Stellar Spectroscopy

Resolved observations of the sun allow us to look at variations across the surface, but we can only look at (almost all) other stars in integrated light.

In the visible, we see the photosphere as a disk (projected hemisphere). The surface temperature is ~5700K compared to 15,000,000K in the core. Photons journey outwards being multiply scattered, absorbed and re-emitted before they emerge. Random walk process $R \sim \lambda N^{1/2}$ and with mean free path, $\lambda \sim$ 1cm in the solar core, $N \sim 10^{20}$ encounters before emerging from the photosphere

The visible continuum spectrum and emission and/or absorption lines inform us about the surface layers.

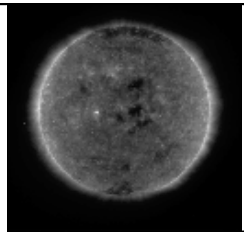
Energy flow and stellar models allow us to infer the interior structure (checked via astroseismology)

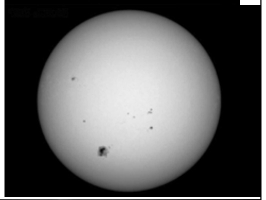
What you see depends on how you look: see EUV, Visible images

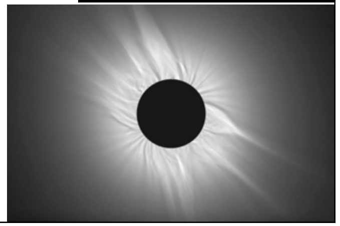
Above the photosphere, the major regions are the Chromosphere and Corona and the solar wind

The temperature in the tenuous Corona is ~ 2 10^6 K with emission lines from highly ionized species.

The heating mechanism is not fully understood, but involves magnetic reconnection.

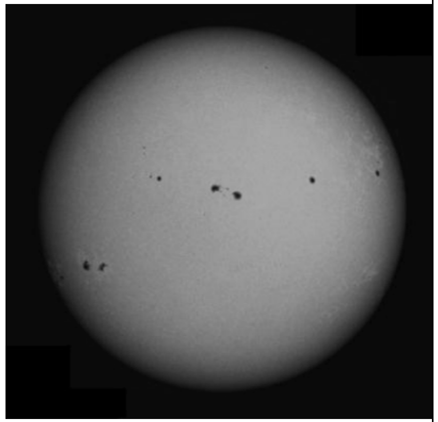






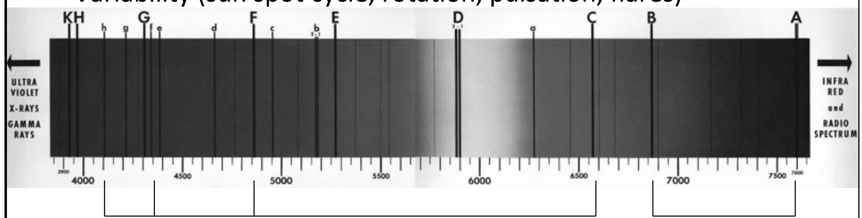
The Sun as a Typical Star

- Solar Interior cannot be probed directly (except neutrinos & helioseismology)
- Emerging radiation from solar atmosphere tells us
  - Total Flux (Luminosity)
  - Photospheric Temperature, Density
  - Surface Abundances
  - Dynamics
- Photosphere well defined layer
  - T~ 5800K, $\rho \sim 10^{-4}$ kg m^{-3}
  - ~300 km thick, (0.0005 $R_{\odot}$)
  - Fraunhofer Spectrum
  - Limb Darkening, Granulation, Sunspots



Stellar Spectra

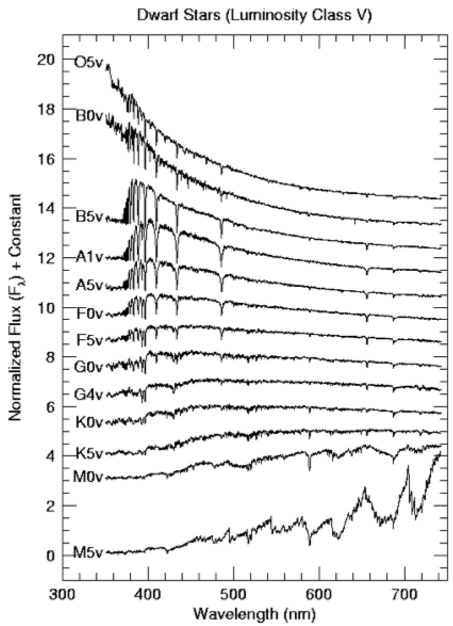
- Stars classified via spectral lines
- HD Spectral Sequence OBAFGKMLT 40,000 -> 1500 K
- Sun is a G2V main sequence star (V= dwarf, high surface gravity)
- Fraunhofer absorption lines - element abundances seen against continuous (approx black body) spectrum
- Note that the density is high and we do not see forbidden lines from the photosphere (but we do from the Corona)
- Variability (sun spot cycle, rotation, pulsation, flares)



