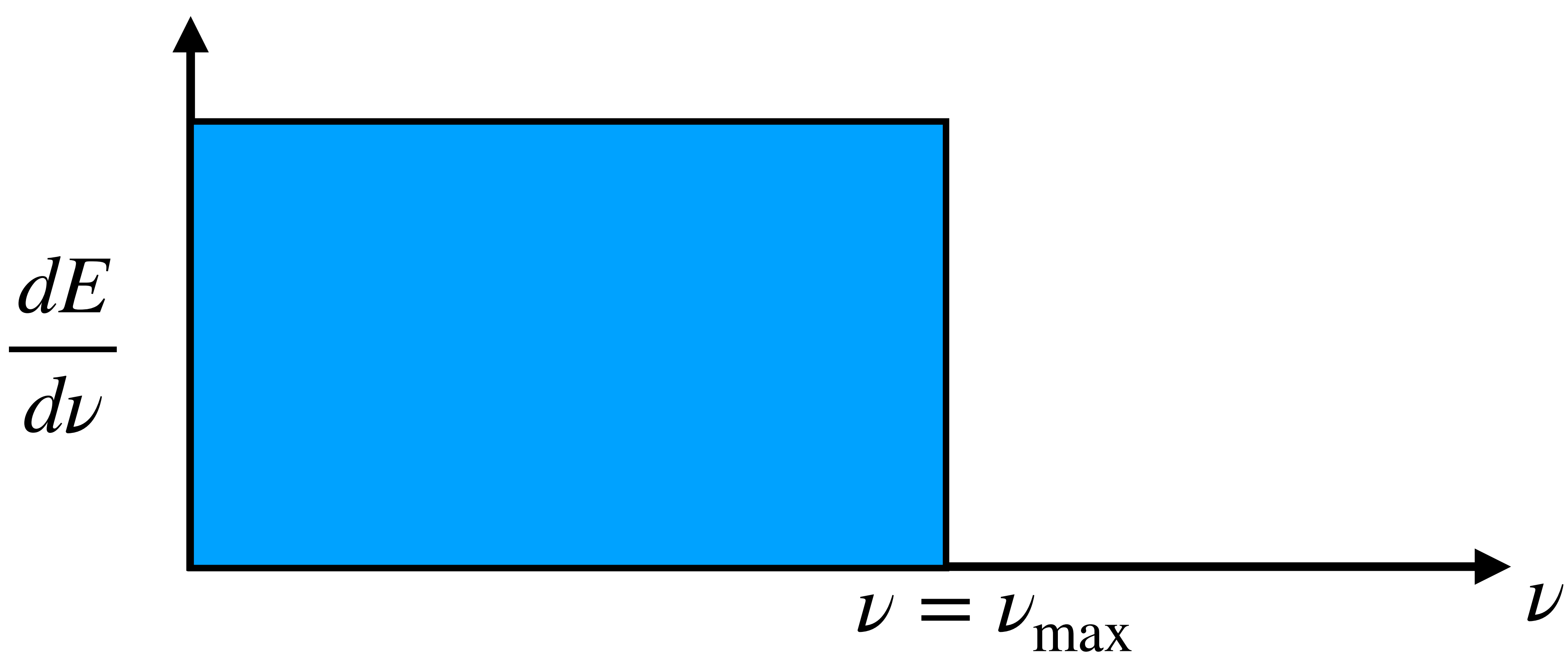

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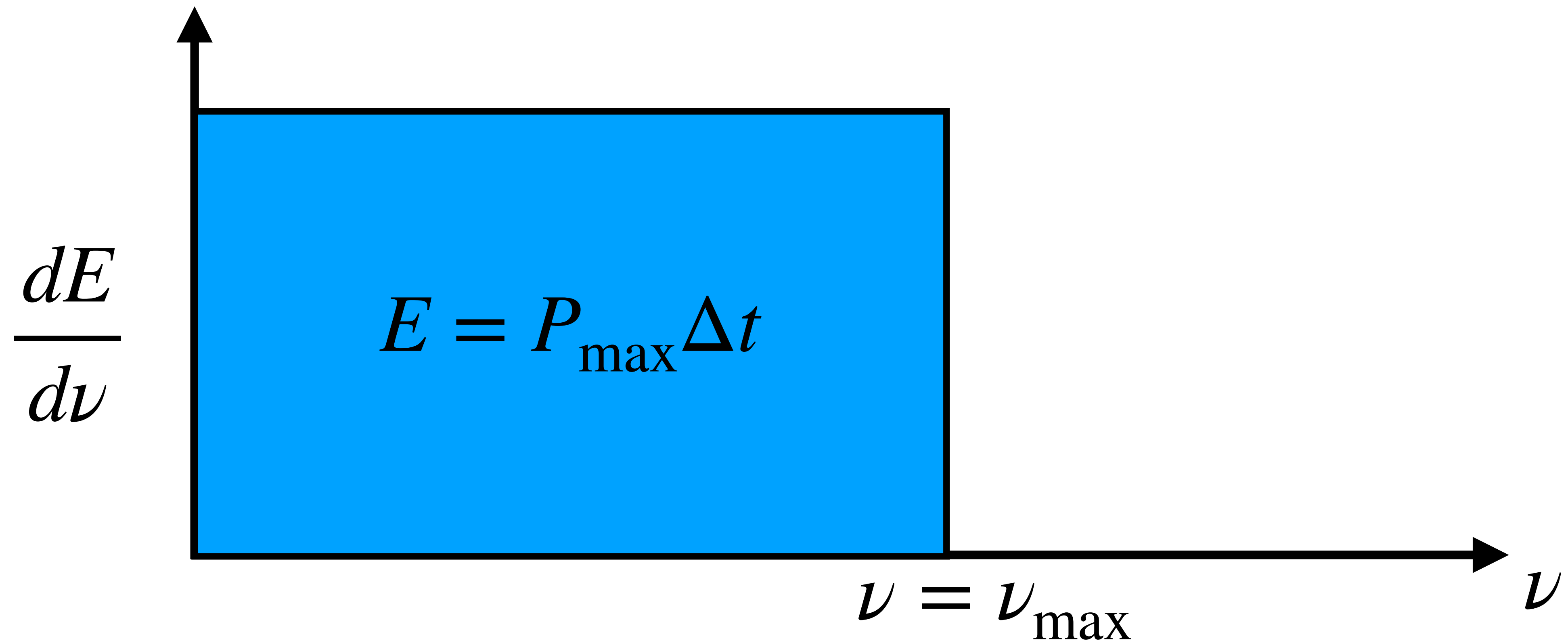
$$E = \int_0^\infty \frac{dE}{d\nu} d\nu = \int_{-\infty}^\infty P(t) dt \approx P_{\rm max} \Delta t$$



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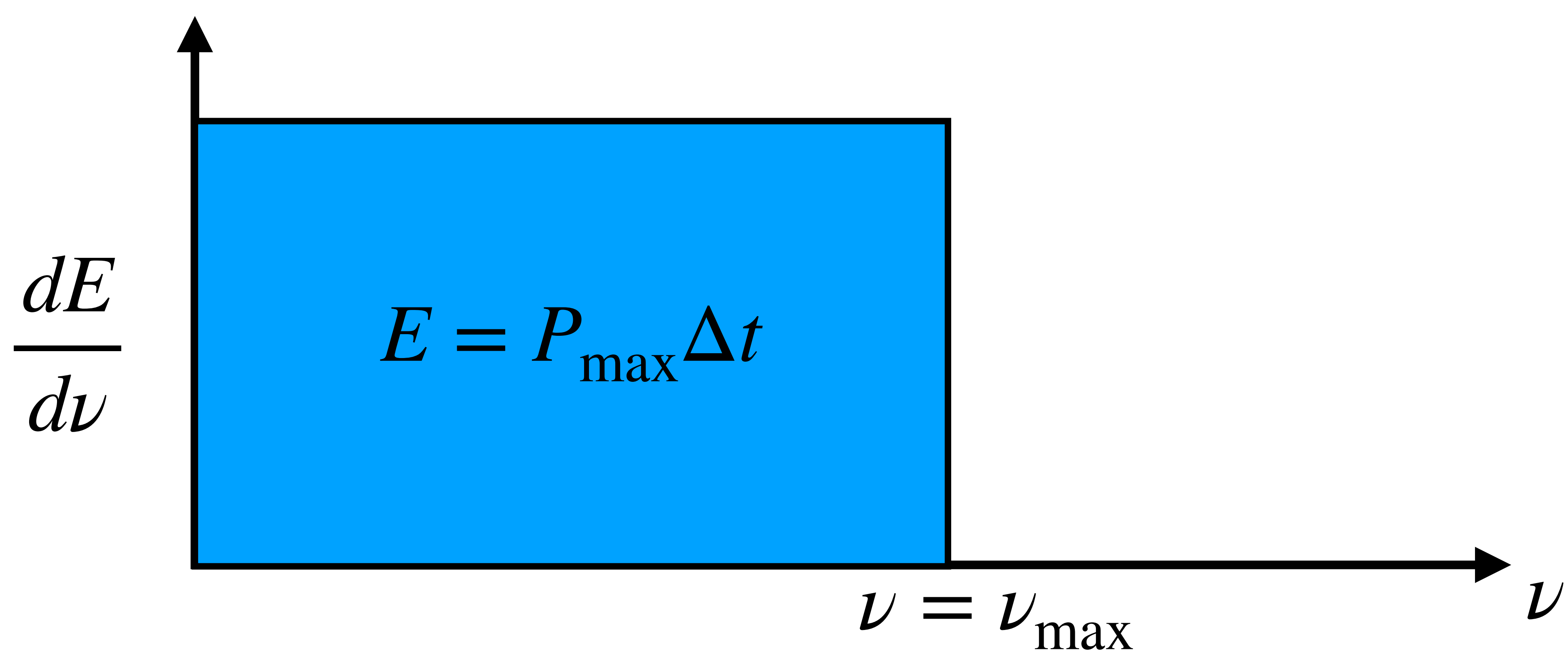

