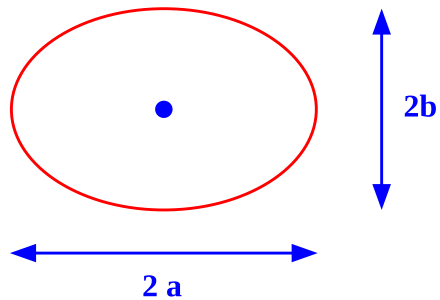
BINARY STARS

- most stars are members of binary systems or multiple systems (triples, quadruples, quintuplets, ...)
- orbital period distribution: $P_{orb} = 11\,\mathrm{min}$ to $\sim 10^6\,\mathrm{yr}$
- the majority of binaries are wide and do not interact strongly
- close binaries (with $P_{orb} \lesssim 10\,\mathrm{yr}$) can transfer mass → changes structure and subsequent evolution
- approximate period distribution: $f(\log P) \simeq \mathrm{const.}$ (rule of thumb: 10 % of systems in each decade of log P from 10^{-3} to 10^7 yr)
- generally large scatter in distribution of eccentricities $e^2 \equiv 1 - b^2/a^2$, a = semi-major, b = semi-minor axis

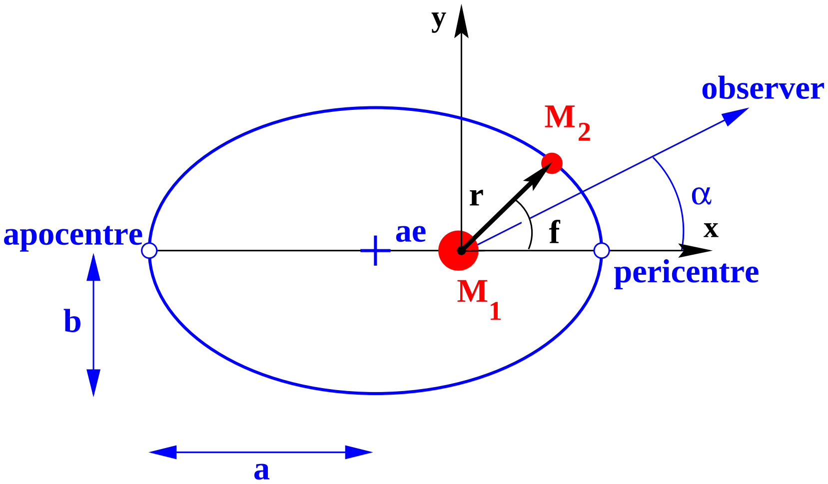


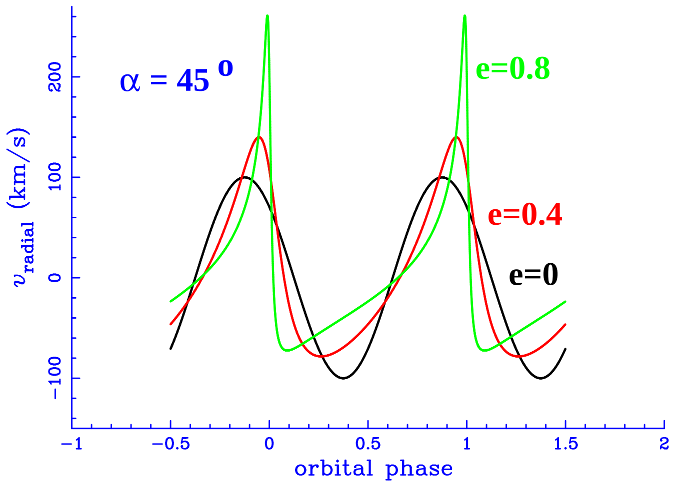
- systems with eccentricities $\lesssim 10\,\mathrm{d}$ tend to be circular → evidence for tidal circularization

Classification

- visual binaries: see the periodic wobbling of two stars in the sky (e.g. Sirius A and B); if the motion of only one star is seen: astrometric binary
- spectroscopic binaries: see the periodic Doppler shifts of spectral lines
  - ▷ single-lined: only the Doppler shifts of one star detected
  - ▷ double-lined: lines of both stars are detected
- photometric binaries: periodic variation of fluxes, colours, etc. are observed (caveat: such variations can also be caused by single variable stars: Cepheids, RR Lyrae variables)
- eclipsing binaries: one or both stars are eclipsed by the other one → inclination of orbital plane $i \simeq 90^\circ$ (most useful for determining basic stellar parameters)

Radial Velocity (eccentric binaries)





for an eccentric binary

$x(t) = a\,(\cos E - e)$ $\qquad \tan\frac{f}{2} = \sqrt{\frac{1+e}{1-e}}\,\tan\frac{E}{2}$

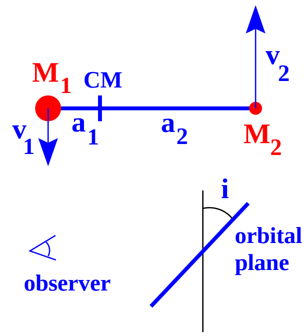
$y(t) = b\sin E$

where the eccentric anomaly E is defined by Kepler's equation

$$E - e\sin E = \frac{2\pi}{P}t = M \quad \text{(mean anomaly)}$$

THE BINARY MASS FUNCTION

- consider a spectroscopic binary
- measure the radial velocity curve along the line of sight from $\frac{v_r}{c} \simeq \frac{\Delta\lambda}{\lambda}$ (Doppler shift)



▷ $M_1 a_1 = M_2 a_2$

▷ $P = \frac{2\pi}{\omega} = 2\pi\frac{a_1\sin i}{v_1\sin i} = 2\pi\frac{a_2\sin i}{v_2\sin i}$

