NEUTRON STARS

1934: Baade and Zwicky proposed that *supernovae* represented the transition of normal stars to neutron stars

1939: Oppenheimer and Volkoff published the first *theoretical model*

1967: *discovery of pulsars* by Bell and Hewish

Maximum mass of a neutron star:

- Neutrons are *fermions: degenerate neutrons* are unable to support a neutron star with a mass above a certain value (c.f. *Chandrasekhar mass* limit for white dwarfs).
- Important differences from white dwarf case:
  (i) *interactions between neutrons* are very important at high densities
  ii) very *strong gravitational fields* (i.e. use *General Relativity*)
  N.B. There is a maximum mass but the calculation is very difficult; the result is very important for black-hole searches in our Galaxy
- Crude estimate– ignore interactions and General Relativity – apply same theory as for white dwarfs: $M_{max} \simeq 6\,M_\odot$
- *interactions between neutrons* increase the theoretical maximum mass
  - ▷ The interaction is attractive at distances $\sim 1.4\,\mathrm{fm}$ but repulsive at shorter distances

  → matter harder to compress at high densities

  - ▷ *but, at high densities,* degenerate *neutrons* are *energetic* enough to produce *new particles (hyperons and pions)* → the pressure is reduced because the new particles only produce a small pressure
- *Effect of gravity* (gravitational binding energy of neutron star is comparable with rest mass):
- Gravity is strengthened at very high densities and pressures. Consider the pressure gradient:

$$\textit{Newton:}\ \frac{dP}{dr} = -\frac{Gm\rho}{r^2}$$

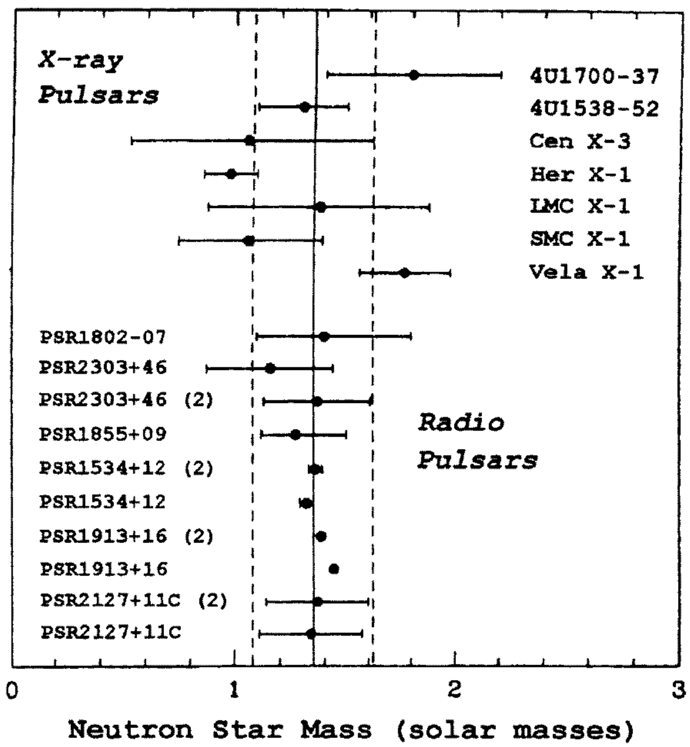
$$\textit{Einstein:}\ \frac{dP}{dr} = -\frac{Gm\rho}{r^2} \times \frac{(1 + P/(\rho c^2))(1 + 4\pi r^3 P/(mc^2))}{1 - 2Gm/(rc^2)}$$

- Pressure P occurs on RHS. *Increase of pressure,* needed to oppose gravitational collapse, leads to *strengthening of gravitational field*
- need an equation of state $P(\rho)$ that takes account of $n - n$ interactions
- Oppenheimer and Volkoff (1939) calculated M_{max} for a star composed of non-interacting neutrons.
  Result: $M_{max} = 0.7\,M_\odot$
- *Enhanced gravity leads to collapse at finite density* when *neutrons* are just becoming relativistic – *not ultrarelativistic*
- Various calculations, using different compressibilities for neutron star matter, predict

$$\boxed{M_{max} \in [1.5, 3]\,M_\odot} \qquad R_{NS} \simeq 7 - 15\,\mathrm{km}$$

