Stellar Structure and Evolution: Syllabus

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Primary Textbooks

- ***ZG:*** Zeilik & Gregory, "Introductory Astronomy & Astrophysics" (4th edition)
- ***CO:*** Carroll & Ostlie, "An Introduction to Modern Astrophysics" (Addison-Wesley)
- also: Prialnik, "An Introduction to the Theory of Stellar Structure and Evolution"

1. Observable Properties of Stars (ZG: Chapters 11, 12, 13; CO: Chapters 3, 7, 8, 9)

1.1 Luminosity, Parallax (ZG: 11; CO: 3.1)
1.2 The Magnitude System (ZG: 11; CO: 3.2, 3.6)
1.3 Black-Body Temperature (ZG: 8-6; CO: 3.4)
1.4 Spectral Classification, Luminosity Classes (ZG: 13-2/3; CO: 5.1, 8.1, 8.3)
1.5 Stellar Atmospheres (ZG: 13-1; CO: 9.1, 9.4)
1.6 Stellar Masses (ZG: 12-2/3; CO: 7.2, 7.3)
1.7 Stellar Radii (ZG: 12-4/5; CO: 7.3)

2. Correlations between Stellar Properties (ZG: Chapters 12, 13, 14; CO: Chapters 7, 8, 13)

2.1 Mass-Luminosity Relations (ZG: 12-2; CO: 7.3)
2.2 Hertzsprung-Russell diagrams and Colour-Magnitude Diagrams (ZG: 13-3; CO: 8.2)
2.3 Globular Clusters and Open (Galactic) Clusters (ZG:13-3, 14-2; OG: 13.4)
2.4 Chemical Composition (ZG: 13-3; CO: 9.4)
2.5 Stellar Populations (ZG: 14-3; CO: 13.4)

3. The Physical State of the Stellar Interior (ZG: P5, 16; CO: 10)

3.1 The Equation of Hydrostatic Equilibrium (ZG: 16-1; CO: 10.1)
3.2 The Dynamical Timescale (ZG: P5-4; CO: 10.4)
3.3 The Virial Theorem and its Implications (ZG: P5-2; CO: 2.4)
3.4 The Energy Equation and Stellar Timescales (CO: 10.3)
3.5 Energy Transport by Radiation (ZG: P5-10, 16-1) and Convection (ZG: 16-1; CO: 9.3, 10.4)

4. The Equations of Stellar Structure (ZG: 16; CO: 10)

4.1 The Mathematical Problem (ZG: 16-2; CO: 10.5)
4.1.1 The Vogt-Russell "Theorem" (CO: 10.5)
4.1.2 Stellar Evolution
4.1.3 Convective Regions (ZG: 16-1; CO: 10.4)
4.2 The Equation of State
4.2.1 Perfect Gas and Radiation Pressure (ZG: 16-1: CO: 10.2)
4.2.2 Electron Degeneracy (ZG: 17-1; CO: 15.3)
4.3 Opacity (ZG: 10-2; CO: 9.2)

5. Nuclear Reactions (ZG: P5-7 to P5-9, P5-12, 16-1D; CO: 10.3)

5.1 Nuclear Reaction Rates (ZG: P5-7)
5.2 Hydrogen Burning
5.2.1 The pp Chain (ZG: P5-7, 16-1D)
5.2.2 The CN Cycle (ZG: P5-9; 16-1D)
5.3 Energy Generation from H Burning (CO: 10.3)
5.4 Other Reactions Involving Light Elements *(Supplementary)*
5.5 Helium Burning (ZG: P5-12; 16-1D)

6. The Evolution of Stars

6.1 The Structure of Main-Sequence Stars (ZG: 16-2; CO 10.6, 13.1)
6.2 The Evolution of Low-Mass Stars (ZG: 16-3; CO: 13.2)
6.2.1 The Pre-Main Sequence Phase
6.2.2 The Core Hydrogen-Burning Phase
6.2.3 The Red-Giant Phase
6.2.4 The Helium Flash
6.2.5 The Horizontal Branch
6.2.6 The Asymptotic Giant Branch
6.2.7 White Dwarfs and the Chandrasekhar Mass (ZG: 17-1; CO: 13.2)

