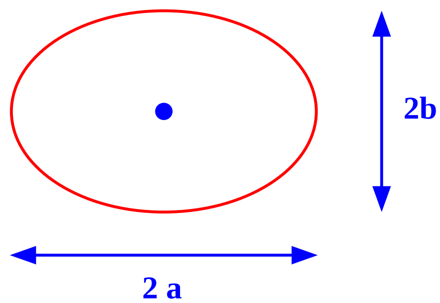
7. BINARY STARS (ZG: 12; CO: 7, 17)

- most stars are members of binary systems or multiple systems (triples, quadruples, quintuplets, ...)
- *orbital period* distribution: $P_{orb} = 11\,\text{min}$ to $\sim 10^6\,\text{yr}$
- the majority of binaries are wide and do not interact strongly
- *close binaries* (with $P_{orb} \lesssim 10\,\text{yr}$) can transfer mass → *changes structure and subsequent evolution*
- *approximate period distribution:* $f(\log P) \simeq$ const. (rule of thumb: 10 % of systems in each decade of log P from 10^{-3} to 10^7 yr)
- generally large scatter in distribution of eccentricities
  $e^2 \equiv 1 - b^2/a^2$,
  a = semi-major,
  b = semi-minor axis

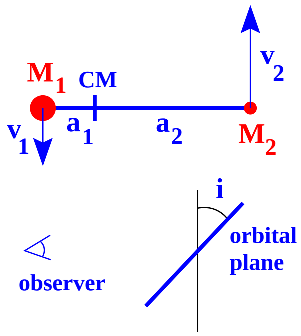


- systems with eccentricities $\lesssim 10\,\text{d}$ tend to be circular → evidence for *tidal circularization*

7.1 Classification

- *visual binaries:* see the periodic wobbling of two stars in the sky (e.g. Sirius A and B); if the motion of only one star is seen: *astrometric binary*
- *spectroscopic binaries:* see the periodic *Doppler shifts* of spectral lines
  - ▷ *single-lined:* only the Doppler shifts of one star detected
  - ▷ *double-lined:* lines of both stars are detected
- *photometric binaries:* periodic variation of fluxes, colours, etc. are observed (caveat: such variations can also be caused by single variable stars: Cepheids, RR Lyrae variables)
- *eclipsing binaries:* one or both stars are eclipsed by the other one → inclination of orbital plane $i \simeq 90^\circ$ (most useful for determining basic stellar parameters)

- consider a *spectroscopic binary*
- measure the *radial velocity curve* along the line of sight from $\frac{v_r}{c} \simeq \frac{\Delta\lambda}{\lambda}$ (Doppler shift)



▷ $M_1 a_1 = M_2 a_2$

▷ $P = \frac{2\pi}{\omega} = 2\pi \frac{a_1 \sin i}{v_1 \sin i} = 2\pi \frac{a_2 \sin i}{v_2 \sin i}$

▷ *gravitational force*
= centripetal force

$$\rightarrow \quad \frac{GM_1M_2}{(a_1+a_2)^2} = \frac{(v_1 \sin i)^2}{a_1 \sin^2 i} M_1, \quad \frac{GM_1M_2}{(a_1+a_2)^2} = \frac{(v_2 \sin i)^2}{a_2 \sin^2 i} M_2$$

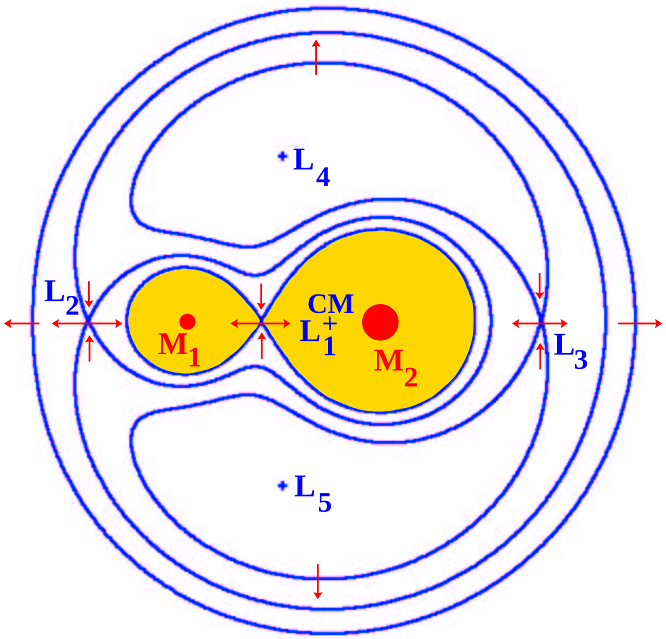
substituting $(a_1+a_2)^2 = a_1^2 (M_1+M_2)^2/M_2^2$, etc.