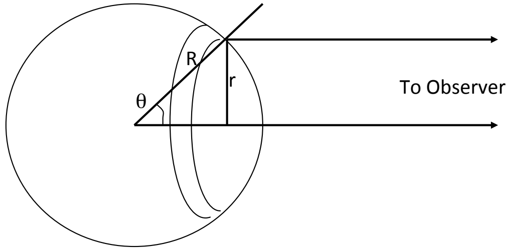
H Balmer series ⊕ Atmospheric absorption

Note the appearance and then disappearance of the break in the spectra near 360nm (the Balmer jump) , the weakening of hydrogen absorption lines in G-type stars and the onset of molecular band emission in the cool M stars

These gross change reflect changes in the dominant source of opacity in the photospheres



Measurement of Stellar Flux



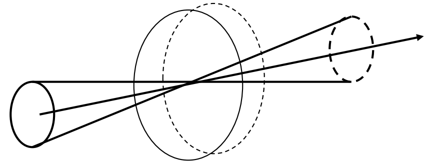
Annulus on stellar surface has an area $2\pi r\ dr = 2\pi R^2 \sin\theta\cos\theta d\theta$ normal to line of sight

and subtends solid angle $d\Omega = 2\pi(R/D)^2 \sin\theta\cos\theta d\theta$ with distance D

Flux measured by observer is $F_\nu = (R/D)^2 F_{\nu(0)}$.

Emission and Absorption

In stars, photons are *absorbed* and *scattered* (absorbed energy may be thermalised before being re-emitted).



Consider radiation of Specific Intensity I_ν and solid angle $d\Omega$ normally incident on a slab of stellar atmosphere with cross section dA, thickness ds and density ρ.

As it propagates through, the beam loses energy through absorption

$$dE_\nu = k_\nu I_\nu \rho ds dA d\Omega d\nu dt$$

where k_ν is the extinction coefficient per unit mass, or opacity, and consists of scattering and and absorption terms $k = \sigma + \alpha$

Radiative Transfer

Energy emitted in the direction of propagation

$$dE_\nu = j_\nu \rho ds dA d\Omega d\nu dt$$

where j_ν is the emission coefficient per unit mass, containing contributions from scattering and thermal emission

The ratio j_ν / k_ν - emissivity/opacity - is known as the Source Function, denoted by S_ν

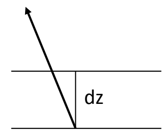
Note that in a purely absorbing atmosphere, $S_\nu = B_\nu$ (T)
- limiting case for thermodynamic equilibrium
 where $j_\nu = k_\nu B_\nu$(T) (Kirchoff's Law)

in a pure scattering atmosphere with no absorption, $S_\nu = J_\nu$.

The difference between energy emitted and absorbed in the element is related to the change in Specific Intensity of the beam:

$$dI_\nu dA d\Omega d\nu dt = (J_\nu \rho ds - k_\nu \rho I_\nu ds) dA d\Omega d\nu dt$$

Optical Depth



Consider a path through the slab $ds = dz/\cos\theta = dz/\mu$

$$\mu dI_\nu / \rho dz = J_\nu - k_\nu I_\nu$$

In general k will vary as a function of x, and the integral of the extinction coefficient w.r.t. distance is the *Optical Depth,* τ.