Useful Numbers

Astronomical unit	$\mathrm{AU} = 1.5 \times 10^{11}\,\mathrm{m}$
Parsec	$\mathrm{pc} = 3.26\ \mathrm{ly}$
	$= 3.086 \times 10^{16}\,\mathrm{m}$
Lightyear	$\mathrm{ly} = 9.46 \times 10^{15}\,\mathrm{m}$
Mass of Sun	$M_\odot = 1.99 \times 10^{30}\,\mathrm{kg}$
Mass of Earth	$M_\oplus = 5.98 \times 10^{24}\,\mathrm{kg}$
	$= 3 \times 10^{-6}\,M_\odot$
Mass of Jupiter	$M_{\mathrm{Jup}} = 10^{-3}\,M_\odot$
Radius of Sun	$R_\odot = 6.96 \times 10^{8}\,\mathrm{m}$
Radius of Earth	$R_\oplus = 6380\,\mathrm{km}$
Radius of Jupiter	$R_{\mathrm{Jup}} = 10^{-3}\,R_\odot$
Luminosity of Sun	$L_\odot = 3.86 \times 10^{26}\,\mathrm{W}$
Effective temperature of Sun	$T_{\mathrm{eff}} = 5780\,\mathrm{K}$
Central temperature of Sun	$T_{\mathrm{c}} = 15.6 \times 10^{6}\,\mathrm{K}$
Distance to the Galactic centre	$R_0 = 8.0\,\mathrm{kpc}$
Velocity of Sun about Galactic centre	$V_0 = 220\,\mathrm{km\,s^{-1}}$
Diameter of Galactic disc	$= 50\,\mathrm{kpc}$
Mass of Galaxy	$= 7 \times 10^{11}\,M_\odot$

Summary of Equations

Equation of Stellar Structure

Equation of Hydrostatic Equilibrium:

$$\frac{\mathrm{d}P_r}{\mathrm{d}r} = -\frac{GM_r\rho_r}{r^2} \quad \text{(page 45)}$$

Equation of Mass Conservation:

$$\frac{\mathrm{d}M_r}{\mathrm{d}r} = 4\pi r^2 \rho_r \quad \text{(page 45)}$$

Energy Conservation (no gravitational energy):

$$\frac{\mathrm{d}L_r}{\mathrm{d}r} = 4\pi r^2 \rho_r \varepsilon_r \quad \text{(page 52)}$$

Energy Transport (Radiative Diffusion Equation):

$$L_r = -4\pi r^2 \frac{4ac}{3\kappa\rho} T^3 \frac{\mathrm{d}T}{\mathrm{d}r} \quad \text{(page 55)}$$

Energy Transport by Convection, Convective Stability:

$$\frac{\mathrm{d}T}{\mathrm{d}r} = \frac{\gamma - 1}{\gamma} \frac{T}{P} \frac{\mathrm{d}P}{\mathrm{d}r} \quad \text{(page 57)}$$

Constitutive Relations

Equation of State, Ideal Gas:

$$P = NkT = \frac{\rho}{\mu m_{\mathrm{H}}} kT \quad \text{(page 65)}$$

Equation of State, Radiation Pressure:

$$P = \frac{1}{3} a T^4 \quad \text{(page 66)}$$

Equation of State, Electron Degeneracy ($T = 0\,K$):

$$P = K_1 \left(\frac{\rho}{\mu_e m_{\mathrm{H}}}\right)^{5/3} \quad \text{(page 66)}$$

(non-relativistic degeneracy)

$$P = K_2 \left(\frac{\rho}{\mu_e m_{\mathrm{H}}}\right)^{4/3} \quad \text{(page 67)}$$

(relativistic degeneracy)

Notes:

Opacity: Thomson (Electron) Scattering:

$$\kappa = 0.020\,\mathrm{m^2\,kg^{-1}}\,(1 + X) \quad \text{(page 69)}$$

Kramer's Opacity:

$$\kappa \propto \rho\, T^{-3.5} \quad \text{(page 69)}$$

Low-Temperature Opacity:

$$\kappa \propto \rho^{1/2}\, T^4 \quad \text{(page 69)}$$

Energy Generation Rates (Rough!)

