C1: Atomic Processes, Appendix A

Collisional excitation and de-excitation coefficients

Electrons are ejected from atoms through photionization and carry energy which is rapidly thermalised through interactions with other particles, leading to a maxwellian velocity distribution. The electrons can excite bound electrons within atoms and ions through collisions; but note that at typical nebular temperatures, they do not have sufficient energy to ionise most species or to populate significantly the first excited state in H. The excited electrons can be collisionally de-excited, or they may decay radiatively back to the lower level. In the limiting case of thermal excitation and de-excitation, energy is conserved and the upwards collisional excitations and balanced by downward collisional de-excitation, producing statistical equilibrium.
We have:

$$\mathrm{N}_1 Q_{2-1}(E')f(E')E'dE' = \mathrm{N}_2 Q_{1-2}(E)f(E)EdE$$

where Q is the collisional cross section, $E' = \frac{1}{2}mv'^2, E = \frac{1}{2}mv^2, and \Delta E = E' - E$

then:

$$\frac{Q_{2-1}(E')}{Q_{1-2}(E)} = \frac{\mathrm{N}_1 f(E)EdE}{\mathrm{N}_2 f(E')E'dE'}$$

$$\frac{Q_{2-1}(E')}{Q_{1-2}(E)} = \frac{\mathrm{N}_1 v\ E\ dE exp(-\frac{E}{kT_e})}{\mathrm{N}_2 v'\ E'\ dE' exp(-\frac{E'}{kT_e})}$$

$$\frac{Q_{2-1}(E')}{Q_{1-2}(E)} = \frac{\mathrm{N}_1 EdE}{\mathrm{N}_2 E'dE'} exp(-\frac{\Delta E}{kT_e})$$

But in a thermal distribution : $\frac{\mathrm{N}_1}{\mathrm{N}_2} = \frac{g_1}{g_2} exp(\frac{\Delta E}{kT_e})$

$$\frac{Q_{2-1}(E')}{Q_{1-2}(E)} = \frac{\mathrm{g}_1 E}{\mathrm{g}_2 E'}$$

The Maxwellian distribution is :

$$f(E) = \frac{2E^{1/2}}{\pi^{1/2}(kT_e)^{3/2}} exp(-\frac{E}{kT_e})$$

The collisional de-excitation rate coefficient C_{2-1} is given by the integral of the cross section, v and the electron speed distribution. i.e.

$$R_{21} = \mathrm{N_e N_2 C_{21}} = \mathrm{N_e N_2} \int_0^\infty Q_{21}(E)v(E)f(E)dE$$

and with the velocity $v(E) = \sqrt{\frac{2E}{m}}$, substituting the Maxwellian distribution for $f(E)$:

$$\mathrm{C}_{21} = \frac{2E^{1/2}}{\pi^{1/2}(kT_e)^{3/2}} \sqrt{\frac{2E}{m}} \int_0^\infty Q_{21} exp(-\frac{E}{kT_e})$$

Now it is usual to replace the collision cross section, Q, by the dimensionless Collision strength Ω defined such that $\Omega_{21} = \Omega_{12}$ according to:

$\Omega_{21} = g_2 Q_{21} E'/\pi a_0^2$ and

$\Omega_{12} = g_1\ Q_{12} E/\pi a_0^2$

where a_0 is the radius of the first Bohr orbit, so that $Q_{21}(E) = (\frac{h^2}{8\pi mE})\frac{\Omega_{21}(E)}{g_2}$

which gives :

$$\mathrm{C}_{21} = (\frac{h^2}{8\pi mg_2})\sqrt{\frac{2}{m}}\frac{2}{\pi^{1/2}(kT_e)^{3/2}}\int_0^\infty \Omega_{21}exp(-\frac{E}{kT_e})dE$$

With a broad range of electron energies, we can define an effective collision strength or Maxwell-averaged collision strength $\bar{\Omega}_{21}$

$$\bar{\Omega}_{21} = \int_0^\infty \Omega_{21}exp(-\frac{E}{kT_e})d(\frac{E}{kT_e})$$

and substituting $\hbar = h/2\pi$ and gathering terms gives:

$$\mathrm{C}_{21} = \sqrt{\frac{2\pi\hbar^4}{km^3}}\frac{1}{\sqrt{T_e}}\frac{\bar{\Omega}_{21}}{g_2}$$

$$\mathrm{C}_{21} = \mathbf{8.6 \times 10^{-12}\ \frac{\bar{\Omega}_{12}}{g_2}T_e^{-\frac{1}{2}}}$$

For collisional excitation, the approach is similar, except that there is a minimum energy or threshold energy needed to raise the electron to the excited state. The integral therefore is from E_{12} to ∞.

$$R_{12} = \mathrm{N_eN_1C_{12}} = \mathrm{N_eN_1}\int_{E_{12}}^\infty Q_{12}(E')v(E')f(E')dE'$$

$$\mathrm{C}_{12} = \int_{E_{12}}^\infty Q_{12}(E')v(E')f(E')dE'$$

$$\mathrm{C}_{12} = \mathbf{8.6 \times 10^{-12}\ \frac{\bar{\Omega}_{12}}{g_1}T_e^{-\frac{1}{2}}exp - (\Delta E/kT_e)}$$

and therefore:

$$\mathrm{C}_{21} = \mathrm{C}_{12}\mathbf{\frac{g_1}{g_2}exp(\Delta E/kT_e)}$$

These relationships between the collisional excitation and de-excitation coefficients are key to interpretation of line emission in nebulae.