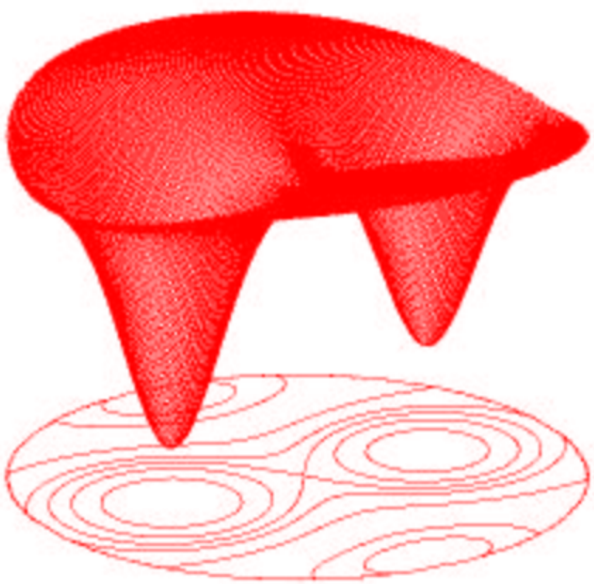
$$\rightarrow \quad f_1(M_2) = \frac{M_2^3 \sin^3 i}{(M_1+M_2)^2} = \frac{P\,(v_1 \sin i)^3}{2\pi G}$$

$$f_2(M_1) = \frac{M_1^3 \sin^3 i}{(M_1+M_2)^2} = \frac{P\,(v_2 \sin i)^3}{2\pi G}$$

- f_1, f_2 *mass functions:* relate observables $v_1 \sin i$, $v_2 \sin i$, P to quantities of interest M_1, M_2, $\sin i$
- measurement of f_1 and f_2 (for double-lined spectroscopic binaries only) determines $M_1 \sin^3 i$, $M_2 \sin^3 i$
  - ▷ if i is known (e.g. for visual binaries or eclipsing binaries) $\rightarrow M_1$, M_2
  - ▷ for $M_1 \ll M_2 \rightarrow f_1(M_2) \simeq M_2 \sin^3 i$ (measuring $v_1 \sin i$ for star 1 constrains M_2)
- for *eclipsing binaries* one can also determine the *radii* of both stars (main source of accurate masses and radii of stars [and luminosities if distances are known])

The Roche Potential





7.3 THE ROCHE POTENTIAL

- ***restricted three-body problem:*** determine the motion of a test particle in the field of two masses M_1 and M_2 in a circular orbit about each other

- equation of motion of the particle in a frame rotating with the binary $\Omega = 2\pi/P$:

$$\frac{d^2\vec{r}}{dt^2} = -\vec{\nabla}\, U_{\rm eff} - \underbrace{2\vec{\Omega}\times\vec{v}}_{\text{Coriolis force}}\ ,$$

where the *effective potential* $U_{\rm eff}$ is given by

$$U_{\rm eff} = -\frac{GM_1}{|\vec{r}-\vec{r}_1|} - \frac{GM_2}{|\vec{r}-\vec{r_2}|} - \underbrace{\frac{1}{2}\Omega^2\,(x^2+y^2)}_{\text{centrifugal term}}$$

- ***Lagrangian points:*** five stationary points of the Roche potential $U_{\rm eff}$ (i.e. where effective gravity $\vec{\nabla}U_{\rm eff} = 0$)

  ▷ 3 saddle points: L_1, L_2, L_3

- ***Roche lobe:*** equipotential surface passing through the *inner Lagrangian point* L_1 (‘connects’ the gravitational fields of the two stars)

- approximate formula for the *effective Roche-lobe radius* (of star 2):

$$R_L = \frac{0.49}{0.6 + q^{2/3}\ln\left(1+q^{-1/3}\right)}\,A,$$

where $q = M_1/M_2$ is the mass ratio, A orbital separation.

