2004/05 4th Year Astrophysics : Problem Set 7 Atomic Processes & Revision

Q1. Sketch the mean variation of the electron temperature T_e with height above the solar photosphere indicating the principal regions and the ranges of T_e that they include. What fundamental problem does this temperature variation present? [8]

Derive an expression relating the flux in a collisionally excited allowed emission line to the emission measure. [8]

In the solar spectrum, allowed transitions to the ground term occur in O IV at 55.4 nm and in Ne IV at 54.4 nm. The optimum temperature of formation for these lines is 1.8×10^5 K and the lines have the same dependence of emissivity on T_e. Use the data below to find the abundance of neon relative to that of oxygen. State and briefly justify any assumptions you make.

	Observed line flux/W m^{-2}	Statistical weight of ground term	$\frac{N_1\Omega_{12}}{N_\mathrm{E}}$
O IV	1.2	6	8.5
Ne IV	0.042	4	1.2

Here N_1 is the population density of the ground term and N_E is the number density of the element. [9]

[The rate coefficient for collisional excitation is

$$C_{12} = \frac{8.63 \times 10^{-12} \Omega_{12} \exp(-W_{12}/k_\mathrm{B}T_\mathrm{e})}{T_\mathrm{e}^{1/2} g_1} \mathrm{m}^3\ \mathrm{s}^{-1},$$

where Ω_{12} is the collision strength, g_1 is the statistical weight of the ground term and W_{12} is the energy of the excited term.]

Q2. Explain what is meant by an *emission measure* and the *emission-measure distribution* in the context of cool-star transition regions. [4]

Show that, for a plane-parallel atmosphere in hydrostatic equilibrium, the electron pressure P_e at any electron temperature T_e can be expressed as

$$P_\mathrm{e}^2(T_\mathrm{e}) = P_\mathrm{e}^2(T_0) - 2\sqrt{2}\,\mu m_\mathrm{H} k_\mathrm{B} g_* \int_{T_0}^{T_\mathrm{e}} Em(0.3)\ \mathrm{d}T_\mathrm{e},$$

where T_0 is a temperature at which the electron density can be measured, μ is the mean relative molecular mass, g_* is the stellar surface gravity and $Em(0.3)$ is the emission measure averaged over a logarithmic temperature range of $\Delta \log T_\mathrm{e} = 0.3$. State clearly any approximations which you make. [12]

For $T_0 \leq T_\mathrm{e} \leq T_\mathrm{corona}$, the emission-measure distribution is described by $aT_\mathrm{e}^{3/2}$, where $a = 4 \times 10^{27}\,\mathrm{K}^{-3/2}\,\mathrm{m}^{-5}$. Given that $\mu = 0.6$, $g_* = 274\,\mathrm{m\,s}^{-2}$, $T_0 = 2 \times 10^5$ K and $P_\mathrm{e}(T_0) = 10^{-2}$ Pa, find the temperature T_1 at which $P_\mathrm{e}^2(T_\mathrm{e}) = 0.81\,P_\mathrm{e}^2(T_0)$. What aspect of the structure of the transition region leads to this small variation of $P_\mathrm{e}(T_\mathrm{e})$? [5]

The energy flux carried by thermal conduction is almost constant for the temperature range over which an emission line is typically formed and is related to the temperature gradient $\mathrm{d}T_\mathrm{e}/\mathrm{d}h$ by $F_\mathrm{c}(T_\mathrm{e}) = -\kappa_0 T_\mathrm{e}^{5/2}(\mathrm{d}T_\mathrm{e}/\mathrm{d}h)$, where $\kappa_0 = 10^{-11}\,\mathrm{W\,m}^{-1}\,\mathrm{K}^{-7/2}$. Find $F_\mathrm{c}(T_\mathrm{e})$ at T_1. [4]

Q3. Matter–Radiation Equality

The present density of matter is $\rho_{M0} \equiv \Omega_M \rho_c$ and the present density of radiation is $\rho_{R0} = \rho_{\gamma 0} + \rho_{\nu 0}$, where $\rho_{\gamma 0} = A{T_0}^4$ is the microwave background ($T_0 = 2.725$ K) and $\rho_{\nu 0} = (21/8)A{T_{\nu 0}}^4$ is the neutrino background (we assume that neutrinos are massless). Here $A = \pi^2/15$, and $T_{\nu 0} = (4/11)^{1/3} T_0$.

