

Michaelmas Term 2011 1st Year Relativity - Problems 2

Special Relativity: Professor K M Blundell

SECTION A – Collision problems, threshold energies, decays, recoils

1. The K^0 meson can decay at rest into a pair of charged π mesons. Given that the rest masses of the K and π are 498 and $140 \text{ MeV}/c^2$ respectively, show that the speeds of the pions are $0.83c$ in the rest frame of the K^0 .

2. *Estimate* the mass lost when 1 million tonnes of TNT explodes. Assume that each chemical reaction between individual molecules involves 10 eV of energy.

3. Energy from the Sun (distance $1.496 \times 10^{11} \text{ m}$ from Earth) arrives at the Earth (above the atmosphere) at a rate of about 1400 W m^{-2} . How fast is the sun losing mass due to energy radiation?

[Ans: $4.4 \times 10^9 \text{ kg s}^{-1}$]

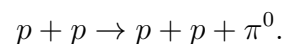
4. An electron accelerated from the Stanford Linear Accelerator (SLAC) in California has a total energy of 50 GeV . (a) How much of this is kinetic energy? (b) What is the momentum of the electron? (c) What is its speed?

[Ans: (a) $0.999999E$, (b) $2.7 \times 10^{-17} \text{ kg m s}^{-1}$, (c) $(1 - (5 \times 10^{-11}))c$]

5. A π^0 meson whose rest mass is $135 \text{ MeV}/c^2$ is moving with a kinetic energy of 1 GeV . It decays in flight into two photons whose paths are along the direction of motion of the meson. Find the energies of the two photons.

[Ans: 4 and 1131 MeV .]

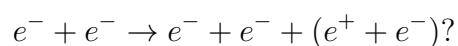
6. Neutral π^0 mesons can be formed by the reaction



What is the threshold kinetic energy for π^0 production by this process if the target proton is stationary? (Masses of π^0 and proton are 135 and $938 \text{ MeV}/c^2$.)

[Ans: 280 MeV .]

7. What is the threshold $K.E.$ for production of an electron pair by collision of an incident electron with a stationary electron *i.e.* for



[Ans: $6m_e c^2$, where m_e is the rest mass of an electron.]

8. The Tevatron at the Fermi National Accelerator Laboratory in the USA carries two beams of 1 TeV (1000 GeV) protons moving in opposite directions, so that for proton-proton collisions the centre of mass frame is also the laboratory frame. What proton energy must an accelerator provide to get the same centre of mass energy when bombarding a stationary hydrogen target?

[Ans: 2.13×10^3 TeV.]

9. (i) Show that a free electron moving in a vacuum at velocity v cannot emit a single photon.
 (ii) A hydrogen atom in an excited state can emit a single photon, why is this different from (i) above?

10. An atom in an excited state of energy Q_0 (as measured in its rest frame) above the ground state moves towards a scintillation counter with speed v . The atom decays to its ground state by emitting a photon of energy Q (as recorded by the counter), coming completely to rest as it does so. If the rest mass of the atom is M , show that $Q = Q_0[1 + (Q_0/2Mc^2)]$.

SECTION B – Some past Prelims questions

11. State the *Lorentz transformations* which relate the space and time coordinates in two inertial frames, S and S' , travelling with relative speed v along the $x(x')$ axis. Use the transformations to derive expressions for the phenomena of *length contraction* and *time dilation*.

A rocket passes close to the earth with relative velocity $0.8c$, at which time observers on the earth and the rocket reset their clocks ($t_E = t_R = 0$).

- (a) After 30 minutes as measured by the rocket's clock, the rocket passes a space-station which is fixed relative to the earth. What time is recorded by the observer on the earth?
- (b) How far is the space-station relative to the earth
 - (i) in earth coordinates,
 - (ii) in rocket coordinates?
- (c) When the rocket passes the space-station, the rocket sends a radio signal back to earth. At what time (by earth time) does the signal arrive?
- (d) The station on earth replies immediately. At what time (by rocket time) is the reply received by the rocket?

12. Write down the relativistic expressions for momentum p and energy E of a particle of rest mass m_0 in terms of its velocity v and show that they satisfy the equation

$$E^2 = p^2c^2 + m_0^2c^4$$

- (a) An electron of energy 9.0 GeV and a positron of energy E collide head-on to produce a B^0 meson and an anti- B^0 meson, each with a mass of $5.3 \text{ GeV}/c^2$. What is the minimum positron energy (threshold energy) required to produce the B^0 -meson pair? [You may neglect the rest-mass energies of the electron and positron.]
- (b) Electrons and positrons collide head-on in the laboratory with beam energies as in part (a), producing B^0 and anti- B^0 mesons at the threshold energy. The B^0 mesons undergo decay with a mean proper lifetime of $1.5 \times 10^{-12} \text{ s}$. Calculate the mean distance that the B^0 mesons travel before decay, as observed in the rest frame of the laboratory.

13. State the *Lorentz transformations* which relate the space and time coordinates in two inertial frames, S and S' , where S' is travelling with velocity v relative to S along the positive x direction. Use the transformations to derive an expression for the phenomenon of *length contraction*.

An observer in S measures the velocity of a body travelling in the positive x direction to be u_x . Show that the velocity of the body measured by an observer in S' is given by

$$u'_x = \frac{u_x - v}{1 - vu_x/c^2}$$

An observer on the earth measures a spaceship to be approaching at a speed of $0.7c$. The same observer measures a second spaceship to be approaching in the opposite direction at a speed of $0.8c$. Each spaceship has a length of 100 m (as measured by observers in the respective spaceships).

- (a) What are the lengths of each spaceship, as measured by the observer on the earth?
- (b) What is the length of each spaceship, as measured by the observer on the other spaceship?

14. Write down the relativistic expressions for momentum p and energy E of a particle of rest mass m_0 in terms of its velocity v and show that they satisfy the equation

$$E^2 = p^2 c^2 + m_0^2 c^4$$

A D^0 meson decays at rest to a \bar{K}^0 meson and a π^0 meson. Calculate the energy and velocity of the π^0 .

The π^0 produced in the D^0 decay subsequently decays into a pair of photons. Calculate the maximum and minimum energies of each photon in the D^0 rest frame.

[The rest masses of the D^0 , \bar{K}^0 and π^0 mesons are $1.86 \text{ GeV}/c^2$, $0.50 \text{ GeV}/c^2$ and $0.13 \text{ GeV}/c^2$, respectively.]