$\tau = \int k(x)\ \rho\ dz$ and Specific Intensity falls off with τ as:

$I = I_0 e^{-\tau}$ and optical depth $\tau = \ln (I_0/I)$ (note: τ is measured inwards) and is a direct measure of the absorptivity of the medium

Purely emitting medium: $\mu dI_\nu / dz = \rho J_\nu$

and no emission : $\mu\ dI_\nu / dz = - k_\nu \rho I_\nu$

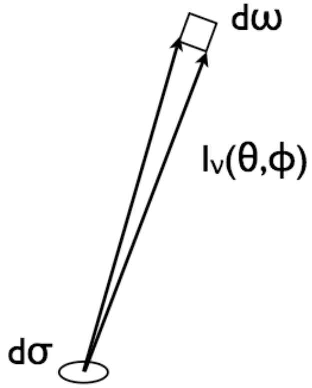
Dividing the top equation by k_ν gives the standard form of the Radiative Transfer Equation :

$$\mu \frac{dI_\nu}{d\tau_\nu} = I_\nu - S_\nu$$

Emergent Flux

Consider radiation of Specific Intensity I_ν and solid angle dω emerging from a surface dσ at an angle θ to the normal.

The flux:



$$\pi F_\nu(\tau_\nu) = \int_0^{4\pi} I_\nu(\tau_\nu,\theta)\cos\theta d\omega$$

$$\pi F_\nu(\tau_\nu) = \int_0^{2\pi} \int_0^{\pi} I_\nu(\tau_\nu,\theta)\cos\theta\sin\theta d\theta d\phi$$

And with d(cosθ)/dθ = -sinθ,

$$\pi F_\nu(\tau_\nu) = 2\pi\int_{-1}^{1} I_\nu(\tau_\nu,\theta)\cos\theta d(\cos\theta)$$

So with a plane-parallel atmosphere

The Grey Atmosphere

Multiplying the equation of Radiative Transfer by integration factor $e^{-\tau_\nu \sec\theta}$

gives $$\frac{dI_\nu}{d(\tau_\nu \sec\theta)} e^{-\tau_\nu \sec\theta} = (I_\nu - S_\nu)e^{-\tau_\nu \sec\theta}$$

or $$(-I_\nu + \frac{dI_\nu}{d(\tau_\nu \sec\theta)})e^{-\tau_\nu \sec\theta} = -S_\nu e^{-\tau_\nu \sec\theta}$$

$$\frac{dI_\nu e^{-\tau_\nu \sec\theta}}{d(\tau_\nu \sec\theta)} = -S_\nu e^{-\tau_\nu \sec\theta}$$

And $$\left[I_\nu e^{-\tau_\nu \sec\theta}\right]_0^\infty = I_\nu(0,\theta) = \int S_\nu e^{-\tau_\nu \sec\theta} d(\tau_\nu \sec\theta)$$

We approximate the depth dependence of the source function S_ν by $S_\nu = a_\nu + b_\nu \tau_\nu$ along the normal direction

So $$I_\nu(\tau_\nu,\theta) = \int a_\nu e^{-\tau_\nu \sec\theta} d(\tau_\nu \sec\theta) + b_\nu \int \tau_\nu e^{-\tau_\nu \sec\theta} d(\tau_\nu \sec\theta)$$

The Grey Atmosphere

$$I_\nu(\tau_\nu,\theta) = \int a_\nu e^{-\tau_\nu \sec\theta} d(\tau_\nu \sec\theta) + b_\nu \int \tau_\nu e^{-\tau_\nu \sec\theta} d(\tau_\nu \sec\theta)$$

$$= -a_\nu\left[e^{-\tau_\nu \sec\theta}\right]_0^\infty - \frac{b_\nu}{\sec\theta}\left[e^{-\tau_\nu \sec\theta}\right]_0^\infty - \frac{b_\nu}{\sec\theta}\left[\tau_\nu \sec\theta e^{-\tau_\nu \sec\theta}\right]_0^\infty$$

$$I_\nu(\tau_\nu,\theta) = a_\nu + b_\nu \cos\theta$$

At the stellar surface, need only consider 0 < cosθ < 1 $$\pi F_\nu(\tau_\nu) = 2\pi\int_0^1 (a_\nu + b_\nu\cos\theta)\cos\theta d(\cos\theta)$$