PP Burning:

$$\varepsilon_{\mathrm{PP}} \propto \rho\, X_{\mathrm{H}}^2\, T^4 \quad \text{(page 79)}$$

CNO Burning:

$$\varepsilon_{CNO} \propto \rho\, X_{\mathrm{H}}\, X_{\mathrm{CNO}}\, T^{20} \quad \text{(page 79)}$$

Helium Burning (triple α):

$$\varepsilon_{3\alpha} \propto X_{\mathrm{He}}^3\, \rho^2\, T^{30} \quad \text{(page 82)}$$

Stellar Timescales

Dynamical Timescale:

$$t_{\mathrm{dyn}} \simeq \frac{1}{\sqrt{4G\rho}} \quad \text{(page 48)}$$

$$\sim 30\,\mathrm{min}\,\left(\rho/1000\,\mathrm{kg\,m^{-3}}\right)^{-1/2}$$

Thermal (Kelvin-Helmholtz) Timescale):

$$t_{\mathrm{KH}} \simeq \frac{GM^2}{2RL} \quad \text{(page 51)}$$

$$\sim 1.5 \times 10^7\,\mathrm{yr}\,(M/M_\odot)^2\,(R/R_\odot)^{-1}\,(L/L_\odot)^{-1}$$

Nuclear Timescale:

$$t_{\mathrm{nuc}} \simeq M_c/M\ \eta\ (Mc^2)/L \quad \text{(page 52)}$$

$$\sim 10^{10}\,\mathrm{yr}\,(M/M_\odot)^{-3}$$

(Radiative) Diffusion Timescale:

$$t_{\mathrm{diff}} = N \times \frac{l}{c} \simeq \frac{R_s^2}{lc} \quad \text{(page 53)}$$

Notes:

Derived Relations

Central Temperature Relation (for Ideal Gas):

$$kT_{\rm c} \simeq \frac{GM_s\,\mu m_H}{R_s} \quad \text{(page 46)}$$

Virial Theorem:

$$3(\gamma - 1)U + \Omega = 0 \quad \text{(page 50)}$$

Mass–Luminosity Relation (for stars $\sim 1\,M_\odot$):

$$L \simeq L_\odot \left(\frac{M}{M_\odot}\right)^4 \quad \text{(page 85)}$$

Mass–MS Lifetime Relation (for stars $\sim 1\,M_\odot$):

$$T_{\rm MS} \simeq 10^{10}\,{\rm yr} \left(\frac{M}{M_\odot}\right)^{-3} \quad \text{(page 85)}$$

Mass–Radius Relation for White Dwarfs (non-relativistic):

$$R \propto \frac{1}{m_e}\,(\mu_e m_{\rm H})^{5/3}\,M^{-1/3} \quad \text{(page 98)}$$

Chandrasekhar Mass for White Dwarfs:

$$M_{\rm Ch} = 1.457 \left(\frac{2}{\mu_e}\right)^2 M_\odot \quad \text{(page 99)}$$

Schwarzschild Radius (Event Horizon) for Black Holes:

$$R_{\rm S} = \frac{2GM}{c^2} \simeq 3\,{\rm km} \left(\frac{M}{M_\odot}\right) \quad \text{(page 112)}$$

Notes:

Miscellaneous Equations

Distance Modulus:

$$(m - M)_V = 5\,\log\,(D/10{\rm pc}) \quad \text{(page 12)}$$

Absolute V Magnitude:

$$M_V = -2.5\,\log\,L/L_\odot + 4.72 + B.C. + A_V \quad \text{(page 12)}$$

Salpeter Initial Mass Function (IMF):

$$f(M)\,{\rm d}M \propto M^{-2.35}\,{\rm d}M \quad \text{(page 15)}$$

Black-Body Relation:

$$L = 4\pi\,R_s^2\,\sigma T_{\rm eff}^4 \quad \text{(page 17)}$$

Kepler's Law:

$$a^3 \left(\frac{2\pi}{P}\right)^2 = G(M_1 + M_2) \quad \text{(page 25)}$$

Notes: