Q1. Explain what is meant by *detailed balance, thermodynamic equilibrium* and *statistical equilibrium* in the context of the number densities of adjacent stages of ionization. [3]

Give an account of the most important ionization and recombination processes that determine the relative number densities of ions in (a) a planetary nebula and (b) a stellar corona. [8]

The tables below give total rate coefficients for recombination *to* and ionization *from* the ions listed, at an electron temperature of 10^6 K. Ions for which data are not given can be neglected. Calculate the relative ion populations and hence find which stage of ionization has the largest relative population at this temperature. In a sketch, show how you would expect $\log_{10}[N(FeX)/N(Fe)]$ and $\log_{10}[N(FeXI)/N(Fe)]$ to vary with temperature between $\log_{10}[T_e(K)] = 5.8$ and 6.2. [10]

$\log_{10}[T_e(K)]$	FeVIII	FeIX	FeX	FeXI
6.0	1.26	1.29	1.55	1.51

Recombination rate coefficients in units of $10^{-16}\,\mathrm{m}^3\,\mathrm{s}^{-1}$.

$\log_{10}[T_e(K)]$	FeVIII	FeIX	FeX	FeXI
6.0	6.03	1.78	0.871	0.447

Ionization rate coefficients in units of $10^{-16}\,\mathrm{m}^3\,\mathrm{s}^{-1}$.

At very high electron densities the contribution from dielectronic recombination is reduced. Discuss qualitatively how the resulting relative ion populations would differ from those calculated. [4]

[The notation $T_e(K)$ represents a number equal to the electron temperature in K.]

Q2. Using a model for a two-level ion (ground state plus one excited state), discuss the processes which should be included when considering the formation of an intersystem (semi-forbidden) line in a cool-star transition region. Hence show that the rate at which energy is emitted in an intersystem line of wavelength λ_{21} is given by

$$E_{21} = \frac{hc}{\lambda_{21}} A_{21} \frac{N_E}{N_H} \int \left(\frac{N_{\rm ion}}{N_E} \right) \frac{f_1(N_e, T_e) N_H}{A_{21} + f_1(N_e, T_e) + f_2(N_e, T_e)} dV \, ,$$

where A_{21} is the spontaneous transition probability, T_e is the electron temperature and N_E, N_H, $N_{\rm ion}$ and N_e are the number densities of the element under consideration, hydrogen, ions and electrons, respectively.

Show that

$$f_1(N_e, T_e) = C_{12} N_e \quad \text{and} \quad f_2(N_e, T_e) = C_{21} N_e,$$

where C_{12} and C_{21} are rate coefficients for collision and excitation. [13]

Intersystem lines of Si III and C III are observed in spectra of cool stars with a range of surface gravities. Assuming that both lines are formed at $T_e = 4.5 \times 10^4$ K, use the data in the table below to calculate the maximum and minimum values of the ratio E(Si III)/E(C III). [7]

In the spectrum of a planetary nebula, the ratio E(Si III)/E(C III) is observed to be less than 0.1. Discuss the differences between the physical conditions in planetary nebulae and cool star transition regions and suggest the main cause of this small energy ratio. [5]

Data for Si III and C III

Ion	Transition	λ (nm)	Ω	A_{21}(s^{-1})	N_E/N_H	$N_{\rm ion}/N_E$
Si III	$3s^2\,{}^1S_0 - 3s3p\,{}^3P^\circ_1$	189.2	2.8	1.5×10^4	3.5×10^{-5}	0.79
C III	$2s^2\,{}^1S_0 - 2s2p\,{}^3P^\circ_1$	190.9	0.32	1.0×10^2	3.5×10^{-4}	0.46

The ionization potentials of Si III and C III are 33.5 eV and 47.9 eV, respectively.

[The rate coefficient for collisional excitation is

$$C_{12} = \frac{8.63 \times 10^{-12} \, \Omega \, 10^{-\left[\frac{6.25 \times 10^6}{\lambda_{21} T_e}\right]}}{g_1 T_e^{1/2}} m^3 \, s^{-1} \, ,$$

where Ω is the collision strength given in the table, g_1 is the statistical weight of the lower level, λ_{21} is in nm and T_e is in K.]

Q3. An element of total number density n_E exists mainly in two stages of ionization, i and $i+1$, with number densities n_i and n_{i+1}. Express n_i/n_E and n_{i+1}/n_E in terms of the ionization rate β_i (from i) and the recombination rate γ_i (to i). In a planetary nebula what processes are the main contributors to β_i and γ_i? How do n_i/n_E and n_{i+1}/n_E vary if the electron number density n_e is increased?