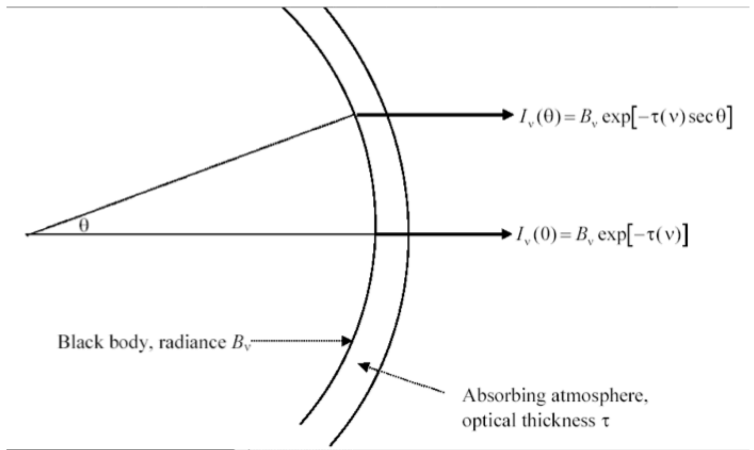
So $$\pi F_\nu(\tau_\nu) = \pi\left[a_\nu + \frac{2}{3}b_\nu\right]$$

and with $S_\nu = a_\nu + b_\nu \tau_\nu$, we see that $F_\nu(0) = S_\nu(\tau_\nu = 2/3)$

This is the Eddington-Barber relation which shows that the flux that emerges from a stellar surface is equal to the Source Function at a depth of τ = ⅔

or that the effective temperature of a star is equal to the temperature at τ = ⅔

Limb Darkening

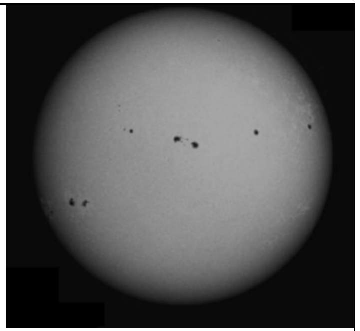


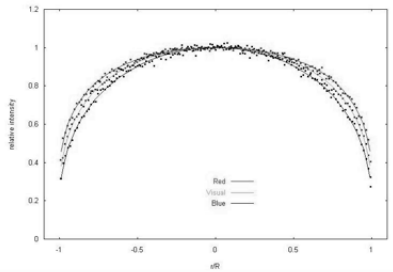
Limb Darkening

As the line of sight moves from the centre to the edge of the stellar disk, it passes through an increasing path length of atmosphere.

Degree of limb darkening depends on the optical depth and temperature gradient. As τ increases, it approximates an opaque surface, with a hard edge, so see into very similar physical depths at centre and edge.

Opacity is higher at infrared wavelengths and the effect of temperature gradient in outer layers is lower.





Solar limb scans at different wavelengths.

Limb Darkening

The emergent intensity at a position on the stellar disk is given by

$$I_\nu(\theta) = \int_0^\infty S_\nu e^{-\tau_\nu \sec\theta} \sec\theta d\tau_\nu$$

and for a grey atmosphere

$$S(\tau) = \frac{3}{4\pi}(\tau + \frac{2}{3})F(0)$$

Note that with B(τ)

$$B(\tau) = \frac{\sigma}{\pi}T^4(\tau) = \frac{3}{4\pi}(\tau + \frac{2}{3})\sigma T_{eff}^4$$

when $\tau=2/3$, $T = T_{eff}$; so that the effective depth at which the continuum is emitted is $\tau=2/3$

and we have S(τ) = B(τ) in the form $S(\nu) = a + b\tau_\nu$

Limb Darkening

$$S(\nu) = a + b\tau_\nu$$

or with $\tau \propto \cos\theta$

$$I_\nu(\theta) = I_\nu(0)(a + b\cos\theta)$$

measurements in the visible approximate with good agreement down to $\cos\theta \sim 0.1$

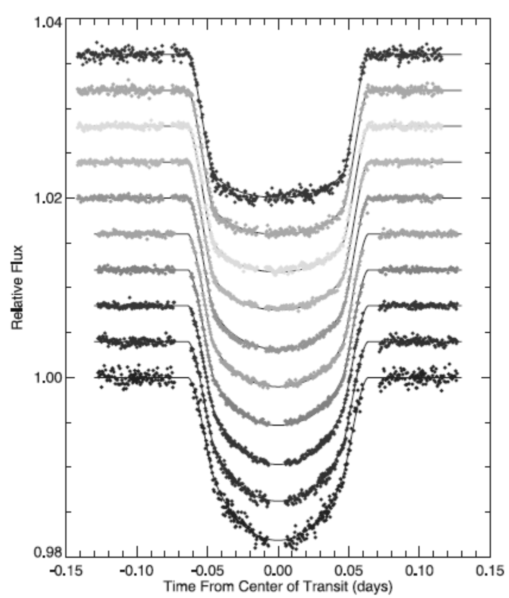
$$\frac{I(\theta)}{I(0)} = (0.4 + 0.6\cos\theta)$$

as we observe closer to the limb of the sun, we see into progressively shallower and cooler regions.

Limb darkening in HD 209458

Transiting planet shows that the limb of HD 209458 is more darkened at short wavelengths, as in the sun.

KNUTSON ET AL.

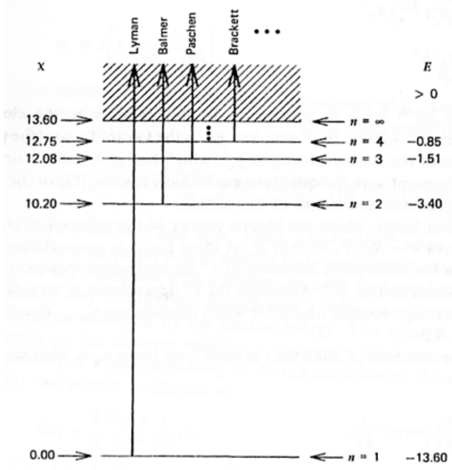


Opacity in Stars

Hydrogen is the most abundant species but in solar-type stars, both the ionization fraction and the populations in excited states are low.

H is predominantly neutral and in the ground state

Bound-free or bound-bound transitions will dominate



Contributions to H opacity

Bound-free absorption:

Ionizations from the ground state (Lyman series) lie in the UV, while the Balmer edge (ionization limit from n=2) occurs at 365nm so cannot contribute to visible opacity

Paschen edge (ionisation limit from n=3) occurs at 820nm so will contribute opacity in the visible

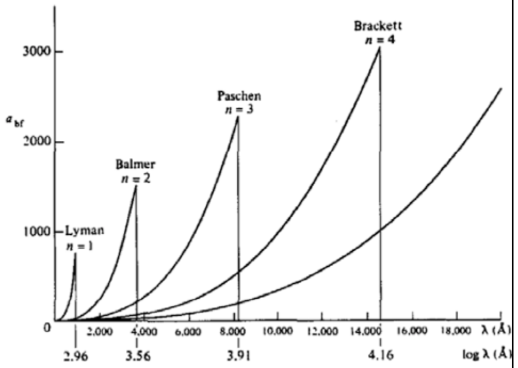


Fig. 3-10 The Bound-Free Absorption Coefficient of a Hydrogen-Like Atom.

Opacity

Paschen edge (ionisation limit from n=3) occurs at 820nm so can contribute opacity in the visible, but we need the n=3 level to be populated for this to contribute significantly.

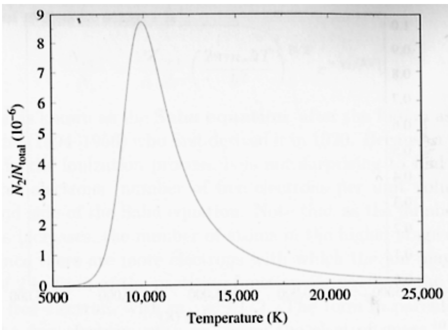
With T<6000K, the population in the n=3 level (12 eV) is very small (estimated from the Boltzmann distribution)

However, in hotter stars the higher n levels are populated and H bound-free opacity becomes important at T > 10000 K.

In even hotter stars, electron scattering (wavelength independent Thompson scattering) becomes dominant

Bound-bound transitions may be important. H has few transitions, but some metals (e.g. Fe) have many.

In cool stars, molecules dominate the spectrum e.g. TiO,VO,CO,C_2



Fraction of H atoms in the n=2 level as a function of temperature

H^- Opacity

Free-free absorption will also give a continuum opacity, but at a low level

Hydrogen can form a negative ion with a proton + 2 electrons. The dissociation energy of H^- is 0.75eV (1.65 μm) and so it can provide continuum opacity in the visible and near-infrared.

H^- will dominate in cool stars, but with increasing photospheric temperature, higher n levels in H will be populated and atomic H bound-free dominates in A-type stars.

In the sun, the Saha equation gives the ionization balance between H and H^-.

$N(H^-)/N(H) \sim 3\ 10^{-8}$.

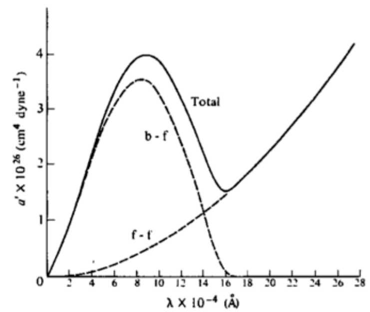


Fig. 3-11 The Absorption Coefficient of the Negative Hydrogen Ion at a Temperature of 6300°K (θ = 0.8) due to Bound-Free and Free-Free Transitions.

