Q1. Explain what is meant by a black-body and describe applications of this concept in astrophysics. [8]

The intensity of radiation emitted per unit wavelength interval by a black-body of temperature T is given as a function of wavelength by

$$B(\lambda) = \frac{2hc^2}{\lambda^5} \frac{1}{exp(hc/\lambda kT) - 1}.$$

Obtain an approximation to this formula appropriate for wavelengths such that $hc/\lambda kT << 1$. In what region of the electromagnetic spectrum would you expect this approximation to apply to the radiant emission from stars? [5]

The apparent magnitude m of a hot star of surface temperature 40,000 K is measured at the two wavelengths, 440 nm and 550 nm, corresponding to the colour filters known as B and V. The colour index, $m_B - m_V$, is found to be –0.35. Compare this value with the theoretical colour index predicted for this star by the black-body distribution. [12]

[The apparent magnitude m is related to the observed flux f by $m = -2.5\log_{10} f + k$, where k is a constant for a given wavelength. The colour index of a star of surface temperature 11,700 K is zero.]

Q2. Explain what is meant by the angular resolution and magnification of a telescope. Show that the limit of resolution is given approximately by λ/D where λ is the wavelength of the incident light and D is the diameter of the telescope aperture. What is the typical angular resolution (in seconds of arc) achieved with a ground-based telescope operating at visible wavelengths? [8]

Give expressions for the resolving power and angular dispersion of a diffraction grating, defining all quantities. [5]

In order to study the internal motions of galaxies, astronomers need to measure velocity differences typically of order 10 km s^{-1}. Consider a grating spectrograph equipped with a camera of focal length 2 m and a CCD detector whose pixel size is 20 μm. The spectrograph is used to study nearby galaxies in the [OIII] emission line whose rest wavelength is 500.7 nm. Given that the light is incident normally on the grating and that the first order spectrum falls on the detector, estimate the minimum grating ruling, in lines per mm, required to give the desired velocity resolution. [12]

Q3. Explain what is meant by the term *hydrostatic equilibrium* in stellar structure and discuss its importance. Derive an expression for the time-scale on which changes occur if equilibrium conditions are disturbed. [8]

A star is completely supported by radiation pressure, and transport of energy is by radiation only. Use the equation of state $p = aT^4/3$, and the radiative transport equation

$$L = -\frac{16\pi r^2 acT^3}{3\kappa\rho}\frac{dT}{dr}$$

(where L is the luminosity, κ the opacity, ρ the density, r the radius, c the speed of light, and a the radiation density constant), to show that in hydrostatic equilibrium the luminosity is given by

$$L = \frac{4\pi cGM}{\kappa}$$

where G is the gravitational constant and M is the mass of the star. [12]

What would happen if L were suddenly increased beyond this value? [5]

Q4. Explain what is meant by *escape velocity* and derive an expression for the escape velocity of a particle at distance R from an object of Mass M. [6]

By assuming that the maximum escape velocity is c, the speed of light, obtain an expression for the radius, R_s of a black hole of mass M. Evaluate this radius for the cases $M = 10$ and $10^6 M_\odot$. [6]

What rate of matter infall (in units of $M_\odot$ per year) would be needed to power a quasar of luminosity 10^{39} W if a black hole of mass $10^6 M_\odot$ lay at the quasar core? Assume conversion of potential energy to light with 100% efficiency. Does your answer support accretion of matter by a black hole as a likely model for quasar energy generation? [13]

Q5. State the cosmological principle and define the terms *homogeneity* and *isotropy*. Give an example showing that a homogeneous universe need not be isotropic. The Friedmann–Robertson–Walker metric for a homogeneous and isotropic universe is given by

$$ds^2 = -c^2dt^2 + a(t)^2\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right],$$

where ds is the proper time interval between two events, t is the cosmic time, k measures the spatial curvature, and r, θ and ϕ are radial, polar and azimuthal co-ordinates respectively. Discuss the physical significance of $a(t)$, the scale factor, sketching its form for the three cases of a universe with positive, zero and negative curvature. [8]

Define the term *luminosity distance*. By considering the amount of radiation which is received in a unit area located a co-moving distance away from a source, show that the luminosity distance $d_{\rm lum}$ is given by the formula

$$d_{\rm lum} = a_0 r_0 (1+z)$$

where r_0 is the co-moving distance and z is the red-shift. [8]

Describe how the luminosity distance of Type Ia supernovae might be used to constrain cosmological parameters, and discuss the observations which are required and any key assumptions of the method. [9]

Q6. Discuss the significance of the observation that the spectral lines of distant galaxies are redshifted compared to their rest wavelengths. [6]

For what range of redshifts would the hydrogen Lyman-α line (rest wavelength 121.6 nm) be redshifted into the visible part (400-700nm) of the electromagnetic spectrum? What particular problems are encountered in attempting to detect Lyman-α from galaxies with redshifts greater than 7? [6]

Assuming the Hubble constant H_0 to be 50 km s^{-1} Mpc^{-1}, determine the range of distances (in Mpc) corresponding to the range of redshifts you have calculated. Compare your result with the size of the observable Universe. The cosmological expression relating distance d to redshift z is

$$d = \frac{cz}{H_0}(1 + \frac{z}{2})/(1 + z)^2.$$

[8]

What sort of celestial objects are observable with redshifts $\gtrsim 2$? Outline further evidence which favours locating these objects at cosmological distances. [5]

Q7. Give an account of the observational evidence for the hot Big Bang model of the Universe. [12]

The Friedmann and fluid equations respectively are given by

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2}$$

and

$$\dot{\rho} + 3\frac{\dot{a}}{a}\left(\rho + \frac{\mathrm{p}}{c^2}\right) = 0,$$

where a is the scale factor, ρ is the density and p is the pressure. ($\dot{a}$ and $\dot{\rho}$ are the derivatives of these quantities with respect to time.) Use these equations to derive the acceleration of the Universe. [7]

Hence demonstrate that if the Universe is homogeneous and the strong energy condition $\rho c^2 + 3\mathrm{p} > 0$ holds, then the Universe must have undergone a Big Bang. [6]