High-Energy Astrophysics Lecture 10: Hot gas in clusters of galaxies and thermal bremsstrahlung

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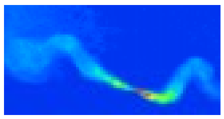


Overview

- Thermal bremsstrahlung
- Introduction to galaxy clusters.
- X-ray emission from clusters and hot plasma properties. Cooling. Interaction of radio sources with cluster plasma.
- Masses of clusters from gravitational lensing.
- The Sunyaev-Zeldovich effect.

Bremsstrahlung (alias free-free emission)

- Caused by acceleration of electrons in the electrostatic fields of ions and nuclei.
- Principal emission mechanism from very high temperature ($T > 10^6$ K) ion plasma (e.g. galaxy halos; clusters of galaxies).
- These plasmas are described as **thermal** because they have a Maxwellian velocity distribution corresponding to a well-defined temperature (cf. A power-law distribution). The emitted spectrum does not look like that of a black body.
- Name from German (*braking radiation*).

Approximate (classical and non-relativistic) approach

- Calculate acceleration of electron in electrostatic field of the nucleus
- Fourier transform of acceleration -> spectrum
- Integrate over impact parameters

Spectrum of emission from an accelerated charge

- Total radiated energy from a charged particle with acceleration a(t) - from Larmor's formula.
- Then take Fourier transform and use Parseval's theorem:

$$E = \int_{-\infty}^{\infty} P dt = \frac{q^2}{6\pi\epsilon_0 c^3}\int_{-\infty}^{\infty} \mid a(t) \mid^2 dt.$$

$$\int_{-\infty}^{\infty} \mid a \mid^2 dt = \int_{-\infty}^{\infty} \mid \tilde{a}(\nu) \mid^2 d\nu$$

Spectrum

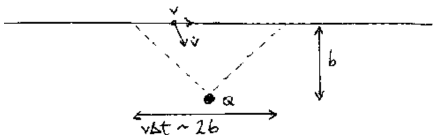
- $E(\nu)$ is energy radiated per unit frequency interval:

$$\begin{aligned} E &= \frac{q^2}{6\pi\epsilon_0 c^3}\int_{-\infty}^{\infty} \mid \tilde{a}(\nu) \mid^2 d\nu \\ &= \frac{q^2}{3\pi\epsilon_0 c^3}\int_{0}^{\infty} \mid \tilde{a}(\nu) \mid^2 d\nu \\ &= \int_0^{\infty} E(\nu) d\nu \end{aligned}$$

$$E(\nu) = \frac{q^2}{3\pi\epsilon_0 c^3} \mid \tilde{a}(\nu) \mid^2$$

Non-relativistic case

- See Longair (vol 1, 3.4) for a fuller discussion.
- Electron moves with a velocity v past an ion, assumed stationary. Experiences **E**, hence acceleration.



Very rough calculation

- Maximum acceleration and rough duration -> short pulse of radiation:

$$a \sim \frac{Qe}{4\pi m_e \varepsilon_0 b^2} \qquad \Delta t \sim \frac{2b}{v} \quad \Rightarrow \quad P \sim \frac{e^2 a^2}{6\pi\varepsilon_0 c^3} \qquad P \sim \frac{Q^2 e^4}{96\pi^3 m_e^2 \varepsilon_0^3 b^4 c^3}$$

Total energy radiated

$$E \sim \frac{Q^2 c\sigma_{\mathrm{T}}}{8\pi^2 \varepsilon_0 b^3 v}.$$

More accurate calculation

- x = r cosθ, b = r sin θ

$$E = \int P dt \simeq \int P(x)\frac{dx}{v}$$

$$\begin{aligned} E &= \frac{Q^2 e^4}{96\pi^3 \varepsilon_0^3 c^3 m_e^2 v}\int_{-\infty}^{\infty}\frac{dx}{r^4} \\ &= \frac{Q^2 e^4}{96\pi^3 \varepsilon_0^3 c^3 m_e^2 v}\int_0^{\pi}\frac{b}{\sin^2\theta}\frac{\sin^4\theta}{b^4}d\theta \\ &= \frac{Q^2 e^4}{96\pi^3 \varepsilon_0^3 c^3 m_e^2 v b^3}\int_0^{\pi}\sin^2\theta\, d\theta \\ &= \frac{Q^2 c\sigma_{\mathrm{T}}}{32\pi b^3 \varepsilon_0 v}. \end{aligned}$$