Starting from the Friedmann equation, find an expression for the age of the Universe, t_{eq}, when $\rho_M = \rho_R$ in terms of the scale factor a, the Hubble constant H_0 and the matter density parameter Ω_M. (Note that in these early times - but not today - you can ignore the curvature and vacuum terms in the Friedmann equation.) [15]

Calculate the age (in years) for the cases $\Omega_M = 0.1$, 0.3 and 1.0 and $H_0 = 70$ km s^{-1}. What was the temperature T_{eq} ? [10]

Q4. Tully Fisher Relation

Suppose that some class of galaxies has a surface brightness that varies with distance from the galaxy centre as $I(R) = I_0 f(R/R_0)$ with all galaxies having the same I_0 and the functional form f, but with different galaxies having different R_0.

If the mass to light ratio is everywhere constant within each galaxy and within the class, show that the total luminosity is related to the characteristic velocity as $L \propto {v_0}^4$. [10]

We can use this relation to measure the distances to galaxies. How would you measure ${v_0}^2$ for spiral galaxies (i.e. what velocity would one measure)? And for elliptical galaxies? What kind of problems do you envisage with this distance indicator? [5]

Q5. Synchrotron radiation

Explain the reasons for concluding that the diffuse component of radio emission observed at a frequency of 408 MHz is due primarily to synchrotron emission from our Galaxy. [5]

Show that the synchrotron radiation from a single ultra-relativistic electron with Lorentz factor $\gamma >> 1$, is strongly peaked around a critical frequency ν_{crit} of about

$$\nu_{\mathrm{crit}} \sim \frac{\gamma^2 eB}{m_{\mathrm{e}}} \sin\theta$$

when the electron is passing through a region of magnetic flux density B at an angle θ to the direction of the magnetic field. [10]

By considering the intensity of radiation emitted by a black body, show how an estimate of the magnetic flux density in a compact radio source can be obtained from a measurement of its surface brightness at a frequency at which the source is optically thick. [10]

[In the Rayleigh Jeans part of the spectrum of a black body at temperature T, the intensity per unit frequency interval S_ν is given by $\mathrm{S}_\nu = 2k_{\mathrm{B}}T\nu^2/c^2$]

Q6. Star Formation

a) Explain the concept of the Jeans mass and its importance for star formation. Consider two dense cores in a cloud of molecular hydrogen (with $\mu = 2$): (i) a cool core with temperature $T = 10\,\mathrm{K}$ and (ii) a warm core with $T = 100\,\mathrm{K}$. Taking the number density of molecular hydrogen to be $n = 10^{10}\,\mathrm{m}^{-3}$ in both cases, estimate the Jeans mass for both cores.

b) Assume that the pre-collapse core is in hydrostatic equilibrium and can be treated as an isothermal sphere, i.e. a sphere of gas at constant temperature T where the supporting thermal pressure is given by $P = \rho\, c_\mathrm{s}^2$ and where $c_\mathrm{s} = \sqrt{kT/\mu m_\mathrm{H}}$ is the isothermal sound speed of the gas. Show that the density as a function of radius r from the centre of the sphere is approximately given by

$$\rho = \frac{c_\mathrm{s}^2}{4\pi G}\frac{1}{r^2},$$

and the mass enclosed within a radius r by

$$M(r) = \frac{c_\mathrm{s}^2}{G}\, r.$$

c) The collapse of an unstable isothermal molecular cloud core occurs from the inside out. Assume that the innermost mass $m(r)$ within a radius r has already collapsed and that the infall velocity at r is given by c_s. Show that the mass-infall rate $\dot{M}$ is given by

$$\dot{M} = \frac{c_\mathrm{s}^3}{G}$$

and is independent of r. What is $\dot{M}$ for the cases (i) and (ii) in part a)?

d*) Now consider an unstable molecular cloud core of $1\,M_\odot$ and initial radius $R = 10^6\,R_\odot$ which is in solid body rotation with angular velocity $\omega = 10^{-13}\,\mathrm{Hz}$. Assuming that the size of the protostar forming at the centre of the collapsing core has a radius of $5\,R_\odot$ and using angular momentum considerations, estimate what fraction of the mass of the core can collapse directly onto the protostar. Estimate the characteristic size of the protostellar disc that forms from the collapse of the bulk of the molecular core. [To obtain these estimates, you may take the density to be constant in the core and ignore factors of order unity.] Comment on the implications of these estimates for the outcome of the star-formation process.