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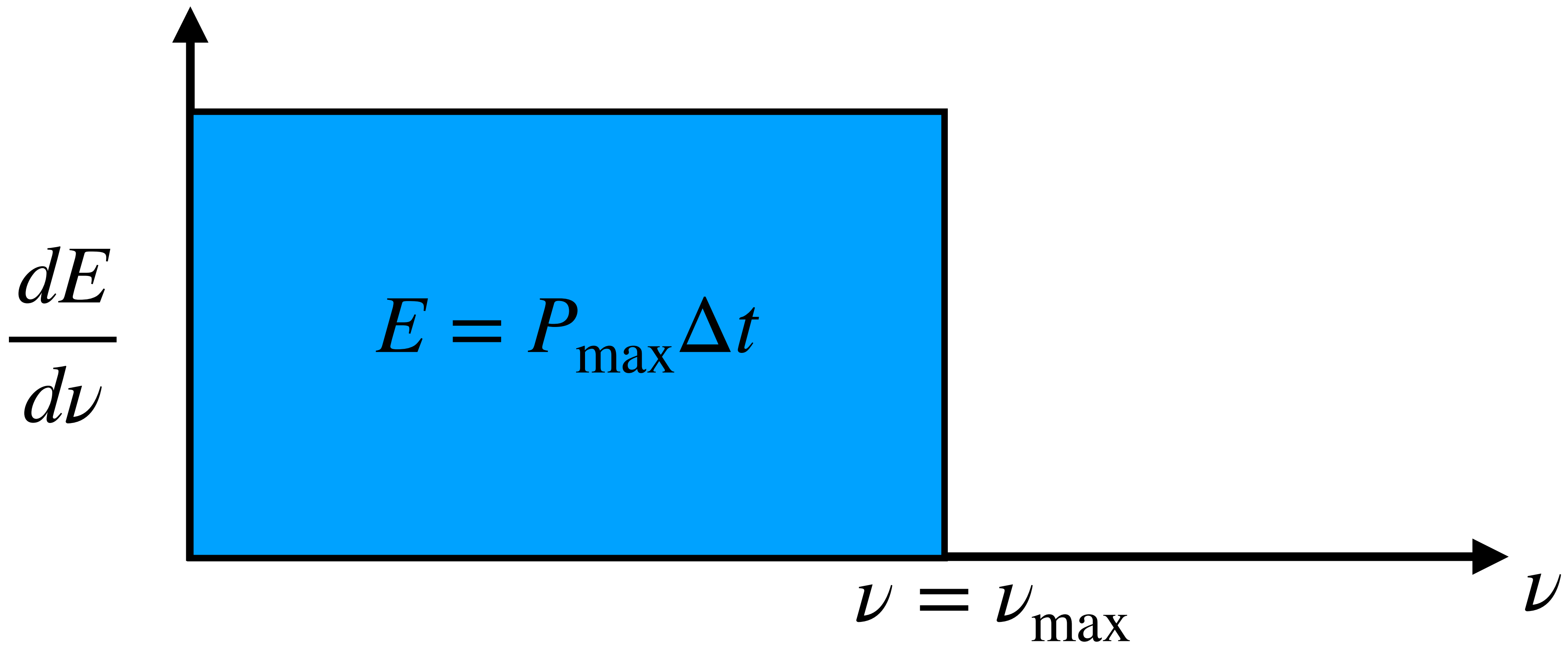

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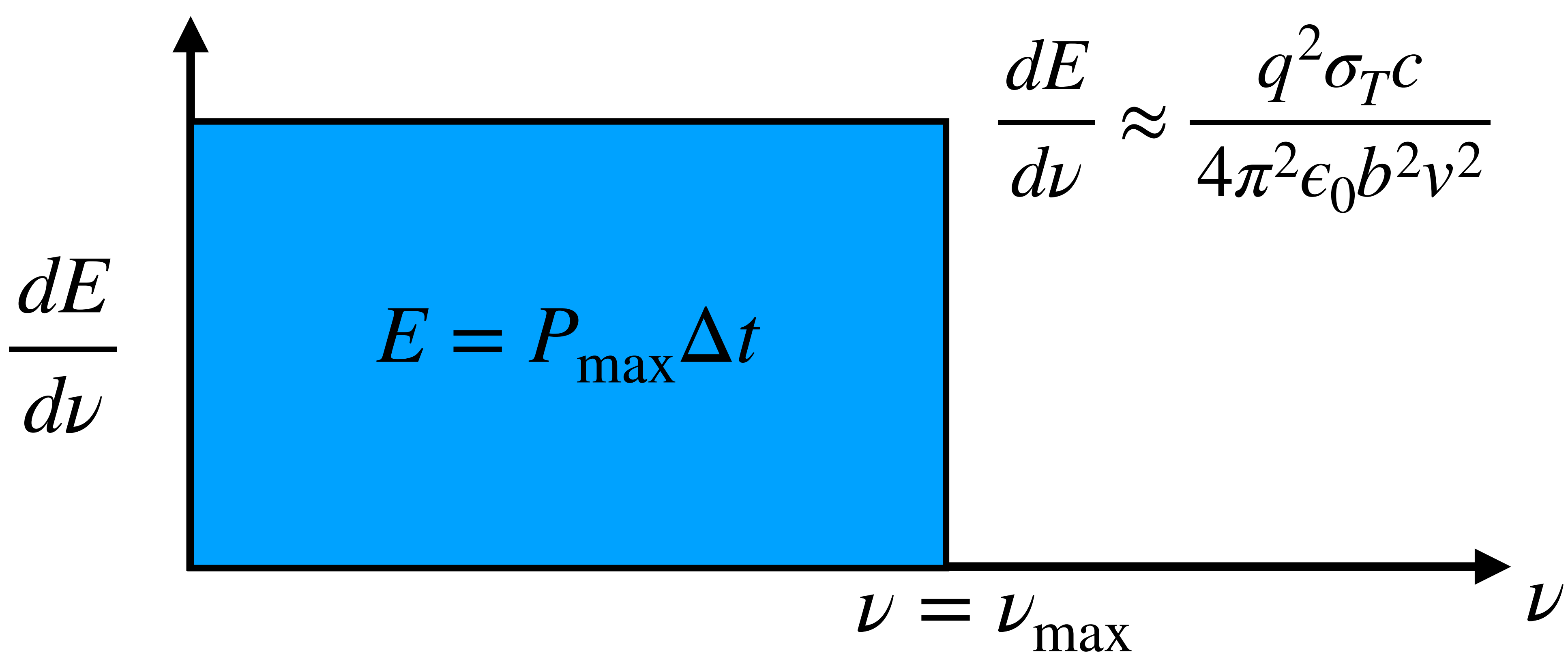
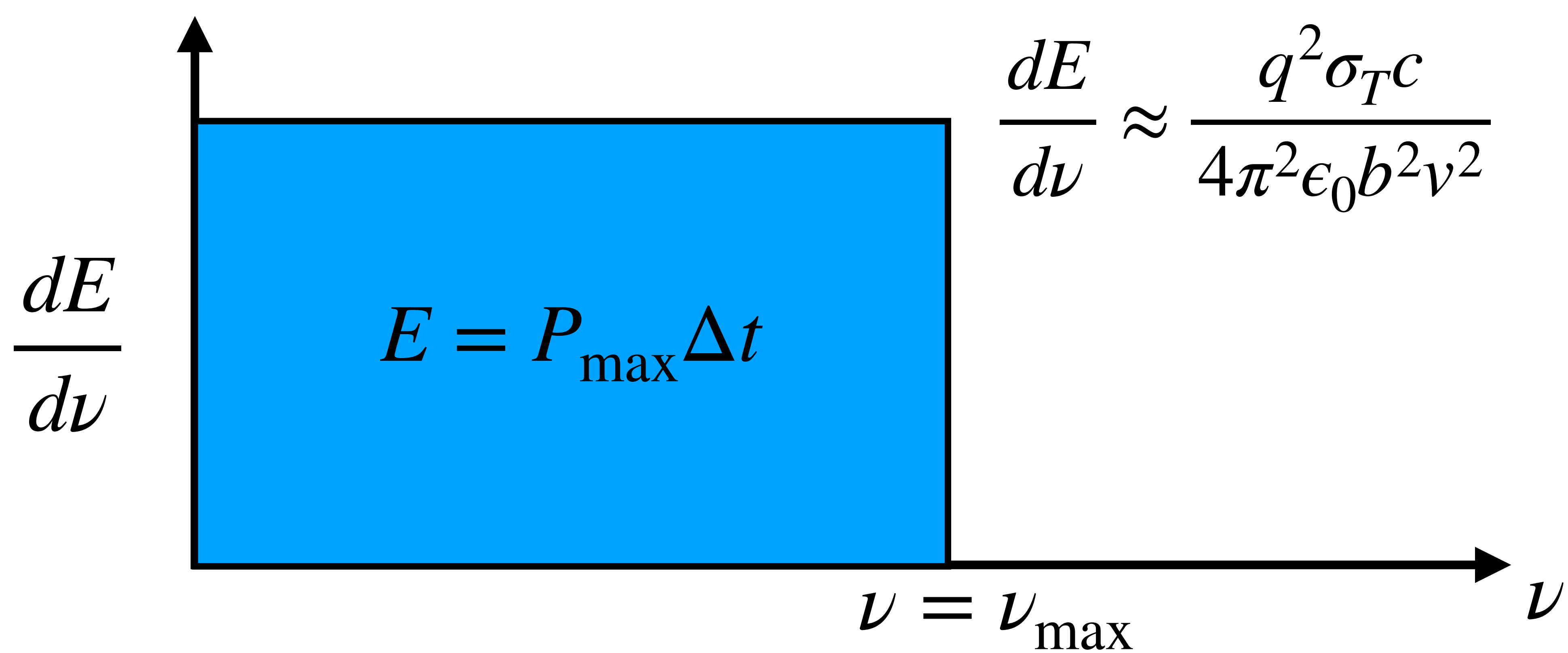