LTE Level Populations

Ionization equilibrium is described by the Saha equation

Atomic/ionic levels are populated thermally (Boltzmann distribution)

Ion and e^- velocity distributions are Maxwellian

The source function is given by the Planck function.

Boltzmann Equation
$$\frac{N_i}{N_1} = \frac{g_i}{g_1} e^{-\chi_i / kT}$$

Where N_i is the population of level i with statistical weight g_i and excitation energy χ_i of ionization stage j

In the solar photosphere, the fraction of H excited to n=2 is $\sim 10^{-8}$

and fraction excited to n=3 is $\sim 6 \cdot 10^{-10}$

Saha Equation

- Derived from the Boltzmann formula
- Gives the balance between successive stages of ionization as a function of temperature:

$$\frac{N_e N_{i+1}}{N_i} = \frac{2}{\Lambda^3} \frac{g_{i+1}}{g_i} e^{-\Delta E / k_B T}$$

where Λ is the thermal deBroglie wavelength of the electron ($\Lambda = \sqrt{h^2/2\pi m_e k_B T}$), N_{i+1} is the number in the state i+1 and g_{i+1} is the statistical weight of that state)

- You have seen this in the cosmology lectures, describing the ionization state of the Universe, and the same expression determines the ionization state of H in stars.

Ionization state in the solar photosphere

- The Saha equation gives
- N(H+)/N(H) = 10^{-4} at T=6,000K and = 0.07 at 10,000K

With increasing temperature, the increasing numbers of H atoms in the n=2 level increase the opacity shortwards of 365nm above that from the H- ion. The increased opacity leads to emission at these wavelengths arising from higher layers, where the temperature is lower, and appears as a break in the spectrum.

In cool stars, the break can be characterised as :

$$\frac{k(365+)}{k(365-)} = \frac{k(H^-)N(H^-)}{k(H^-)N(H^-) + k(H)N_H(n=2)}$$

While in hot stars, H^- becomes insignificant and the opacities arise from bound-free absorption from n=2 shortwards of 365nm and n=3 longwards of 365nm, and so the Balmer jump is a good diagnostic of temperature.

$$\frac{k(365+)}{k(365-)} = \frac{k(H)N_H(n=3)}{k(H)N_H(n=2)}$$

Absorption Lines

Continuum opacity, resulting from the energy levels in Hydrogen (and other species) dictates the overall shape of the observed spectra.

Absorption lines arise in regions of enhanced opacity, corresponding to transitions from populated levels

Heavy elements with low lying levels above the ground state can produce absorption lines in the visible, even in relatively cool stars. e.g. the Alkali metals (the Na D lines 2p-2s have ΔE ~2eV and lie in the visible)

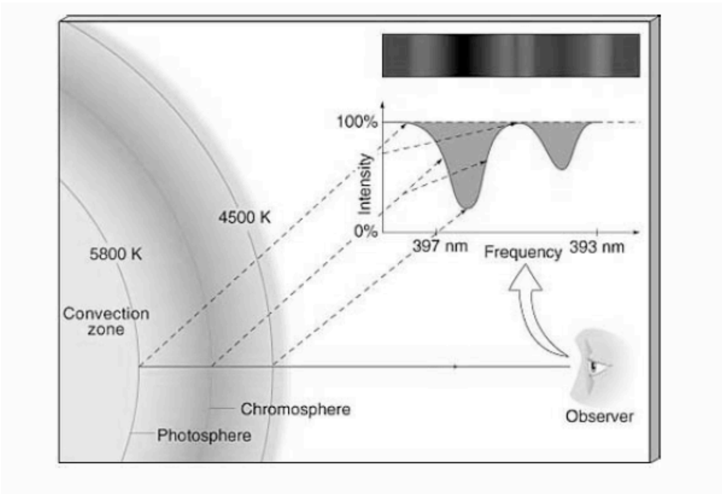
Ca is predominantly singly ionized : the Saha equation gives

