C1: Astrophysics Major Option

Problem Set 4: Galaxy Evolution, Supernovae, Pulsars

(S. Yi & Ph. Podsiadlowski, Weeks 5 and 8, MT04)

1 The Initial Mass Function (IMF) [20 points]

a) Describe *briefly* what is meant by the initial mass function. What is the Salpeter IMF? What are typical values for the lower and upper limits of the stellar mass in the IMF?

b) For a simple stellar population (where all stars are born in an instant starburst) with a total mass of $M_{\rm tot} = 10^6\,M_\odot$, find the scaling constant K in the Salpeter IMF.

c) Find the mass below which the integrated stellar mass becomes half of the total initial mass.

d) Approximating the stellar lifetime by

$$\frac{t_{\rm MS}}{10\,{\rm Gyr}} = \left(\frac{M}{M_\odot}\right)^{-2.5} \tag{1}$$

derive the expression for the maximum (individual) stellar mass as a function of age of the population, $m_{\rm up}(t)$. What are the values of the maximum stellar mass for ages of 0.1, 1, and 10 Gyr?

e) Find the fractions of the total mass that still remain in the hydrogen burning phase (i.e., are still alive) when the age of the population is 0.1, 1, and 10 Gyr, respectively.

2 Galaxy luminosity/colour evolution [20 points]

A large fraction of the UB-band light in relatively young populations comes from main-sequence stars. The luminosity depends on mass in the following forms:

$$L \propto M^\beta \tag{2}$$

where $\beta = 5.0$ for the U-band and 4.5 for the B-band.

a) Using the Salpeter IMF, calculate the total U-band and B-band luminosity change/evolution (in magnitudes) between ages 1 and 10 Gyr.

b) What is the colour change/evolution in the $U-B$ colour, $\Delta(U-B)$, during this period?

c) More sophisticated population synthesis models suggest $\Delta(U-B) \approx 0.3$. Compare your result to this and comment on possible origins for the difference.

d) Explain briefly how such models can be used for studying galaxy evolution (give two examples).

3 Core-Collapse Supernovae [20 points]

Consider the final iron core of a massive star with a mass $M_{\rm Fe} \simeq 1.5\,M_\odot$ and radius $R_{\rm Fe} \simeq 3 \times 10^6$ m, spinning with a spin period $P \simeq 500$ s and having a magnetic field at its outer edge $B_{\rm Fe} \simeq 2 \times 10^3$ T.

a) Stating your assumptions, estimate the final spin and the strength of the magnetic field of the neutron star that forms from the collapse of such a core. Compare the spin period to the maximum spin period for a neutron star.

During the collapse phase, the initial collapse stops when the central core with a mass $M_{\rm core} \simeq 0.7\,M_\odot$ reaches a mass density $\rho \simeq 3 \times 10^{16}\,{\rm kg\,m^{-3}}$. At this density the core bounces driving a shock with an energy $E_{\rm bounce} \sim 10^{44}$J into the infalling outer core.

b) Estimate the energy that is required to photodissociate $0.1\,M_\odot$ of Fe into neutrons and protons. Compare this energy to the bounce shock energy and comment on the fate of the shock.

c) In the proto-neutron star (with an initial radius ~ 30 km), the mean free path of neutrinos is $l_\nu \sim 0.3$ m. Estimate the diffusion time for neutrinos to escape from the proto-neutron star and hence estimate the neutrino luminosity during the initial neutron-star cooling phase.

d) Assuming that 5 to 10 % of the neutrino luminosity is absorbed by the infalling outer core, estimate how long it takes to absorb enough neutrino energy to reverse the infall of the outer core and drive a successful supernova explosion (with a typical explosion energy of 10^{44} J). Compare this time to the dynamical timescale of the proto-neutron star.