# Bremsstrahlung

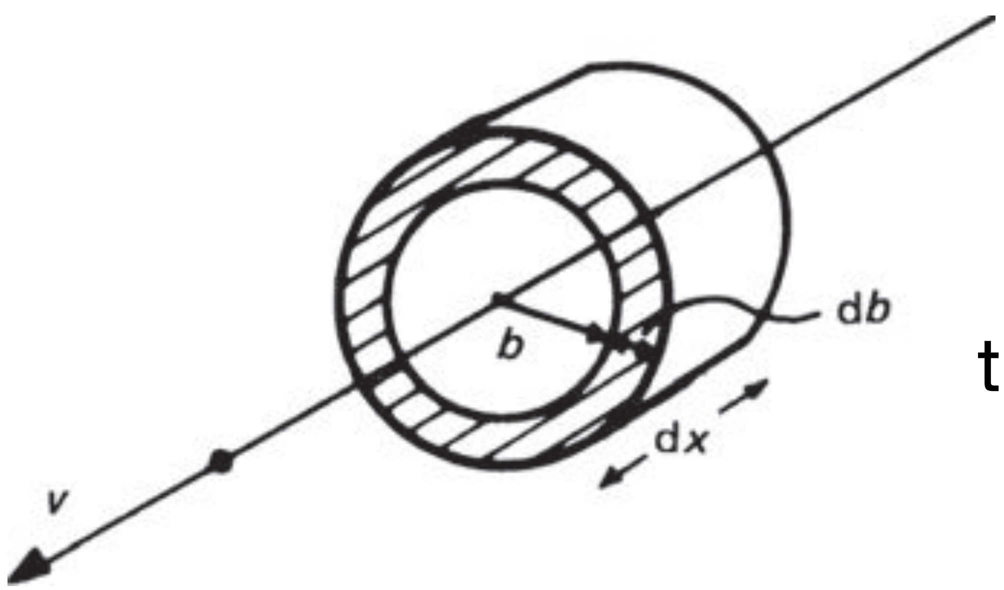
- This is the spectrum for a single event, whereby the electron has a particular speed v and impact parameter b.



# Bremsstrahlung

- This is the spectrum for a single event, whereby the electron has a particular speed v and impact parameter b.
- We need the overall spectrum for the whole velocity distribution, integrated over all impact parameters.
- Number of collisions with impact parameter between b and b+db that a given ion will have is:

$$\frac{dN}{dt} = n_e v 2\pi b db$$




In time interval dt, the volume swept out by this range of impact parameters is 2π b db dx. Volume swept out per unit time is 2π b db v.

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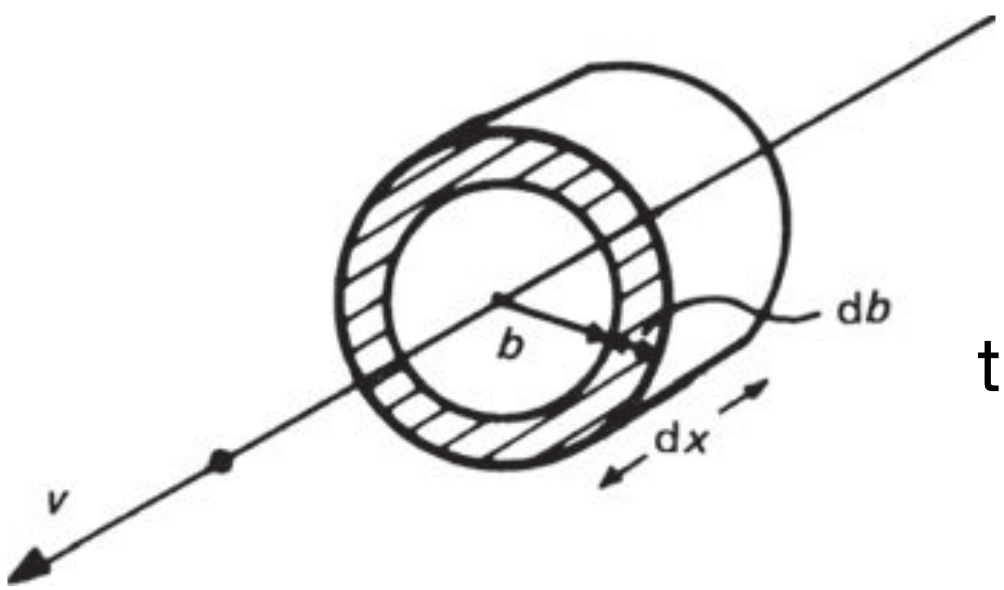


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- Total number of collisions per unit time per unit volume with impact parameter between b and b+db:

$$\frac{dN}{dVdt} = n_e n_i v 2\pi b db$$

# Bremsstrahlung

- Therefore energy radiated per unit frequency per unit time per unit volume:

$$\frac{dE}{d\nu dt dV} = \int_{b_{\rm min}}^{b_{\rm max}} P_{\rm max}(\Delta t)^2 \ . \ n_e v 2\pi b db$$

Energy per frequency per event

Events per time per volume

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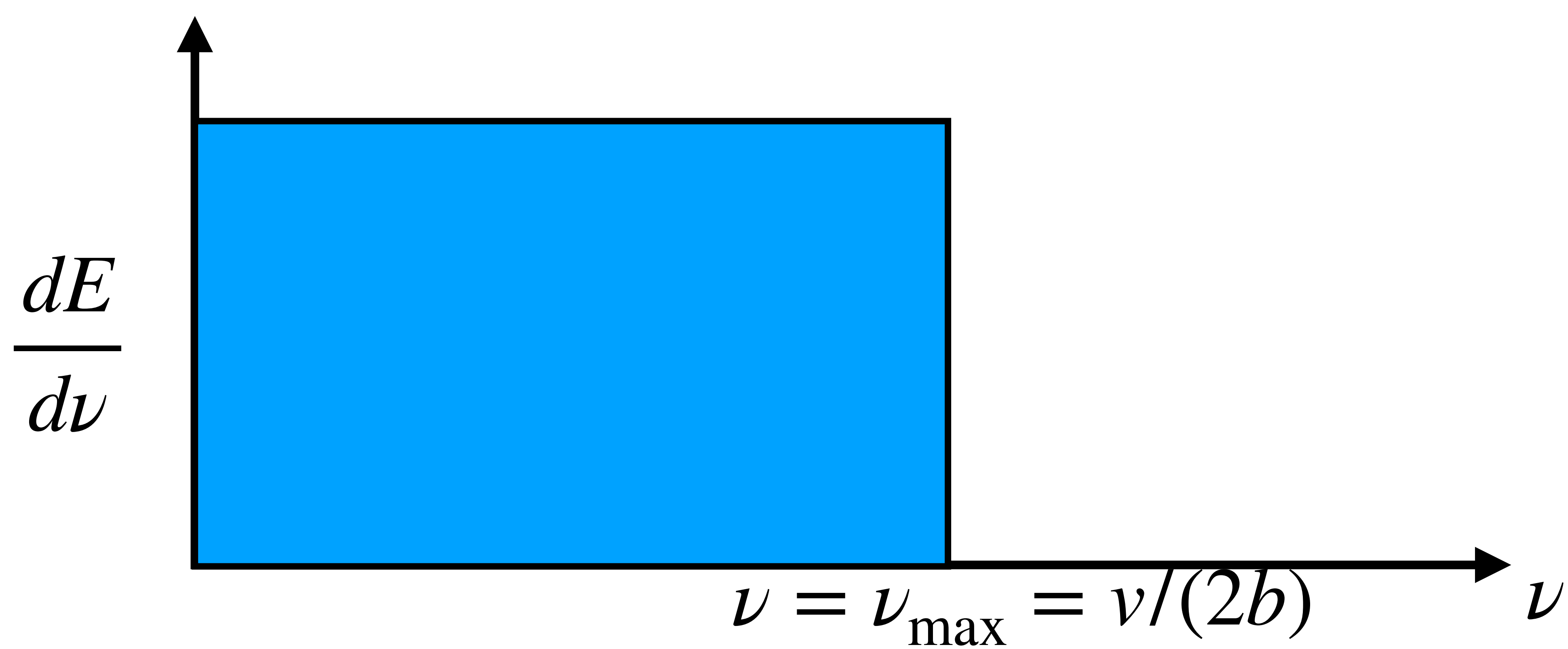