Single-particle spectrum

- Fourier transform of a sharp pulse is a roughly flat frequency spectrum up to $\nu_{max} \sim 1/\Delta t$, so energy radiated per unit frequency below this frequency in a single electron-ion collision is:

$$E_\nu d\nu \sim \frac{Q^2 c\sigma_{\mathrm{T}}}{16\pi \varepsilon_0 b^2 v^2} d\nu$$

Integrate over particles

(n_i = ion density; n_e = electron density)

$$N\,db = n_e v 2\pi b\,db$$

Number of collisions per unit time at impact parameter b per ion

$$N' db = 2\pi b v n_e n_i db$$

Ditto for all ions

$$I(\nu)d\nu \sim \int_{b_{min}}^{b_{max}} \frac{Q^2 c n_e n_i \sigma_{\mathrm{T}}}{8\varepsilon_0 b v} db\, d\nu$$

Power per unit volume and frequency

$$I(\nu)d\nu \sim \frac{Q^2 c n_e n_i \sigma_{\mathrm{T}}}{8\varepsilon_0 v}\ln\left(\frac{b_{max}}{b_{min}}\right)d\nu$$

Impact parameter b

- Maximum b from requirement that the frequency produced is $<\nu_{max}$:

  $b_{max} \sim v/2\nu$

- Minimum b from condition that energy of electron after collision cannot be more than value for complete rebound:

  $b_{min} \sim eQ/8\pi\varepsilon_0 m_e v^2$

  (for low velocities)

  At higher velocities, encounter quantum-mechanical effects (Uncertainty Principle):

  $b_{min} \sim h/4\pi m_e v$

Finally ...

$$I(\nu)d\nu \sim \frac{Q^2 e n_e n_i \sigma_T}{8\varepsilon_0 c} \ln\left(\frac{m v^2}{2h\nu}\right) d\nu$$

- Requires full quantum-mechanical treatment of radiation field, but gives essentially the same result.
- H and He dominate bremsstrahlung from high-temperature plasma, as they are fully ionised.

  H $\propto e^2 n_H n_e$

  He $\propto 4e^2 n_{He} n_e$ (fractional abundance 0.08 by number)

Thermal Bremsstrahlung

$$N(v)dv \propto \left(\frac{m}{kT}\right)^{3/2} v^2 e^{-\frac{mv^2}{2kT}} dv$$

Temperature T; Maxwellian velocity distribution

$$\frac{1}{2} m_e v^2 \sim \frac{3}{2} kT$$

Typical electron velocity

$$I(\nu) \propto T^{-0.5}$$

At low frequencies, since $I(\nu) \propto 1/v$

$$I(\nu) \propto T^{-\frac{1}{2}} e^{-\frac{h\nu}{kT}}$$

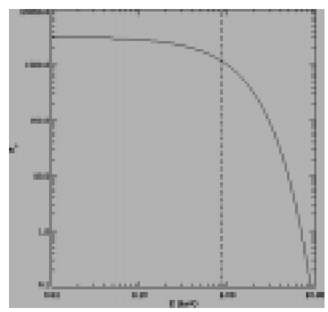
At high frequencies

Hence most sensitive to plasma with $h\nu \sim kT$

Alternative terminology

- **Emissivity** $\propto n_i n_e g(\nu,T) T^{-1/2} \exp(-h\nu/kT)$, where n_i and n_e are the ion and electron number densities and $g(\nu,T) \propto \ln(kT/h\nu)$ is the Gaunt factor.