- *Observed neutron star masses* (from analysis of binary systems) are mostly around $1.5\,M_\odot$

- Modern calculations suggest that M_{max} is probably less than $3M_\odot$ and definitely less than $5M_\odot$. See Phillips, "The Physics of Stars", for an example calculation: incompressible star of constant density



PULSARS

Expected properties of neutron stars:

(a) *Rotation period* (c.f. white dwarfs, e.g. 40 Eri B, $P_{WD} = 1350\,s$)

From simple theory : $\dfrac{R_{WD}}{R_{NS}} \simeq \dfrac{m_n}{2^{5/3} m_e} \simeq 600$

conservation of angular momentum: with $M_{WD} \sim M_{NS}$ and $I = (2/5)MR^2$ (uniform sphere)

$$I_i \omega_i = I_f \omega_f$$

$$\omega_f = \omega_i (R_i/R_f)^2$$

$$P_{NS} \simeq 3 \times 10^{-6}\, P_{WD} \simeq 4\,ms$$

$\rightarrow$ neutron stars *rotate rapidly* when they form

- ▷ but angular momentum is probably lost in the supernova explosion
- ▷ rotation is likely to slow down rapidly

(b) *magnetic field*

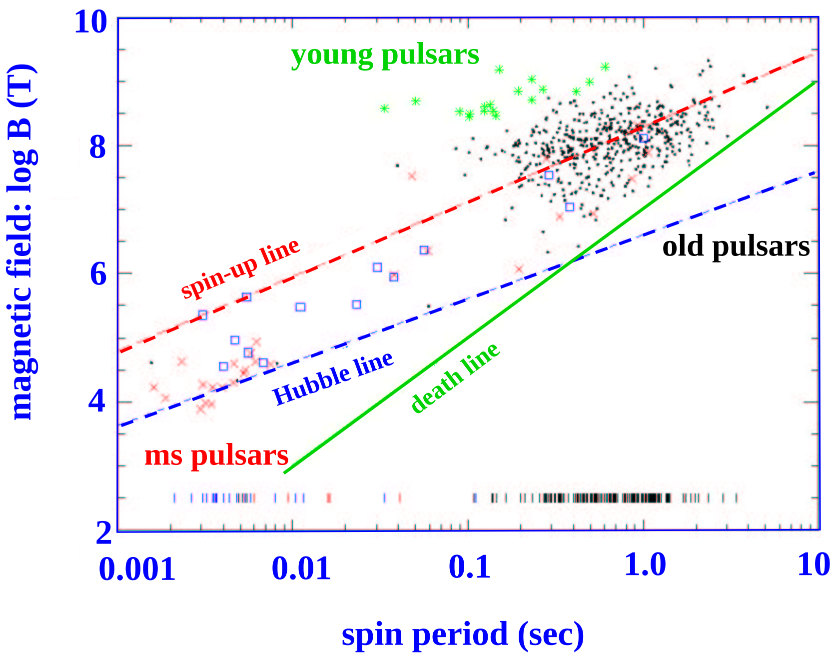
- ▷ *Flux conservation* requires $\int B\, dS$ = constant

$$B_i\, 4\pi R_i^2 = B_f\, 4\pi R_f^2$$

- ▷ Take largest observed white dwarf field, $B_{WD} \simeq 5 \times 10^4$ T

$$B_{NS} \simeq B_{WD} (R_{WD}/R_{NS})^2 \simeq 10^{10}\,T \qquad \text{(upper limit)}$$

Radio Pulsars: the P-B Diagram



(c) *Luminosity*

- ▷ neutron star forms at $T \sim 10^{11}\,K$ but T drops to $\sim 10^{9}\,K$ within 1 day
- ▷ main *cooling process: neutrino emission* (first $\sim 10^{3}\,yr$), then *radiation*
- ▷ after a few hundred years, $T_{internal} \sim 10^{8}\,K$, $T_{surface} \sim$ a few $\times\, 10^{6}\,K$.
- ▷ star cools at constant R for $\sim 10^{4}\,yr$ with $T_{surface} \sim 10^{6}\,K$

$$L \sim 4\pi R^2 \sigma T_s^4 \sim 10^{26}\,W \;(\text{mostly X}-\text{rays},\; \lambda_{max} \sim 3\text{ nm})$$