N(Ca II) / N(Ca I) ~ 900 for T= 5800 K - I.P. = 6.11 eV

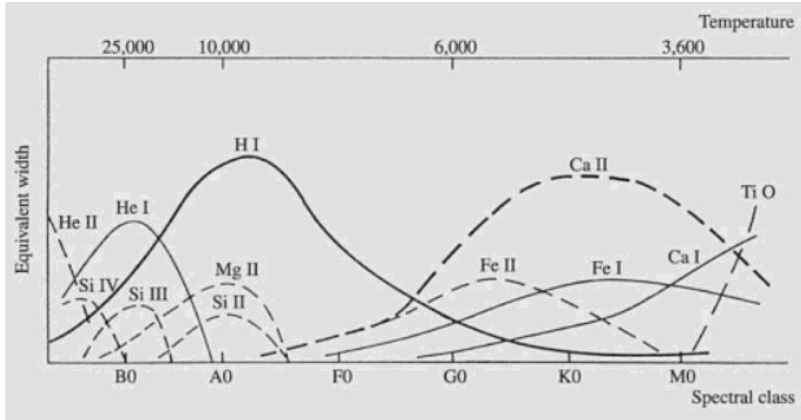
and almost all Ca II ions are in the ground state : $N_2/N_1 \sim 1/270$.

So although the abundance is only 7 10^{-5} N_H, almost all Ca is in the ground state of the Ca^+ ion and the total number is much greater than the number of H atoms in n=2 (Balmer series) at T~ 6000K

Absorption line formation



Prominent Stellar Absorption Lines



Line opacities depend on the ionization state and level populations, and so depend on the temperature of the region where the optical depth ~1

Stars

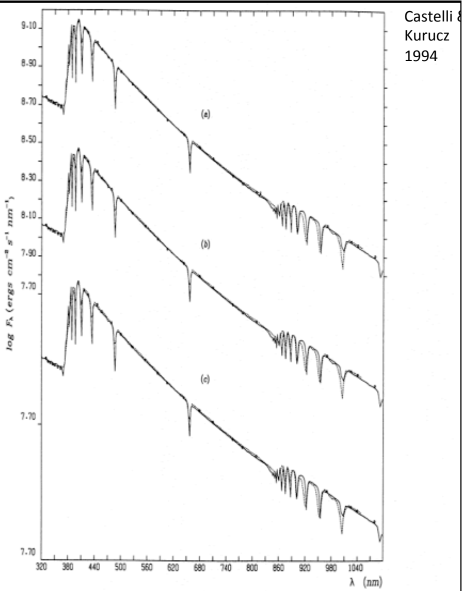
Dense, optically thick objects. At a given wavelength, we see in to a layer corresponding to $\tau \sim 1$

In a region of enhanced opacity (line transition) we do not probe as deeply as at an adjacent continuum wavelength, and so the temperature will be lower, producing an absorption line.

In a real star, there may be complex regions of high and low temperature that will complicate the interpretation (Corona, Chromosphere etc) and/or stellar winds and outflows.

Detailed modeling is needed for a complete picture

Comparison with models permits refinement of photospheric temperature, surface gravity, rotational and turbulent motions and element abundances



Castelli & Kurucz 1994

Self-consistent Spectral energy distributions

Multiple components stellar populations, gas,

dust, extinction, ionization, abundances, geometries….

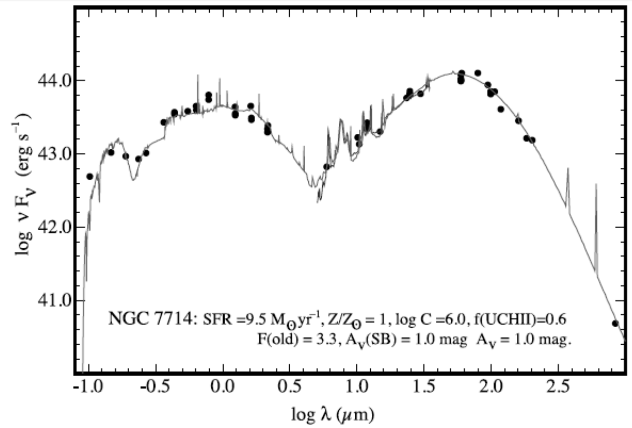


Fig. 5 Groves et al. (2008) model fit (*blue curve*) of the starburst galaxy NGC 7714 SED (*black points* and *red curve* mid-IR spectra), demonstrating the determination of physical galaxy properties such as star-formation rate (SFR) and metallicity (as labelled, see Groves et al. 2008, for full description of parameters) [Courtesy M. Dopita]