Consider a forbidden transition in the optical spectrum of a planetary nebula. The transition occurs between an excited level (2), with number density n_2, and the ground level (1), with number density n_1. Write down the appropriate form of n_2/n_1. Hence explain what is meant by the *critical density*. [10]

Two such lines occur in an ion, from levels 3 and 2 to level 1. Assuming that the wavelengths $\lambda_{21} \simeq \lambda_{31}$, show that their flux ratio is given by

$$\frac{F_{31}}{F_{21}} \simeq \frac{g_3}{g_2}\frac{A_{31}}{A_{21}}\frac{(n_e^*(2)+n_e)}{(n_e^*(3)+n_e)},$$

where A_{ji} is the spontaneous transition probability, g_j is the statistical weight of level j and $n_e^*(j)$ is the critical density for a transition from level j. [5]

Using the data for singly ionized sulphur (S II) given in the table below, find

(a) the values of F_{31}/F_{21} when (i) $n_e \gg n_e^*(2)$ and (ii) when $n_e \ll n_e^*(3)$

(b) the value of n_e at which F_{31}/F_{21} has its greatest dependence on $\ln(n_e)$, and the corresponding value of F_{31}/F_{21}.

Show that if this value of F_{31}/F_{21} can be measured to within $\pm 10\%$ then $\log_{10}(n_e)$ can be determined to within ± 0.18. [10]

Table: Data for S II lines.

λ_{ji}/nm	Transition (ji)	A_{ji}/s^{-1}	Critical density $n_e^*(j)/\mathrm{m}^{-3}$
673.1	${}^2D_{3/2} - {}^4S_{3/2}$ (21)	1.7×10^{-3}	3×10^{10}
671.6	${}^2D_{5/2} - {}^4S_{3/2}$ (31)	6.3×10^{-4}	1×10^{10}

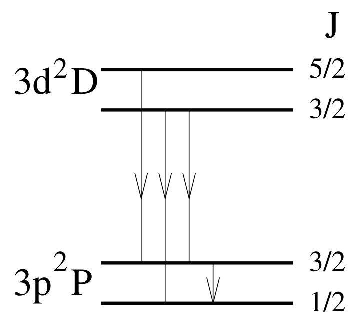
[At a fixed electron temperature T_e the collisional excitation rate is

$$C_{ij} \propto \frac{\exp(-hc/\lambda_{ij} k_B T_e)}{g_i}]$$

Q4. Discuss what is meant by *detailed balance* and *the coronal approximation* in the context of processes that determine the number density N_2 of an excited state in a two level atom. In each case give the expression for N_2/N_1, where N_1 is the ground state number density. [5]

Emission lines arising from permitted and spin-forbidden electric dipole transitions to a common atomic energy level are observed in stellar transition regions. Explain how their relative intensities can be used to determine the electron number density N_e. Explain why there is an electron density below which this method cannot be used. [7]

The diagram shows some transitions observed in Fe XIV.



Transition	λ/nm	C_{ij} (relative)	A_{ji} (relative)
$^2D_{5/2} \rightarrow {}^2P_{3/2}$	21.9	9	6
$^2D_{3/2} \rightarrow {}^2P_{1/2}$	21.1	10	5
$^2D_{3/2} \rightarrow {}^2P_{3/2}$	22.0	1	1

Using the relative collisional excitation rate coefficients C_{ij} and the relative transition probabilities A_{ji} given in the table, derive an expression for the relative intensities of the lines at wavelengths $\lambda = 21.9$ nm and $\lambda = 21.1$ nm in terms of the population ratio $N(^2P_{3/2})/N(^2P_{1/2})$. What relative intensity would be expected if the $^2P_{3/2}$ and $^2P_{1/2}$ level number densities were determined by detailed balance at a temperature $T_e = 2 \times 10^6$ K? [7]

In a solar active region the observed intensity ratio of the above lines is 0.15. Use the collisional excitation rate coefficient $C_{ij} = 7 \times 10^{-15}\, m^3\, s^{-1}$ and the transition probability $A_{ji} = 60\, s^{-1}$ for the $^2P_{3/2} \rightarrow {}^2P_{1/2}$ transition to show that $N(^2P_{3/2})/N(^2P_{1/2})$ depends on N_e. Find the value of N_e. [6]

[The relation between the collisional de-excitation and excitation rate coefficients is $C_{ji} = (g_i/g_j)C_{ij} \exp(W_{ij}/k_BT_e)$ where g_i and g_j are the statistical weights of the lower and upper levels respectively and W_{ij} is the excitation energy of level j above level i.]