4 The Binary Pulsar PSR J0737-3039: Supernova Kicks [40 points]

Recently, the first binary pulsar was discovered (Lyne, A.G. et al. 2004, Science, 303, 1153), which provides a rare laboratory for relativistic physics. The system consists of two pulsars (A and B) in a mildly eccentric orbit with an orbital period $P_{\rm orb} \simeq 2.4$ hr and eccentricity $e \simeq 0.088$. The spin periods and spin-down rates of the two pulsars have been measured to be $P_{\rm A} \simeq 22.7$ ms, $P_{\rm B} \simeq 2.77$ s, $\dot{P}_{\rm A} \simeq 1.7 \times 10^{-18}\,{\rm s\,s^{-1}}$ and $\dot{P}_{\rm B} \simeq 0.88 \times 10^{-15}\,{\rm s\,s^{-1}}$ and the masses have been determined to be $M_{\rm A} \simeq 1.34\,M_\odot$ and $M_{\rm B} \simeq 1.25\,M_\odot$, respectively.

a) Making reasonable assumptions about the pulsar properties, estimate the spin-down luminosities and the spin-down ages (i.e. $P/2\dot{P}$) for both pulsars. Considering the evolutionary history of the system, explain why the spin-down ages should roughly agree.

b) Assuming that the spin-down is caused entirely by magnetic dipole radiation, show that the magnetic field of the pulsars can be estimated from

$$B \simeq \frac{1}{R^3 \sin\theta} \sqrt{\left(\frac{3c^3\mu_0}{32\pi^3}\right) P\dot{P}I},$$

where R is the radius of the pulsar, θ the (generally unknown) inclination of the magnetic axis with respect to the rotation axis and I is the moment of inertia of the pulsar (μ_0 is the magnetic permeability and c the speed of light in vacuo). Estimate the magnetic fields of the two pulsars.

c) It is believed that Pulsar A was spun up by accretion from the progenitor of Pulsar B. Neglecting magnetic fields during the accretion phase, estimate how much mass Pulsar A would have had to accrete from an accretion disc to be spun-up to the observed spin period. How does the actual magnetic field of Pulsar A affect this estimate?

It is reasonable to assume that before the second supernova, in which Pulsar B was formed, the immediate pre-supernova binary system was circular and had an orbital separation $a_0 \simeq 1.4\, R_\odot$.

d) Assuming that in the second supernova, an amount of mass ΔM was instantaneously ejected and that Pulsar B did not receive a recoil in its own frame, show that the post-supernova eccentricity e, post-supernova semimajor axis $a_{\rm PSN}$ and post-supernova system velocity $v_{\rm sys}$ (i.e. the velocity of the new centre-of-mass (CM) frame defined by the two pulsars relative to the pre-supernova CM frame) are given by

$$e = \frac{\Delta M}{M_{\rm A} + M_{\rm B}},$$

$$a_{\rm PSN} = \frac{a_0}{1-e},$$

$$v_{\rm sys} = v_{\rm orb}^0 \, \frac{\Delta M}{M_{\rm A} + M_{\rm B}} \, \frac{M_{\rm A}}{M_{\rm He} + M_{\rm A}},$$

where $M_{\rm He}$ is the mass of the progenitor of Pulsar B just before the supernova (i.e. $M_{\rm B} + \Delta M$) and $v_{\rm orb}^0$ is the pre-supernova orbital velocity. Determine ΔM assuming that the post-supernova eccentricity was $e \simeq 0.1$ and estimate $v_{\rm sys}$.
[Hint: You need to compare the energies and momenta of the system before and after the supernova. The eccentricity e and semi-major axis a of an eccentric orbit are related to the distance of closest approach, the periastron separation, $r_{\rm p}$ by $r_{\rm p} = (1-e)\,a$, and the total energy of an eccentric binary is

$$E_{\rm binary} = -\frac{GM_1M_2}{2a} = -\frac{GM_1M_2}{r} + \frac{1}{2}\frac{M_1M_2}{M_1+M_2}\, v^2,$$

where r is the separation and v the relative orbital velocity at a particular binary phase, and M_1 and M_2 are the masses of the two components. See, e.g., Carroll & Ostlie, Chapter 2.3.]

e) Show that in the limit, where there is no mass loss associated with the second supernova but where Pulsar B received an asymmetric supernova kick of magnitude $v_{\rm kick}$, the post-supernova system velocity is given by

$$v_{\rm sys} = \frac{M_{\rm B}}{M_{\rm A} + M_{\rm B}}\, v_{\rm kick}.$$

What is $v_{\rm sys}$ in this case for a typical $v_{\rm kick} \simeq 250\,{\rm km\,s^{-1}}$?

f) Discuss how the observed eccentrities and system velocities of systems like the double pulsar may be used to constrain supernova kicks.