Discovery of neutron stars

- The *first pulsar* was discovered by Bell and Hewish at Cambridge in 1967
  - ▷ A radio interferometer (2048 dipole antennae) had been set up to study the scintillation which was observed when radio waves from distant point sources passed through the solar wind. Bell discovered a signal, regularly spaced radio pulses 1.337 sec apart, coming from the same point in the sky every night.
- Today, about *1500 pulsars* are known
  - ▷ Most have periods between *0.25 s and 2 s* (average 0.8 s)
  - ▷ *Extremely well defined pulse periods* that challenge the best atomic clocks
  - ▷ Periods increase very gradually
  - ▷ *spin-down timescale* for young pulsars $\sim P/\dot{P} \sim 10^{6} - 10^{7}\,yr$

SUPERNOVA REMNANTS

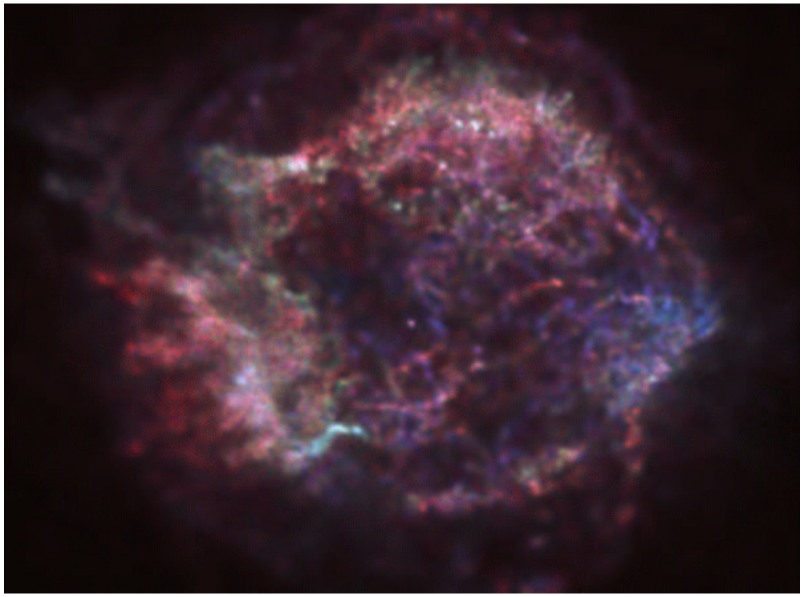
The Crab Nebula (plerionic/filled)

VLT

Chandra

(X-rays)

Cassiopeia A (shell-like)



The Crab Pulsar

- The *Crab nebula* is the *remnant* of a *supernova* explosion observed optically in 1054 AD.
- The *Crab pulsar* is at the centre of the nebula, emitting X-ray, optical and radio pulses with $P = 0.033\,s$.
- The Crab nebula is morphologically different from two other recent supernova remnants, *Cas A* and *Tycho* (both $\sim 400\,yr$ old) which are *shell-like.*
- The present *rate of expansion* of the nebula can be measured: uniform expansion extrapolates back to 90 years *after* the explosion, i.e. the expansion must be accelerating
- The observed *spectrum* is a *power law* from $\sim 10^{14}\,Hz$ (IR) to $\sim 1\,MeV$ (hard X-rays); also, in the extended nebulosity, the *X-rays* are 10-20 % *polarised* $\rightarrow$ signature of *synchrotron radiation* (relativistic electrons spiralling around magnetic field lines with $B \sim 10^{-7}\,T$).
- Synchrotron radiation today requires *(i) replenishment of magnetic field* and *(i) continuous injection of energetic electrons.*
- *Total power* needed $\sim 5 \times 10^{31}\,W$
- *Energy source* is a *rotating neutron star* $(M \simeq 1.4\,M_\odot,\ R \simeq 10\,km)$

$$U = (1/2)\, I\omega^2 = 2\pi^2\, I/P^2$$

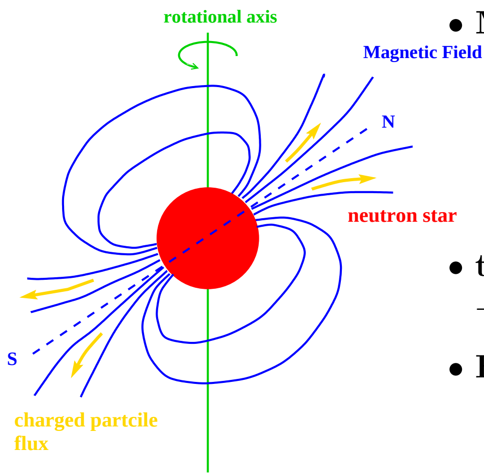
$$\frac{dU}{dt} = -\frac{4\pi^2 I\dot{P}}{P^3}$$

Taking $I = (2/5)\,MR^2 \sim 1 \times 10^{38}\,kg\,m^2;\ P = 0.033\,s;$

$\dot{P} = 4.2 \times 10^{-13} \quad \rightarrow \quad dU/dt \simeq 5 \times 10^{31}\,W$

A simple pulsar model

- A pulsar can be modelled as a *rapidly rotating neutron star* with a strong *dipole magnetic field* inclined to the rotation axis at angle θ.
- Pulsar emission is *beamed* (like a lighthouse beam) $\rightarrow$ observer has to be in the beam to see *pulsed emission.*



- Magnetic dipole radiation:

$$\frac{dU_{mag}}{dt} = -\frac{2}{3c^3}\left(\frac{\mu_0}{4\pi}\right) m^2\omega^4 \sin^2\theta$$

$$= -\frac{32\pi^4}{3c^3}\left(\frac{\mu_0}{4\pi}\right)\frac{m^2\sin^2\theta}{P^4}$$

- taking $dU/dt = -5\times10^{31}\,\mathrm{W}$
  $\rightarrow \quad m\sin\theta \simeq 4\times10^{27}\,\mathrm{A\,m^2}$.
- Hence, *surface magnetic field*

$$B \simeq \frac{\mu_0 m}{4\pi R^3} \simeq 10^8\,\mathrm{T}.$$

- Further argument for magnetic dipole radiation:

$$-\frac{dU_{rot}}{dt} = -I\omega\frac{d\omega}{dt} \propto \omega^4$$

$$\rightarrow \quad \frac{d\omega}{dt} = -C\omega^3.$$

$$\text{Integrate} \quad t = \frac{1}{2C}\left(\frac{1}{\omega^2} - \frac{1}{\omega_0^2}\right)$$

So $t < \dfrac{1}{2C\omega^2}$ with $C = 3.5\times10^{-16}\,\mathrm{s}$ and $\omega = 190\,\mathrm{s^{-1}}$

$\rightarrow t < 1250\,\mathrm{yr}$ (comparable to the known age $\simeq 950\,\mathrm{yr}$)

N.B. The physics underlying pulsar emission mechanisms is very complicated and not well understood

Pulsar dispersion measure

- Consider an electromagnetic wave of the form $E = E_0\cos(kx \pm wt)$ propagating through an *ionised medium* where the number density of electrons is n_e.
- The *dispersion relation* is

  $\omega^2 = k^2c^2 + \omega_p^2, \quad \omega_p = (n_e e^2/(m\epsilon_0))^{1/2}$ (plasma frequency).

- If $\omega < \omega_p \rightarrow$ *no wave propagation* (e.g. $\sim$ few MHz cut-off to radio waves through the ionosphere). If $\omega > \omega_p$, *propagation* occurs with *group velocity* v_g given by:

$$v_g = \frac{d\omega}{dk} = c\left(1 + \frac{w_p^2}{c^2k^2}\right)^{-1/2} = c\left(1 + \frac{\omega_p^2}{\omega^2 - \omega_p^2}\right)^{-1/2}$$

- i.e. *frequency dependent:* longer wavelength has lower velocity.
- A pulse of radiation travels a distance l in time $t = l/v_g$.
- Frequency dependence of arrival time is given by:

$$\frac{\Delta t}{\Delta\omega} = -\frac{l}{v_g^2}\frac{\Delta v_g}{\Delta\omega} \simeq -\frac{l}{c^2}\frac{\Delta v_g}{\Delta\omega} \quad \text{since } v_g \simeq c \text{ for } \omega >> \omega_p.$$

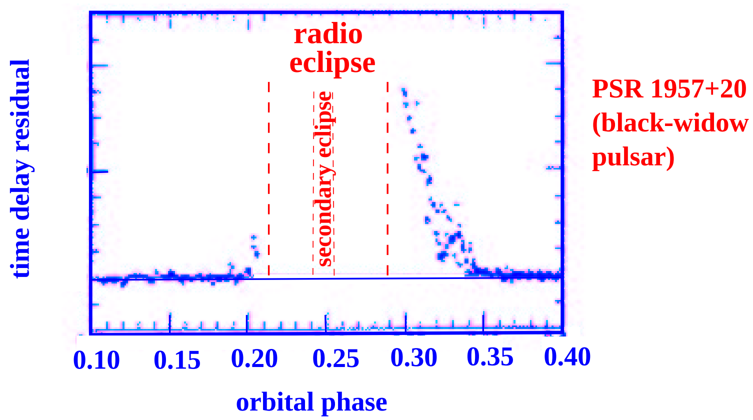
$$\text{But} \quad \frac{\Delta v_g}{\Delta\omega} = \frac{\omega\omega_p^2 c}{(\omega^2-\omega_p^2)^2}\left(1 + \frac{\omega_p^2}{\omega^2-\omega_p^2}\right)^{-3/2} \simeq \frac{\omega_p^2 c}{\omega^3}.$$

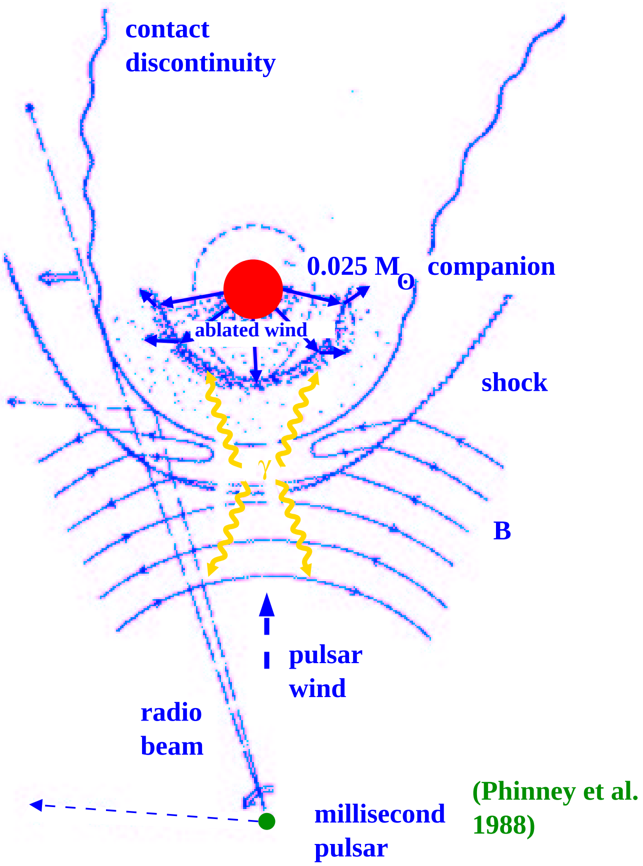
$$\text{Therefore} \quad \frac{\Delta\nu}{\Delta t} = \frac{1}{2\pi}\frac{\Delta\omega}{\Delta t} = -\frac{4\pi^2 mc\ \epsilon_0}{e^2}\frac{\nu^3}{n_e l}$$

$$\text{Strictly} \quad \frac{\Delta\nu}{\Delta t} = -\frac{4\pi^2 mc\ \epsilon_0}{e^2}\frac{\nu^3}{\int n_e dl} \quad \text{since } n_e \text{ varies with } l.$$

- $\int n_e dl$ is known as the ***DISPERSION MEASURE.*** It is a useful distance indicator if n_e is uniform (typical value for the Galactic plane: $1-3\times10^5\,\mathrm{m^{-3}}$)

(Fruchter, Stinebring, Taylor 1988)

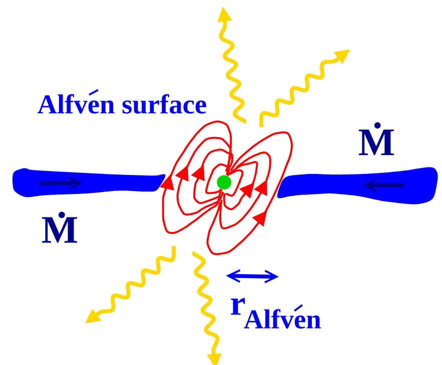




(Phinney et al. 1988)

MILLISECOND (RECYCLED) PULSARS

- a group of ~ 100 radio pulsars with very *short spin periods* (shortest: 1.6 ms) and *relatively weak magnetic fields* ($\mathrm{B} \lesssim 10^6\,\mathrm{T}$)
- they are preferentially members of *binary systems,*
- they have *spin-down timescales* comparable or longer than the Hubble time (age of the Universe)
- *standard model*
  - ▷ these pulsars are neutron stars in *binary systems* that *spin-down* first, *lose their strong magnetic field* (due to accretion?)
  - ▷ and are *spun-up* by *accretion* from a companion



  - ▷ *magnetospheric accretion:* magnetic field becomes dominant when magnetic pressure > ram pressure in flow → flow follows *magnetic field lines* (below $\mathrm{r_A}$)
  - ▷ spin-up due to accretion of *angular momentum*
- *equilibrium spin period:* $\mathrm{v_{rot}(r_A) = v_{Kepler}(r_A)}$

$$\rightarrow \boxed{\mathrm{P_{eq}} = \simeq 2\,\mathrm{ms}\,\left(\mathrm{B}/10^5\,\mathrm{T}\right)^{6/7}\,\left(\dot{\mathrm{M}}/\dot{\mathrm{M}}_{\mathrm{Edd}}\right)^{-3/7}}$$

- a significant fraction of millisecond pulsars are *single*

→ *pulsar radiation* has *evaporated the companion*

  - ▷ example: *PSR 1957+20 (the black-widow pulsar):* companion mass: only $0.025\,\mathrm{M}_\odot$
  - ▷ direct evidence for an *evaporative wind* from the *radio eclipse* (much larger than the secondary)

→ *comet-like* evaporative tail

SCHWARZSCHILD BLACK HOLES

- *event horizon:* (after Michell 1784)
  - ▷ the *escape velocity* for a particle of mass m from an object of mass M and radius R is $v_{esc} = \sqrt{\frac{2GM}{R}}$ ($11\,km\,s^{-1}$ for Earth, $600\,km\,s^{-1}$ for Sun)
  - ▷ assume *photons* have *mass:* $m \propto E$ (Newton's corpuscular theory of light)
  - ▷ photons travel with the *speed of light c*

  → photons cannot escape, if $v_{esc} > c$

  → $\boxed{R < R_s \equiv \frac{2GM}{c^2}}$ *(Schwarzschild radius)*

  - ▷ $R_s = 3\,km\,(M/M_\odot)$

    Note: for neutron stars $R_s \simeq 5\,km$; only a factor of 2 smaller than R_{NS} → GR important
- *particle orbits* near a black hole
  - ▷ the *most tightly bound circular orbit* has a radius $R_{min} = 3\,R_s = (6GM)/c^2$ (defines inner edge of accretion disk)
  - ▷ for a black hole *accreting* from a thin disk, the *efficiency* of energy generation is (usually) determined by the binding energy of the inner most stable orbit ($\sim 6\,\%$ for a Schwarzschild black hole)
- *no hair theorem:* the structure of a black hole is completely determined by its mass M, angular momentum L and electric charge Q