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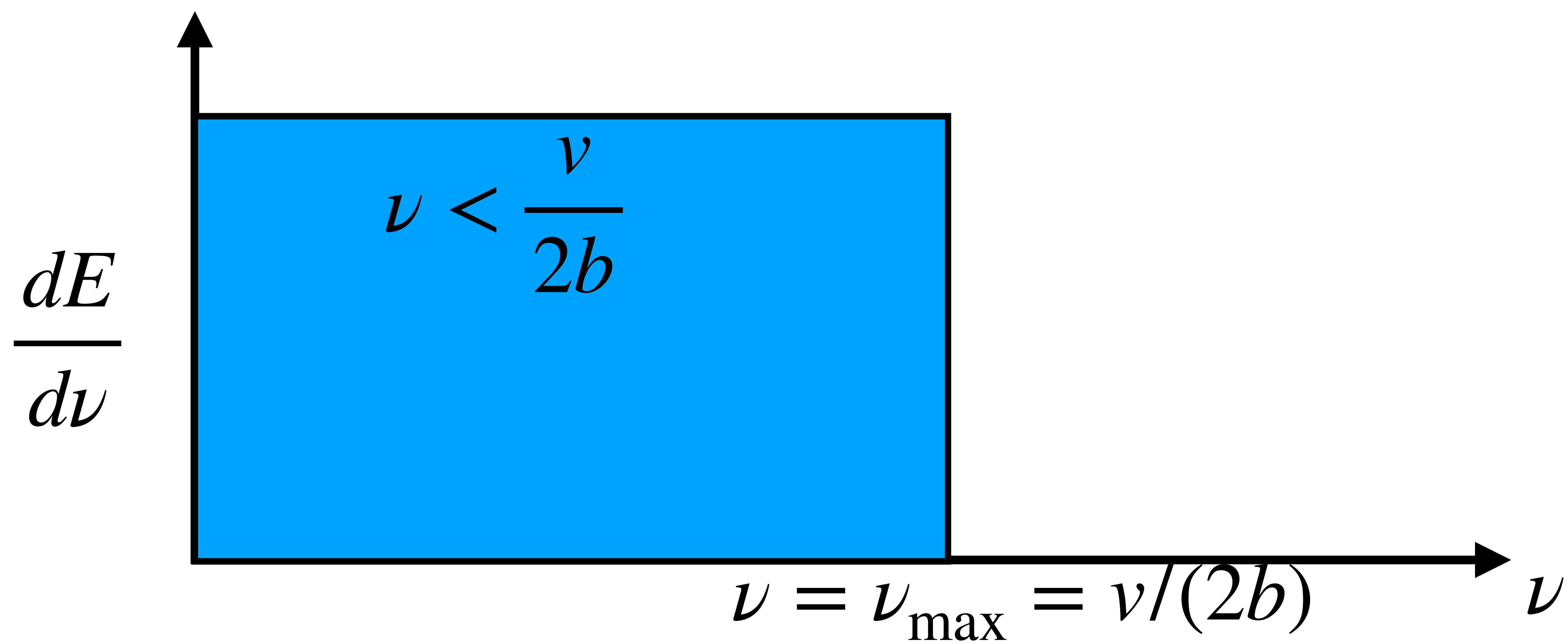

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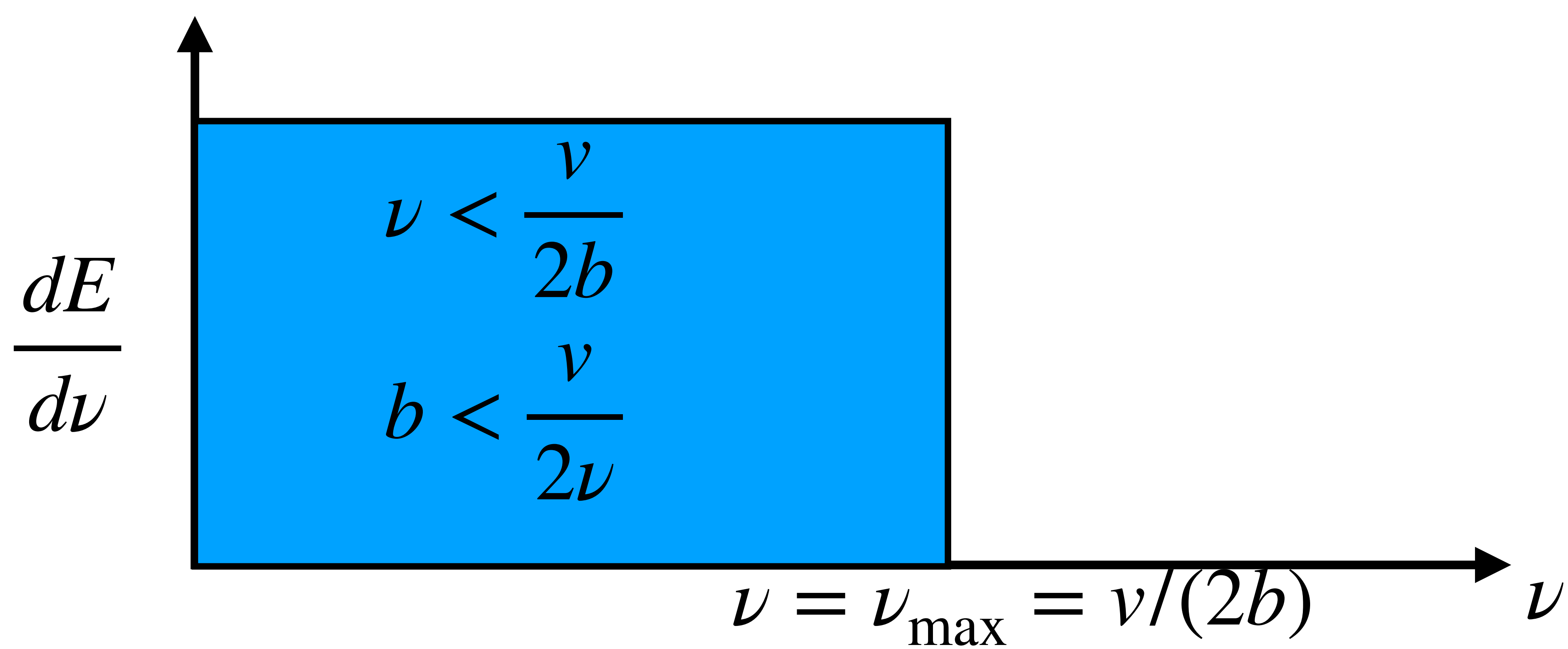

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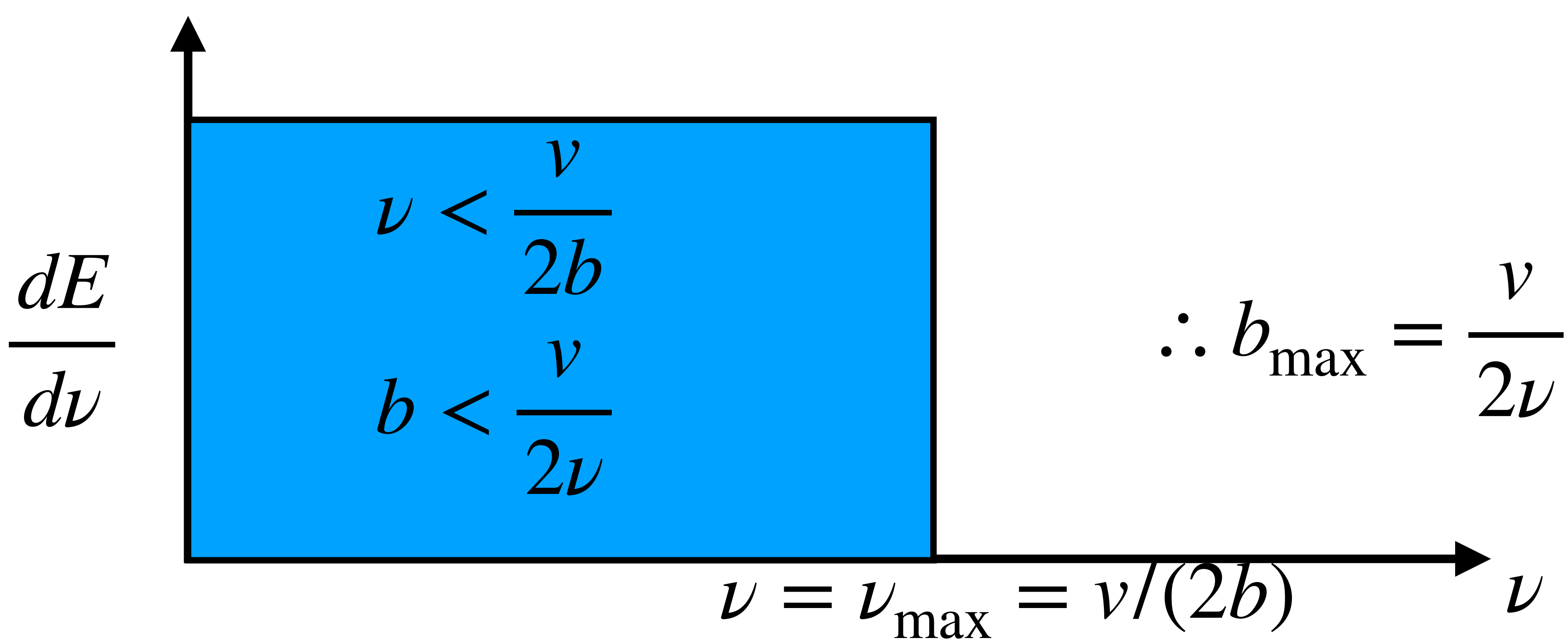

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Energy per frequency per event

Events per time per volume

- Now just need to work out limits $b_{min}$ and $b_{max}$.

- Max: $b_{\max} = v/(2\nu)$

- Minimum impact parameter given by the quantum limit:

$$m_e v \, . \, b_{\min} = \hbar$$

$$\therefore b_{\min} = \hbar/(m_e v)$$

# Bremsstrahlung

$$P_{\max} = \frac{q^2 \sigma_T c}{16\pi^2 \epsilon_0 b^4} \implies$$

$$\frac{dE}{d\nu dt dV} \approx \int_{b_{\min}}^{b_{\max}} P_{\max} (\Delta t)^2 \, . \, n_e v 2\pi b db = \frac{q^2 c n_e n_i \sigma_T}{4\pi \epsilon_0 v} \ln\left(\frac{b_{\max}}{b_{\min}}\right)$$

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$$b_{\max} = v/(2\nu) \qquad\qquad b_{\min} = \hbar/(m_e v)$$

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- Thermal distribution of electrons: $\frac{1}{2} m_e v^2 \sim k_B T_e$

(recall from lecture 6 that we are ignoring a pre-factor of ~3/2 on the RHS here.)

# Bremsstrahlung

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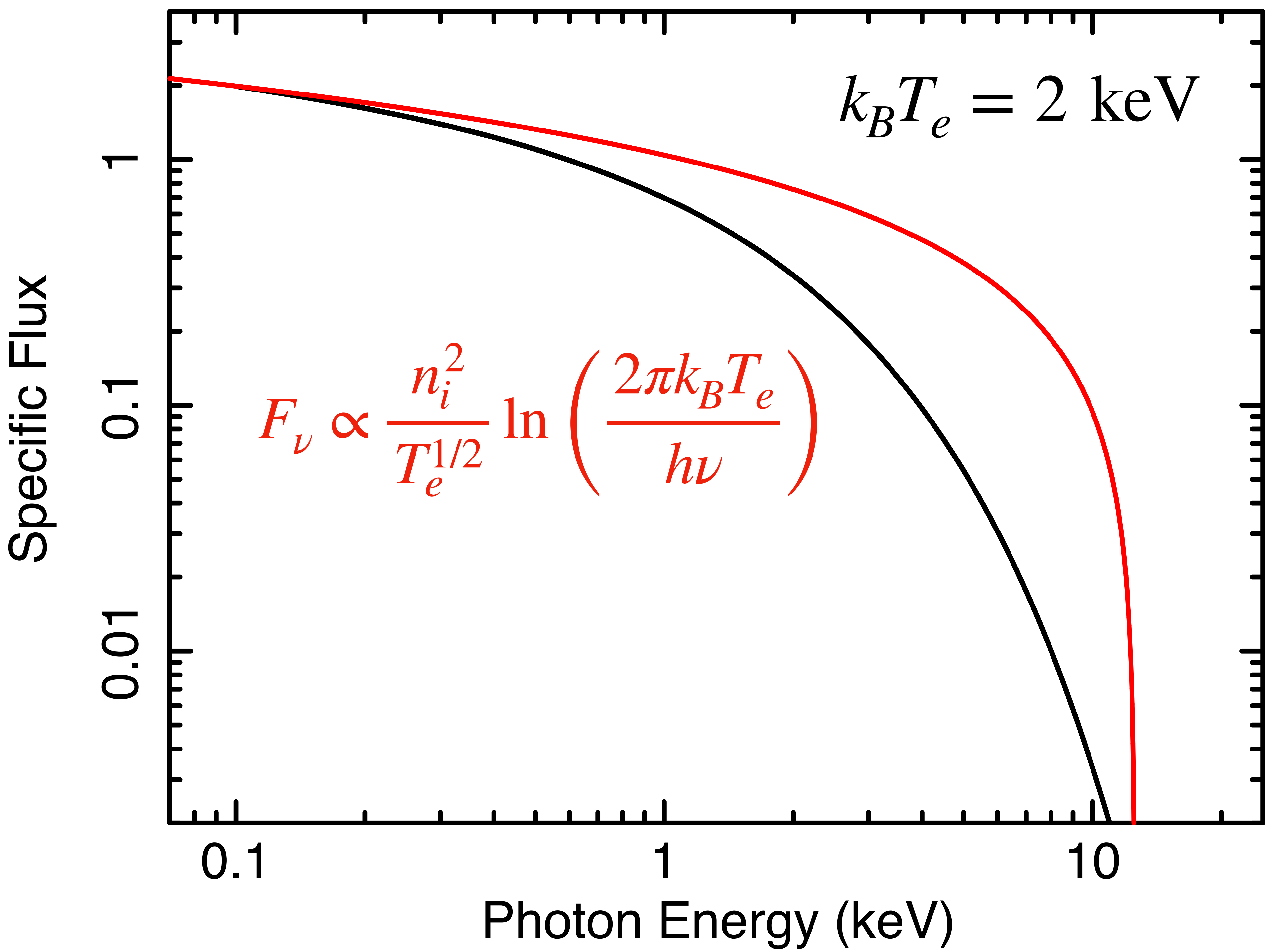
- Thermal distribution of electrons: $\frac{1}{2}m_e v^2 \sim k_B T_e$
- Ion density: $n_i \propto n_e$
- Therefore spectrum:

$$F_\nu \propto \frac{dE}{d\nu dt dV} \propto \frac{n_i^2}{T_e^{1/2}} \ln\left(\frac{2\pi k_B T_e}{h\nu}\right)$$

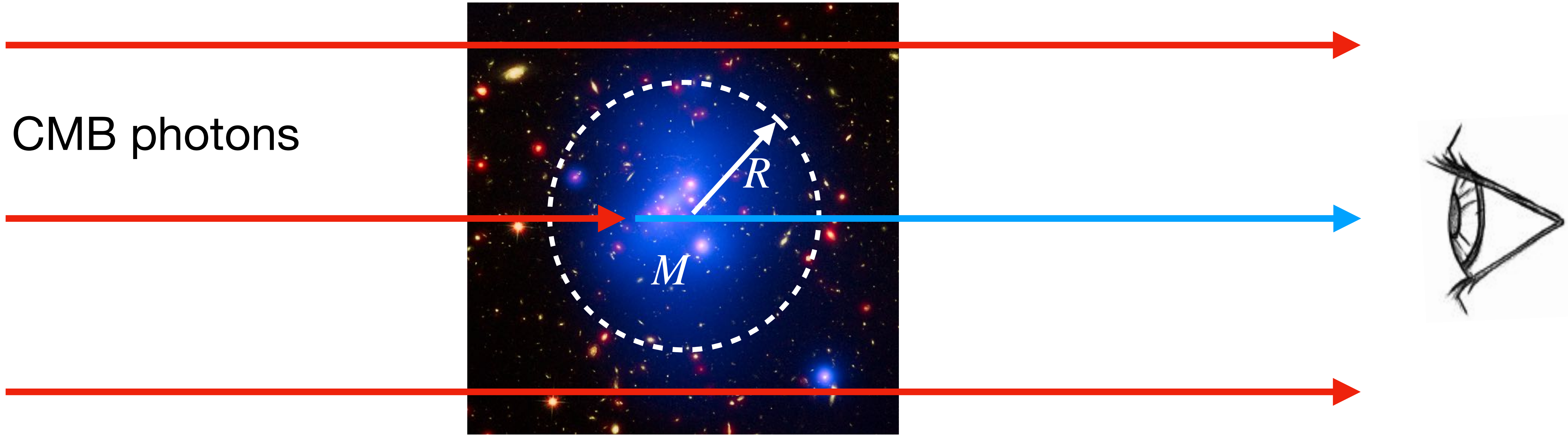
# Bremsstrahlung

- Simple calculation (red) gets the basic characteristics (n & T dependence plus cut-off energy), but misses exact shape (black).
- Can measure cluster temperature and density from X-ray spectrum.



# Sunyaev-Zeldovich Effect

- A fraction of Cosmic Microwave Background (CMB) photons passing through a cluster with be Compton up-scattered by hot electrons in the ICM.
- CMB photons have long wavelengths and the electrons are reasonably hot ($kT_e$ ~ few keV), therefore photons gain energy (let's say fractional gain is x)



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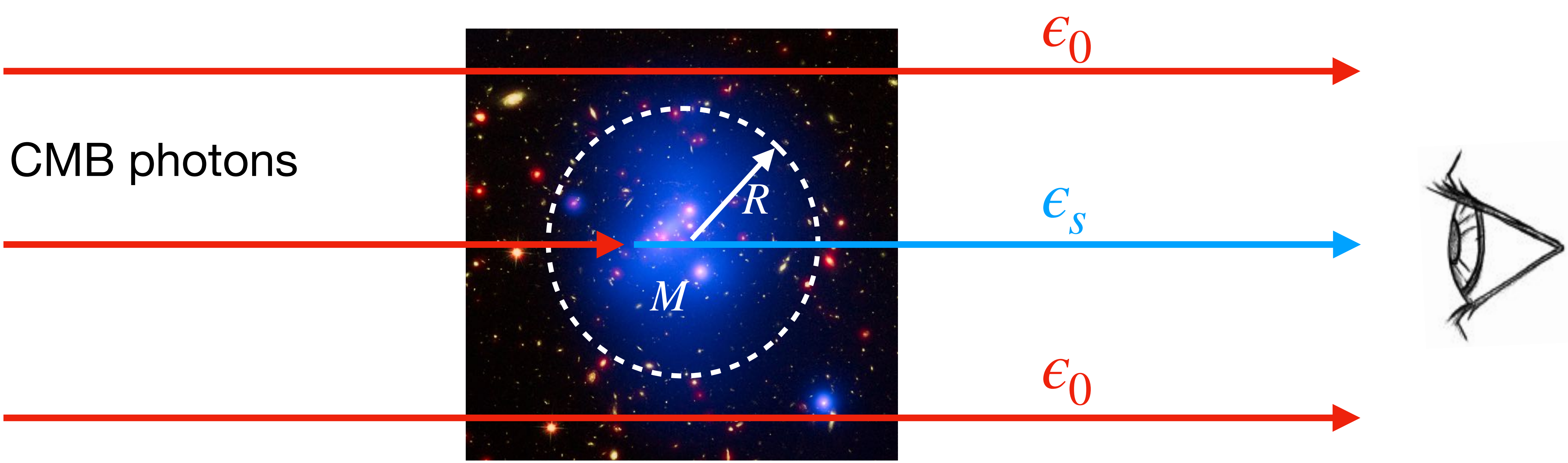
$$\epsilon_s = x\epsilon_0 = (1 + \Delta\epsilon/\epsilon)\epsilon_0$$



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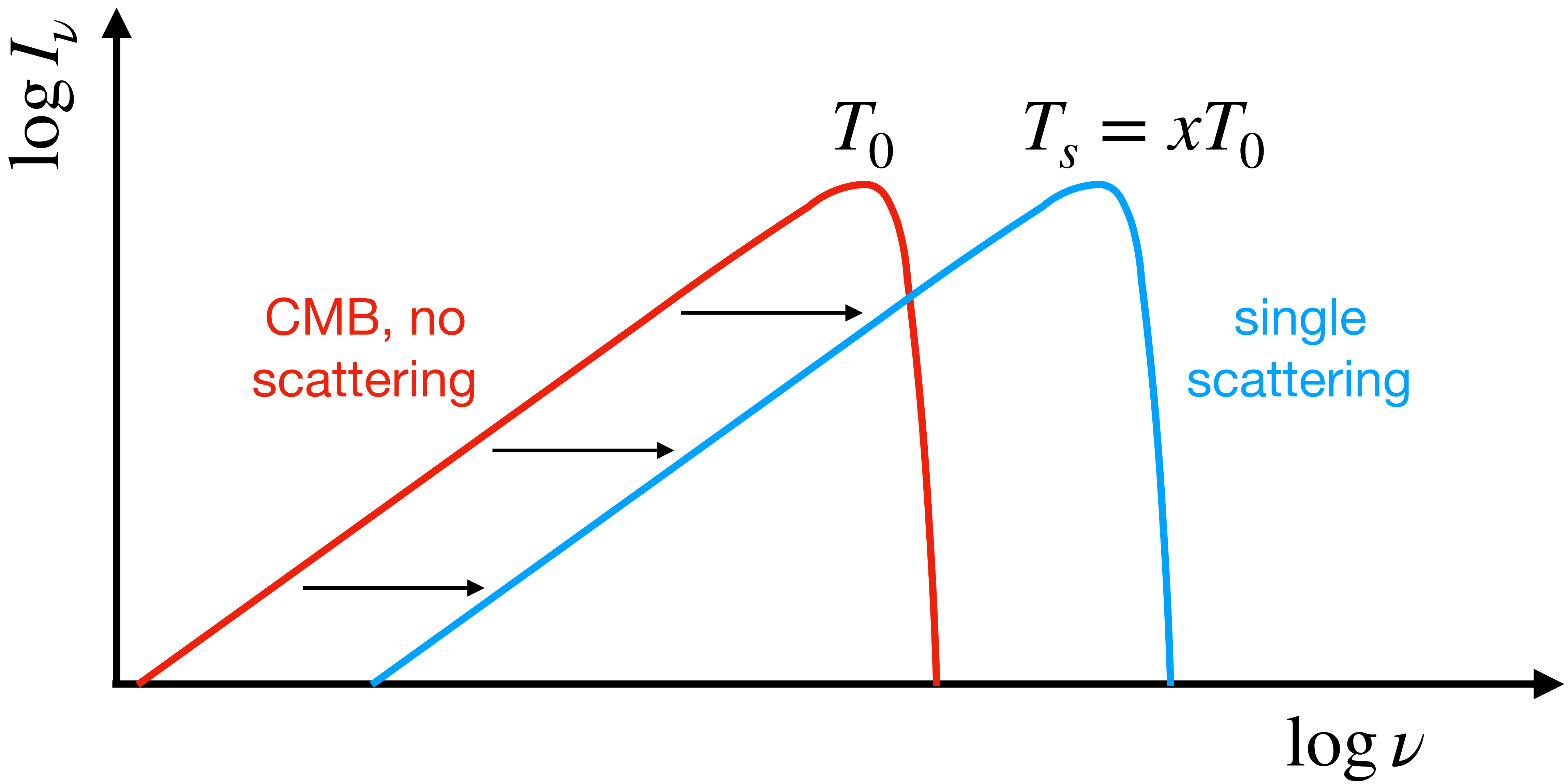
- Since the scattered photons on average gain energy, the blackbody function of the scattered photons has a higher effective temperature.



# Sunyaev-Zeldovich Effect

- Scattering conserves photons, so if the input spectrum were $I_\nu^0 = B_\nu(T_0)$ and each photon were scattered once, the output spectrum would be:

$$I_\nu^s = AB_\nu(T_s)$$

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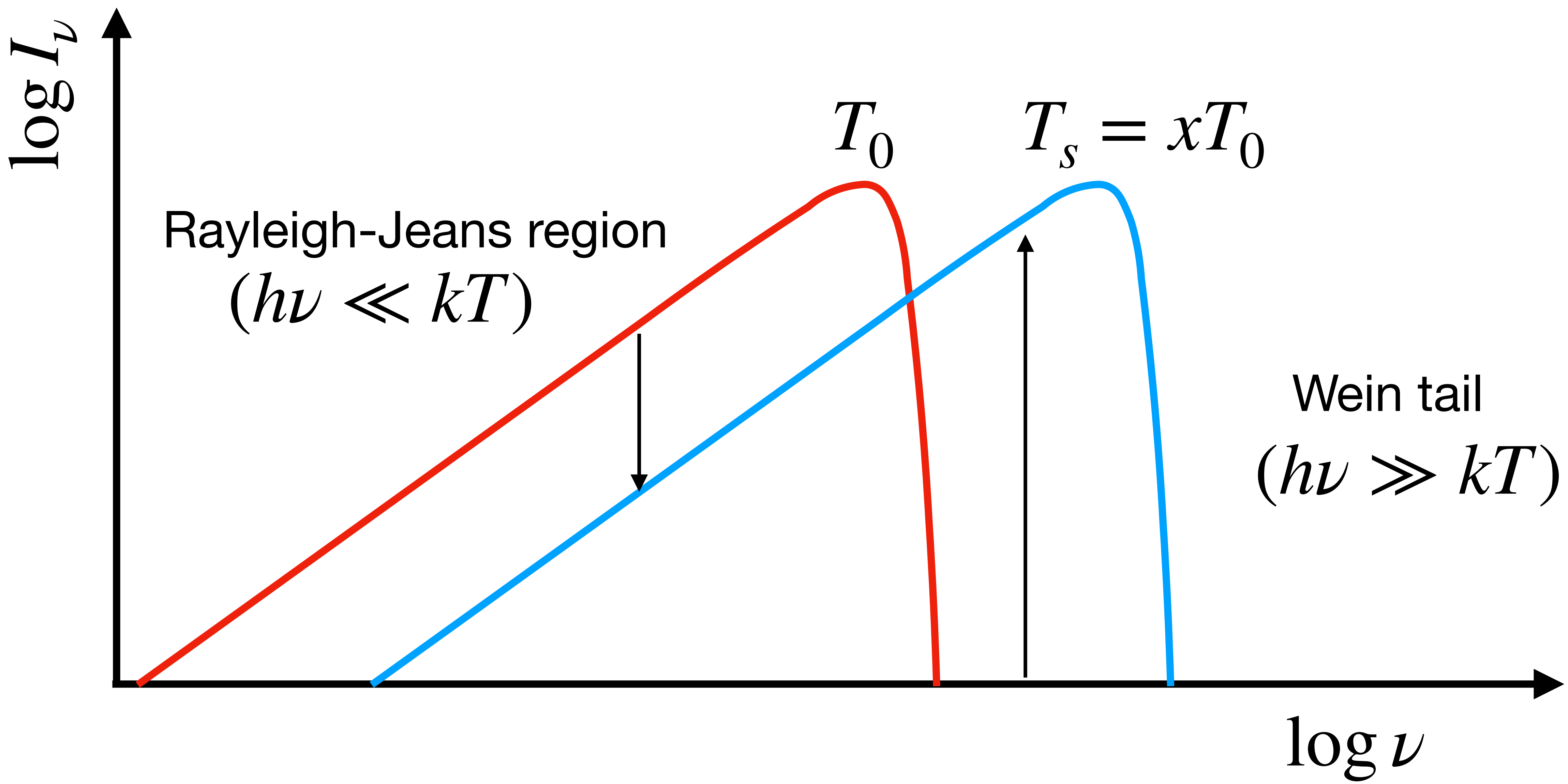
$$\therefore A = 1/x^3 \qquad \therefore I_\nu^s = x^{-3} B_\nu(T_s)$$