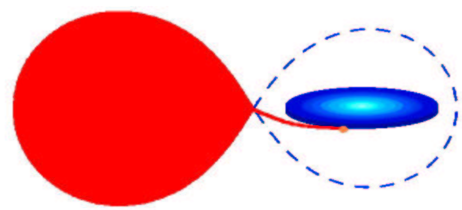
Classification of close binaries

- ***Detached binaries:***
  - ▷ both stars *underfill their Roche lobes,* i.e. the photospheres of both stars lie beneath their respective Roche lobes
  - ▷ *gravitational interactions* only (e.g. tidal interaction, see Earth-Moon system)
- ***Semidetached binaries:***
  - ▷ one *star fills its Roche lobe*
  - ▷ the Roche-lobe filling component *transfers matter* to the detached component
  - ▷ *mass-transferring binaries*
- ***Contact binaries:***
  - ▷ *both stars fill or overfill their Roche lobes*
  - ▷ formation of a common photosphere surrounding both components
  - ▷ *W Ursae Majoris stars*

7.4 BINARY MASS TRANSFER

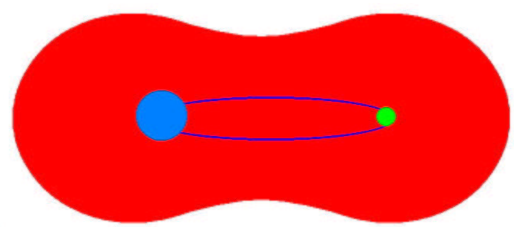
- *30 - 50 %* of all stars experience mass transfer by *Roche-lobe* overflow during their lifetimes (generally in late evolutionary phases)

