Extra Problems

1. (a) Estimate the height of the Coulomb barrier between two protons and compare this with the mean thermal energy of a proton at temperature $T \simeq 10^7$ K. Explain how thermonuclear reactions can take place in stellar interiors at such temperatures.

[10]

(b) Describe the principal reaction chains by which hydrogen is converted into helium in (i) a star with mass 1 $M_\odot$ (ii) a star with mass 10 $M_\odot$. Comment on the timescales of these processes and the temperature sensitivity of the overall energy generation rates.

[12]

(c) The energy generation rate for the triple-α reaction converting helium to carbon is given by

$$\epsilon(T) = \frac{A}{T^3} \exp\left(-\frac{42.9 \times 10^8}{T}\right),$$

where A is a function of density and composition. Develop expressions for the ratio $\epsilon(T)/\epsilon(T_0)$ at a temperature $T_0 + \delta T$ close to T_0 using (i) the above energy generation rate and (ii) the relation $\epsilon(T) = \epsilon(T_0)(T/T_0)^n$. Hence obtain a value of n for which the two expressions are identical to first order in δT and evaluate n for $T_0 = 10^8$ K.

[8]

(d) Comment briefly on the significance of your results for helium ignition in a stellar core supported by degenerate electrons.

[5]

2. (a) The deceleration parameter is defined by

$$q_0 = -\frac{\ddot{a}(t_0)}{a(t_0)} \frac{1}{H_0^2} \ .$$

Use the acceleration equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right)$$

to show that a radiation-dominated Universe at critical density (radiation pressure is $p_{rad} = c^2\rho_{rad}/3$) has $q_0 = \Omega_0$.

[10]

(b) Certain models of the early Universe permit an expansion rate $\alpha \propto t^m$ where m is an arbitrary positive constant. What range of values of m corresponds to an inflationary expansion (defined as an evolution of the Universe where the scale factor is accelerating)?

[10]