# Sunyaev-Zeldovich Effect

- Scattering *reduces* the specific intensity for low photon frequencies (Rayleigh-Jeans) and *increases* it for high photon frequencies (Wien).

$$I_\nu^0 = B_\nu(T_0) \qquad\qquad I_\nu^s = x^{-3} B_\nu(T_s)$$



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- The CMB spectrum we see from the cluster includes some photons that underwent no scatterings and some that underwent one (plus higher orders, but assume low optical depth).
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- Let's look at radio frequencies, well below CMB peak:

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- Therefore for radio frequencies:

$$\frac{\delta I_\nu}{I_\nu} \approx \tau\left(-1 + x^{-3}\frac{xT_0}{T_0}\right) = \tau(x^{-2}-1)$$

# Sunyaev-Zeldovich Effect

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- Confusing jargon warning: radio astronomers are mad-keen on brightness temperature, so it is common to hear that the SZ effect reduces the CMB temperature. It doesn't really, it just decreases the *brightness temperature* inferred by observing only in a radio band (the colour temperature increases).
- This is what the homework problem means about a "diminution" of temperature by a cluster:

$$\frac{\delta T_b}{T_b} = \frac{\delta I_\nu}{I_\nu}$$

# Sunyaev-Zeldovich Effect

$$\boxed{\therefore \frac{\delta I_\nu}{I_\nu} \approx -2\tau \frac{\Delta \epsilon}{\epsilon}}$$

- Electrons are thermal and so we can go back to our thermal Comptonisation discussion from lecture 6:

$$\left\langle \frac{\Delta \epsilon}{\epsilon} \right\rangle = 4\frac{kT_e}{m_e c^2} - \frac{h\nu}{m_e c^2} \approx 4\frac{kT_e}{m_e c^2}$$

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- The optical depth of the cluster is: $\tau \approx 2Rn_e\sigma_T$, where R is radius of the cluster
- Therefore:

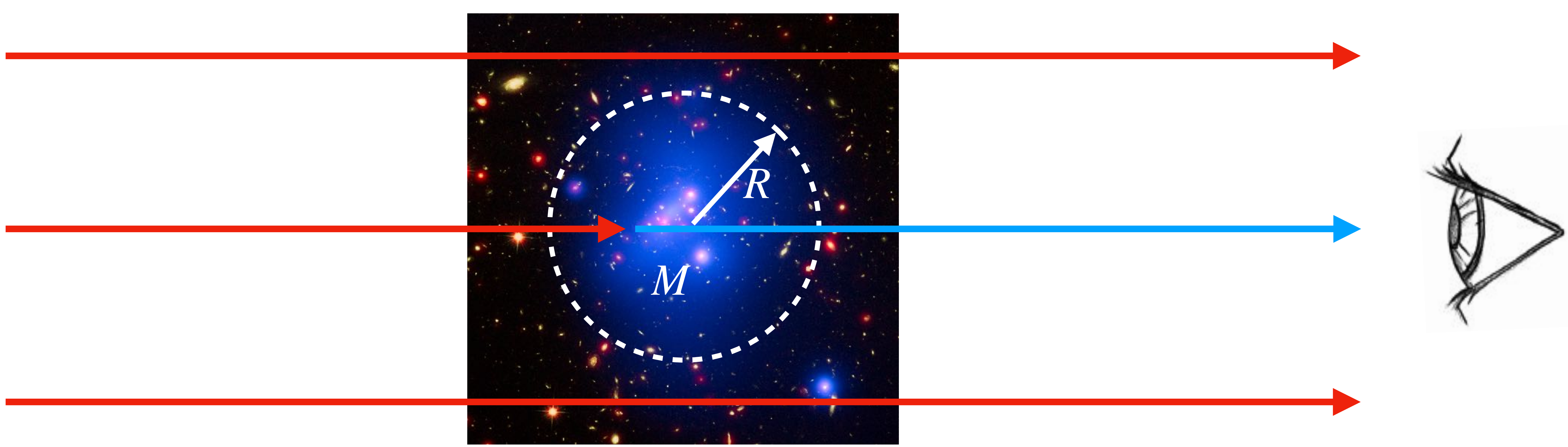
$$\therefore \frac{\delta I_\nu}{I_\nu} \approx -4Rn_e\sigma_T \frac{kT_e}{m_e c^2}$$

# Sunyaev-Zeldovich Effect

$$\frac{\delta I_\nu}{I_\nu} \approx -4Rn_e\sigma_T\frac{kT_e}{m_ec^2}$$

- We can be a little more precise by allowing for changes of density and electron temperature within the cluster:

$$\frac{\delta I_\nu}{I_\nu} \approx -2\sigma_T\int_0^\infty n_e\frac{kT_e}{m_ec^2}d\ell$$



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- The beauty of this is that we can measure the cluster density and temperature from X-ray observations of the hot gas, and then use the SZ effect to infer the radius of the cluster, R.

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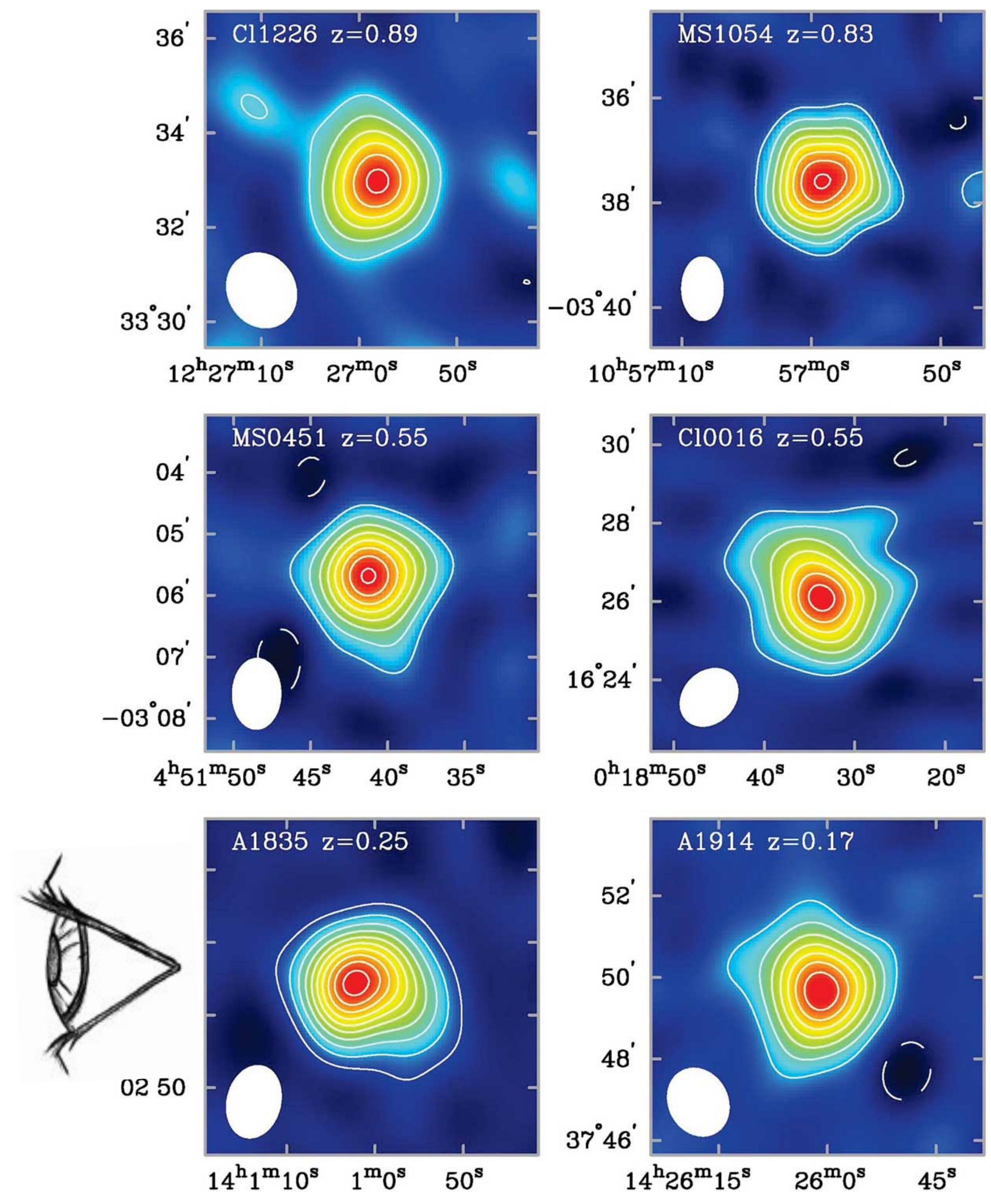