a) (quasi-)*conservative mass transfer*

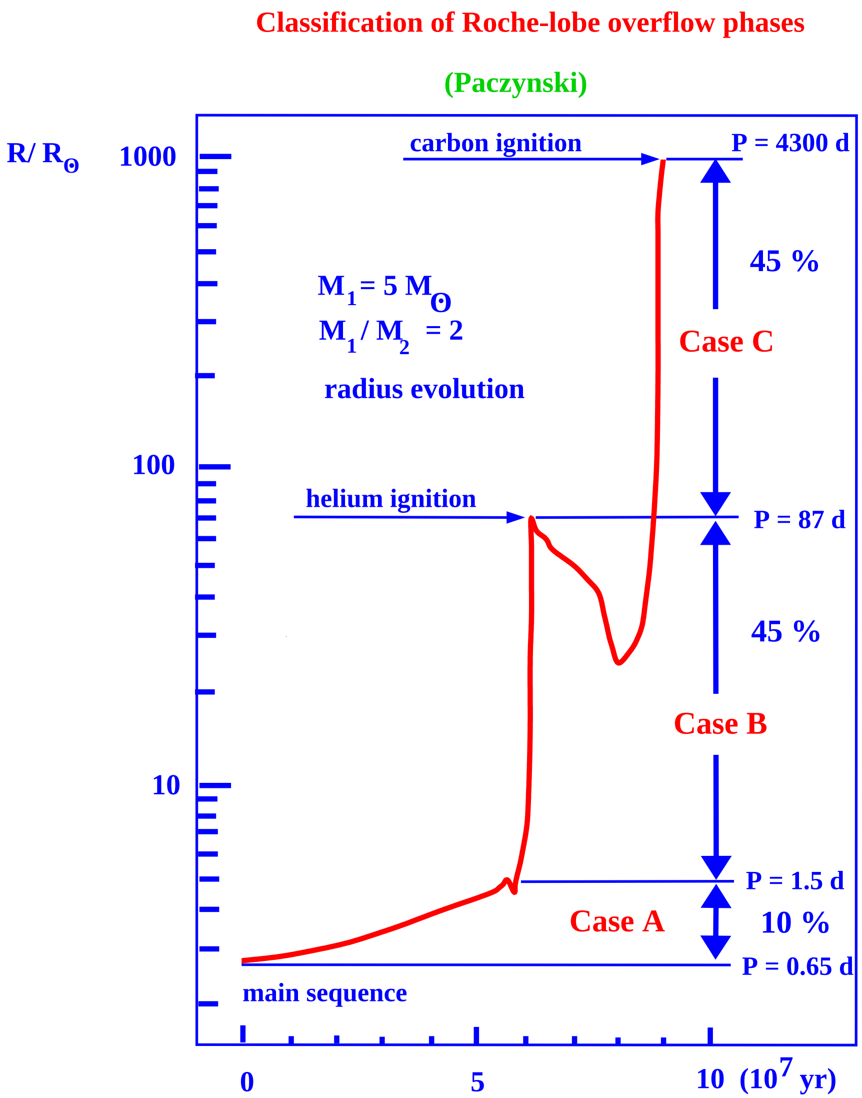


- ▷ *mass loss + mass accretion*
- ▷ the mass loser tends to lose most of its envelope → formation of *helium stars*
- ▷ the accretor tends to be *rejuvenated* (i.e. behaves like a more massive star with the evolutionary clock reset)
- ▷ *orbit* generally *widens*

b) *dynamical mass transfer* → *common-envelope* and spiral-in phase (mass loser is usually a red giant)



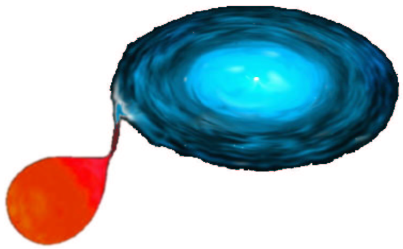
- ▷ accreting component also fills its Roche lobe
- ▷ mass donor (primary) *engulfs secondary*
- ▷ *spiral-in* of the core of the primary and the secondary immersed in a *common envelope*
- ▷ if *envelope ejected* → *very close binary* (compact core + secondary)
- ▷ otherwise: *complete merger* of the binary components → *formation of a single, rapidly rotating star*



Algols and the Algol paradox

- *Algol* is an eclipsing binary with orbital period 69 hr, consisting of a B8 dwarf ($M = 3.7\,M_\odot$) and a K0 subgiant ($M = 0.8\,M_\odot$)
- the *eclipse* of the B0 star is *very deep* → *B8 star more luminous* than the more evolved K0 subgiant
- the *less massive star is more evolved*
- inconsistent with stellar evolution → *Algol paradox*
- explanation:
  - ▷ the K star was *initially the more massive* star and evolved more rapidly
  - ▷ *mass transfer* changed the mass ratio
  - ▷ because of the added mass the B stars becomes the more luminous component

Interacting binaries containing compact objects (Supplementary)



- short orbital periods (11 min to typically 10s of days) → requires *common-envelope* and *spiral-in* phase

Cataclysmic Variables (CV)

- main-sequence star (usually) transferring mass to a *white dwarf* through an *accretion disk*
- *nova outbursts: thermonuclear explosions* on the surface of the white dwarf
- orbit shrinks because of angular-momentum loss due to *gravitational radiation and magnetic braking*

X-Ray Binaries

- *compact component: neutron star, black hole*
- *mass donor* can be of low, intermediate or high mass
- very luminous *X-ray sources* (accretion luminosity)
- neutron-star systems: luminosity distribution peaked near the *Eddington limit,* (accretion luminosity for which radiation pressure balances gravity) $L_{\mathrm{Edd}} = \frac{4\pi c G M}{\kappa} \simeq 2 \times 10^{31}\,\mathrm{W} \left(\frac{M}{1.4\,M_\odot}\right)$
- accretion of mass and angular momentum → *spin-up* of neutron star → formation of *millisecond pulsar*
- *soft X-ray transients:* best *black-hole* candidates (if $M_X >$ max. neutron-star mass $\sim 2-3\,M_\odot$ → likely black hole [but no proof of event horizon yet])

Formation of Low-Mass X-Ray Binaries (I)