Thermal bremsstrahlung spectrum



Clusters of galaxies

- Richest clusters have size scales ~1 Mpc.
- Dominated by elliptical and S0 galaxies.
- Central, massive cD galaxies with extended stellar envelopes are always elliptical.
- Deep gravitational potential wells.
- Typical galaxy velocity dispersion $\sigma \sim 1000$ km s^{-1}
- Crossing time

  $t_{cross} \sim r/\sigma \sim 10^9$ years < Hubble time

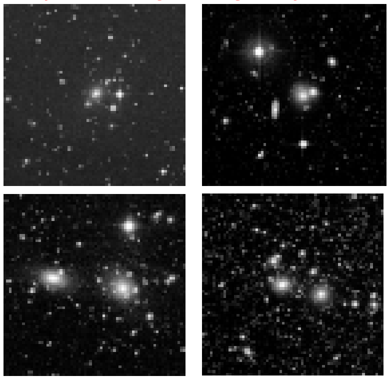
  so clusters have time to relax dynamically.

Clusters of galaxies - 2

- Assuming virial equilibrium, typical mass is

  $M \approx R\sigma^2/G \approx (R/1\ \mathrm{Mpc})(\sigma/1000\ \mathrm{kms}^{-1})^2 \times 10^{15}$ solar masses.
- This is much greater than that associated with stars or plasma, indicating that the potential wells of clusters are dominated by dark matter.

Optical images of galaxy clusters



X-ray properties of clusters of galaxies

- **Mass** ~15% of the mass of galaxy clusters is in the form of hot, diffuse plasma filling its potential well.
- **Temperature** If plasma has the same dynamics as the member galaxies, then expect

  $kT \approx \mu m_p \sigma^2 \approx 7 \times 10^7 (\sigma/1000 \text{ kms}^{-1})^2$ K

  (σ is the cluster velocity dispersion, m_p is the proton mass and μ is the mean molecular weight).
- This relation is observed, so clusters are reasonably relaxed structures in which gas and stars feel the same dynamics.

Coma cluster in X-rays

Hydrostatic equilibrium in clusters

- Equation of hydrostatic equilibrium:

  $dp/dr = -GM(<r)\rho(r)/r^2$
- Perfect gas $p = \rho kT/\mu m_p$
- Hence total gravitating mass within radius r:

  $M(>r) = -(kTr/G\, \mu m_p)(d \ln\rho/d \ln r + d \ln T/d \ln r)$
- Isothermal case: plasma and stars in equilibrium in gravitational potential ϕ. Then Boltzmann distributions for gas and galaxies are

  $\rho_{gas} \propto \exp(-\mu m_p \phi /kT)$

  $\rho_{galaxies} \propto \exp(-\phi /\sigma^2)$

Hydrostatic equilibrium in clusters - 2

- Therefore

  $\rho_{gas} \propto \rho_{galaxies}{}^{\beta}$

  with $\beta = \mu m_p \sigma^2/kT$. $\beta = 1$ if gas and galaxies have the same spatial distribution and mean velocity dispersion.
- Density distribution often described by

  $\rho(r) = \rho(0)[1 + (r/r_c)^2]^{-3\beta/2}$

  (hydrostatic equilibrium for isothermal gas in a potential well associated with a King dark-matter density profile). Observed $\beta = 0.7 - 0.9$.
- $r_c \approx 300$ kpc for a typical rich cluster.

Cooling

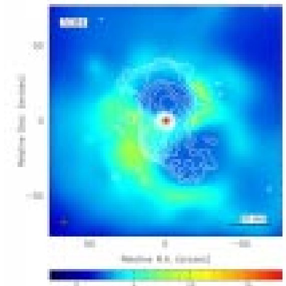
- For bremsstrahlung emission, the cooling time is

  $t_{cool} \approx 8.5 \times 10^{10} (n/1000 \text{ m}^{-3})^{-1} (T/10^8 \text{ K})^{1/2}$ years
- Therefore, the cooling time in central cluster regions can be shorter than the age of the Universe.
- Although temperature gradients are seen in clusters, the catastrophic cooling predicted by this argument is not observed.
- There must be a **feedback mechanism** to heat the gas. This is a subject of debate, but energy input from active galaxies in general and radio sources in particular is favoured.

Interaction of radio sources with gas in galaxy clusters

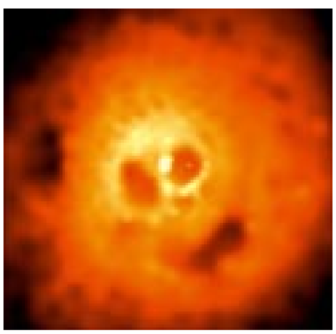
- Ram-pressure of radio source expansion gives kinetic energy to gas near the cluster centre and moves it to large radii.
- Radio-source bow shocks heat the gas.
- Onset of radio-source activity may be triggered by cooling and infall of material into the cluster centre giving a feedback mechanism.

Central galaxy of the Perseus cluster



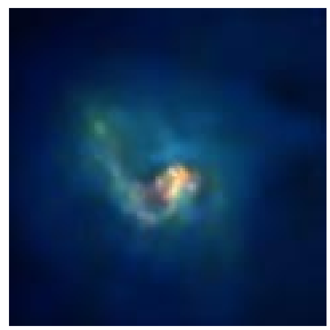
3C84 (NGC1275): X-ray false colour on radio contours

The skull in the Perseus cluster



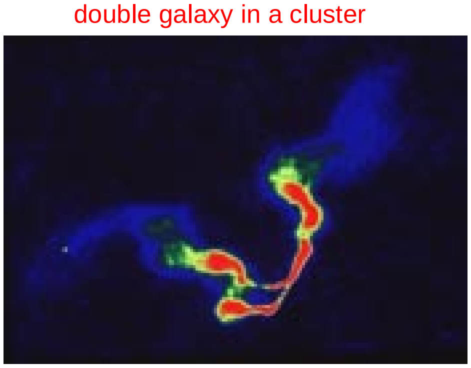
Note the outer dark regions

Centaurus cluster in X-rays

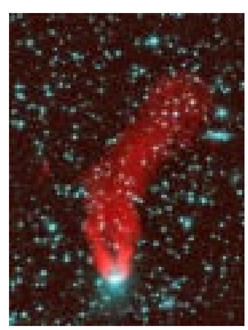


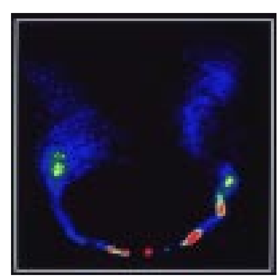
Temperature is colour-coded (orange hot; blue cooler)

3C75: FRI sources in both nuclei of a double galaxy in a cluster

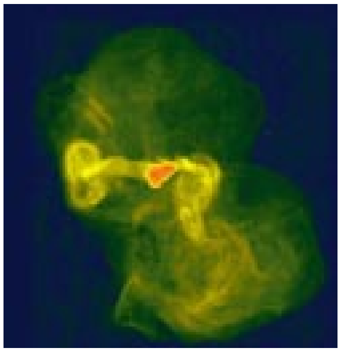


The effects of motion and buoyancy in a galaxy cluster

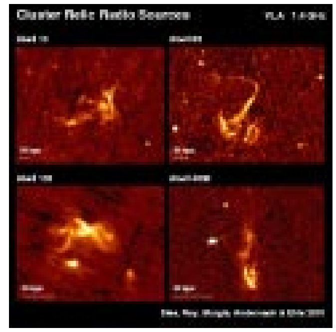




M87 - central galaxy of the Virgo cluster



Relic radio sources in clusters



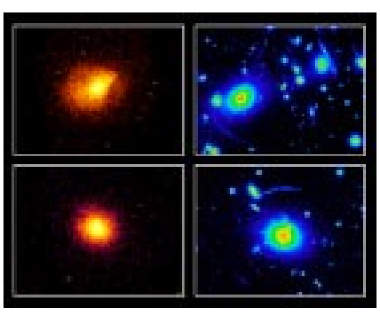
Mass distribution in clusters from gravitational lensing

- Luminous arcs seen in clusters of galaxies.
- These are images of background objects gravitationally lensed by the cluster mass (and are indeed observed to have spectra of background objects).
- Consider an isothermal sphere model for the cluster with $M(<r) = 2\sigma^2 r/G$
- The gravitational deflection angle is $\alpha = 4\pi(\sigma/c)^2$. A source on the line of sight through the centre of an axially symmetric mass is seen as an **Einstein ring** of angle θ with $\alpha d_{\text{lens-source}} = \theta d_{\text{observer-source}}$; hence derive the equivalent velocity dispersion and mass.

HST image of a cluster of galaxies, showing effects of gravitational lensing

X-ray emission from hot gas in lensing clusters

X-ray **Optical**



Masses from gravitational lensing and X-ray emission

- In reasonable agreement, although some tendency for lensing masses to be higher (probably because the hot plasma has substructure).
- Both these indicators (and the cluster velocity dispersions) show that the mass distributions of clusters of galaxies are dominated by **dark matter**.
- The mass of X-ray emitting gas is ~15% of the total.

The Sunyaev-Zeldovich (SZ) Effect

- **Spectral distortion** of the cosmic microwave background (CMB) spectrum caused by the inverse Compton scattering of CMB photons off a thermal distribution of high-energy electrons.
- Temperature change ΔT at dimensionless frequency $x = h\nu/kT$:

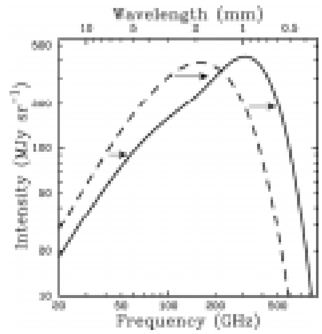
  $\Delta T/T = f(x) \int (n_e k T_e / m_e c^2) \sigma_T \, dl$

  $f(x) = [\, x(e^x + 1)/(e^x - 1) - 4]$

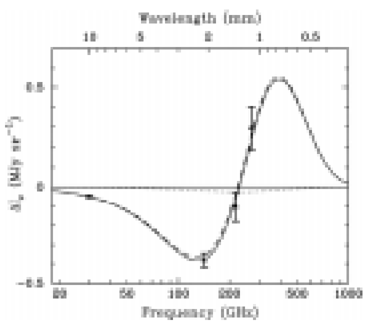
  (neglecting relativistic corrections)

  $f(x) \to -2$ in the Rayleigh-Jeans limit
- See Longair, vol 1, 4.3.

Sunyaev-Zeldovich Effect

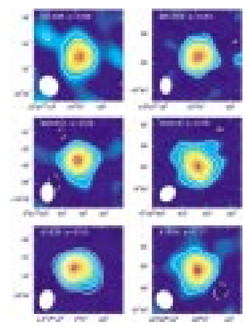


Spectrum of the SZ effect



Galaxy cluster Abell 2163

Images of the SZ effect



SZ effect - 2

- $\Delta T/T$ is independent of redshift - hence very important in studying high-redshift Universe.
- $\Delta T \approx 1$mK for galaxy clusters.
- Unique spectral signature.
- $\Delta T/T \propto$ cluster pressure integrated along line of sight.
- $\Delta T/T \propto n_e T_e$ whereas bremsstrahlung emission $\propto n_e T_e^2$. Hence an estimate of the distance.

Distances from SZ effect + X-ray emission