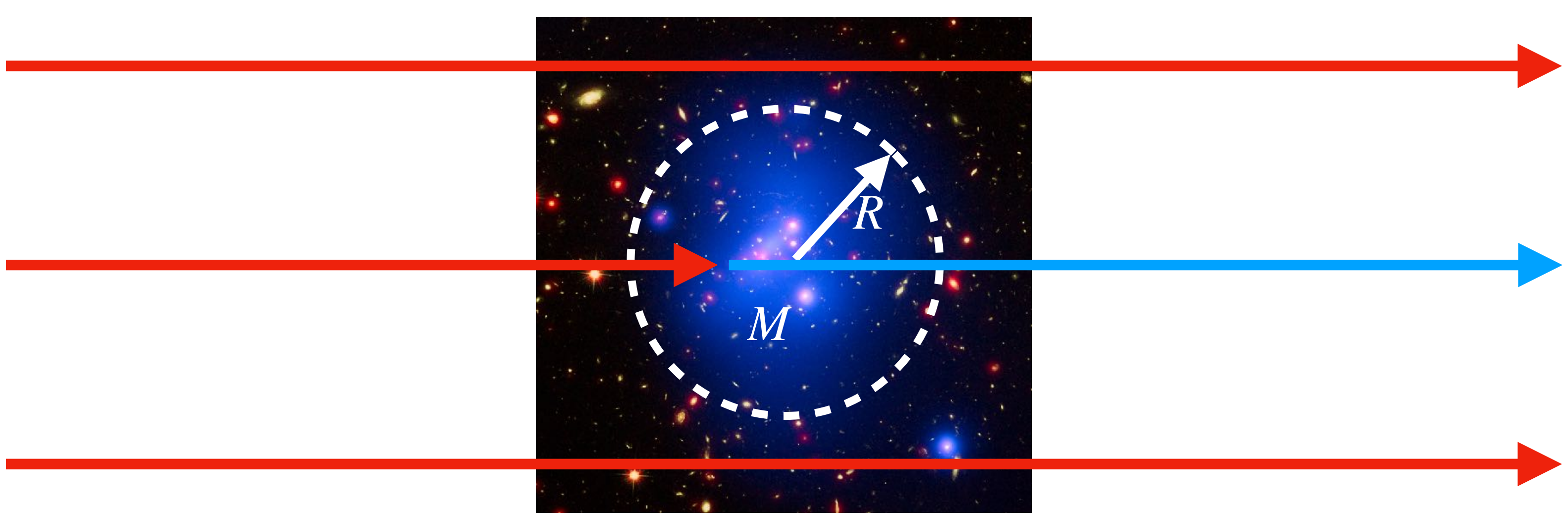

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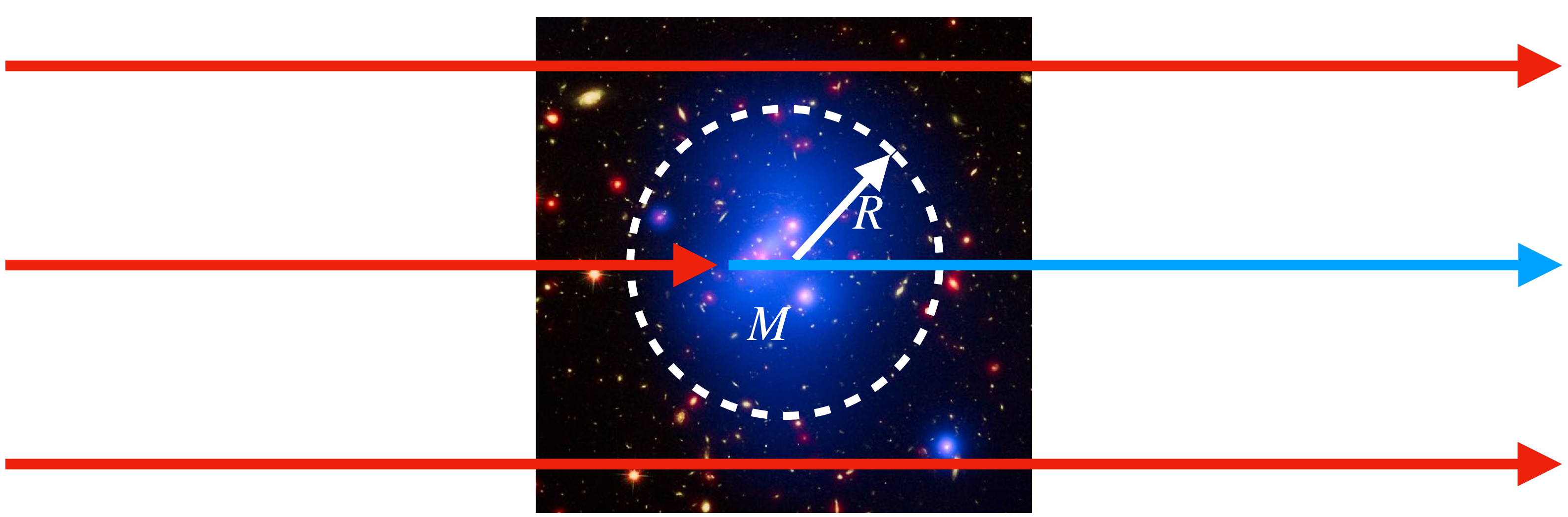


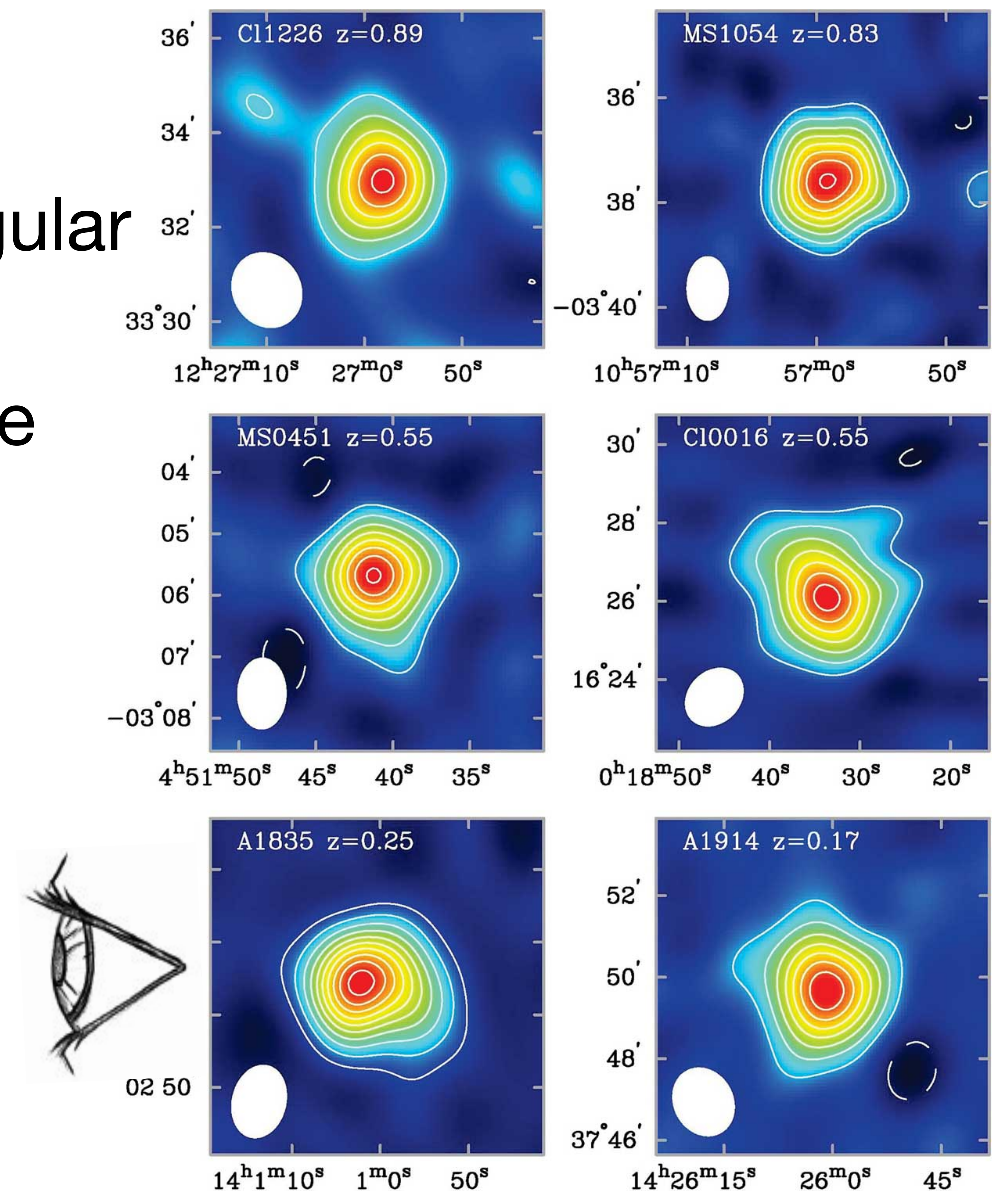

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- Non-spherical, but averages out if we do this for many clusters.