▷ gravitational force $=$ centripetal force

$$\rightarrow \quad \frac{GM_1M_2}{(a_1+a_2)^2} = \frac{(v_1\sin i)^2}{a_1\sin^2 i}M_1, \quad \frac{GM_1M_2}{(a_1+a_2)^2} = \frac{(v_2\sin i)^2}{a_2\sin^2 i}M_2$$

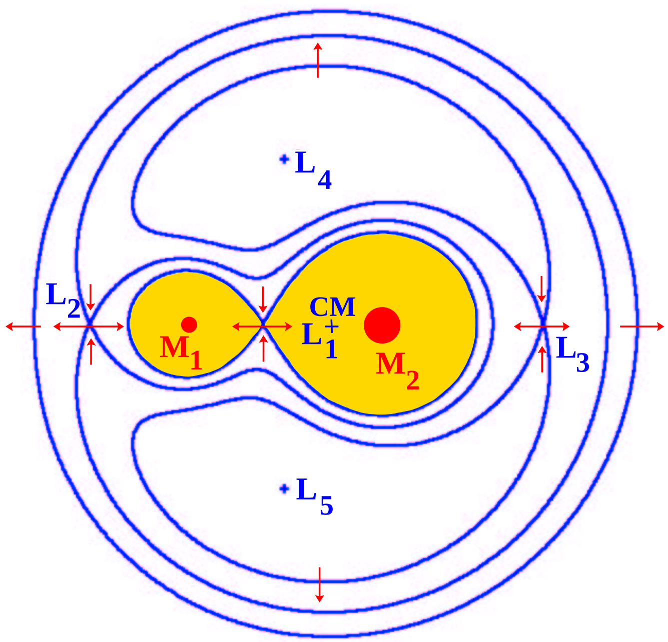
substituting $(a_1+a_2)^2 = a_1^2\,(M_1+M_2)^2/M_2^2$, etc.

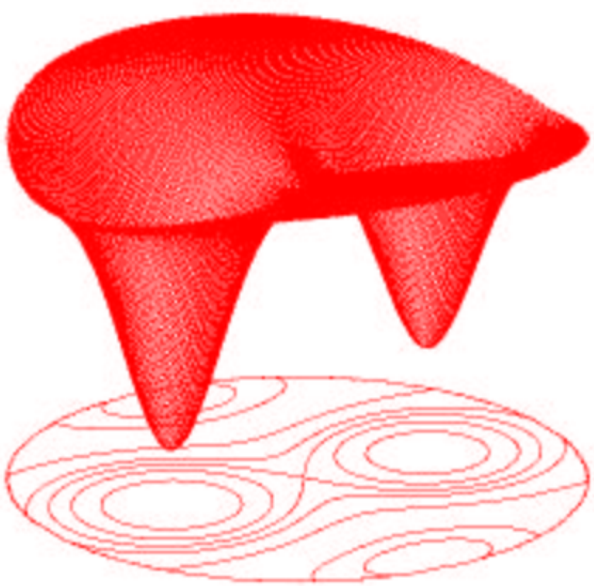
$$\rightarrow \quad f_1(M_2) = \frac{M_2^3\sin^3 i}{(M_1+M_2)^2} = \frac{P\,(v_1\sin i)^3}{2\pi G}$$

$$f_2(M_1) = \frac{M_1^3\sin^3 i}{(M_1+M_2)^2} = \frac{P\,(v_2\sin i)^3}{2\pi G}$$

- f_1, f_2 mass functions: relate observables $v_1\sin i$, $v_2\sin i$, P to quantities of interest M_1, M_2, $\sin i$
- measurement of f_1 and f_2 (for double-lined spectroscopic binaries only) determines $M_1\sin^3 i$, $M_2\sin^3 i$
  - ▷ if i is known (e.g. for visual binaries or eclipsing binaries) $\rightarrow M_1, M_2$
  - ▷ for $M_1 \ll M_2 \quad \rightarrow \quad f_1(M_2) \simeq M_2\sin^3 i$ (measuring $v_1\sin i$ for star 1 constrains M_2)
- for eclipsing binaries one can also determine the radii of both stars (main source of accurate masses and radii of stars [and luminosities if distances are known])

The Roche Potential





THE ROCHE POTENTIAL

- **restricted three-body problem:** determine the motion of a test particle in the field of two masses M_1 and M_2 in a circular orbit about each other
- **equation of motion of the particle in a frame rotating with the binary** $\Omega = 2\pi/P$:

$$\frac{d^2\vec{r}}{dt^2} = -\vec{\nabla}\,U_{eff} - \underbrace{2\vec{\Omega}\times\vec{v}}_{\text{Coriolis force}}\ ,$$

where the effective potential U_{eff} is given by

$$U_{eff} = -\frac{GM_1}{|\vec{r}-\vec{r}_1|} - \frac{GM_2}{|\vec{r}-\vec{r_2}|} - \underbrace{\frac{1}{2}\Omega^2\,(x^2+y^2)}_{\text{centrifugal term}}$$

- **Lagrangian points:** five stationary points of the Roche potential U_{eff} (i.e. where effective gravity $\vec{\nabla}U_{eff} = 0$)
  - ▷ 3 saddle points: L_1, L_2, L_3
- **Roche lobe:** equipotential surface passing through the inner Lagrangian point L_1 (‘connects’ the gravitational fields of the two stars)
- **approximate formula** for the effective Roche-lobe radius (of star 2):

$$R_L = \frac{0.49}{0.6 + q^{2/3}\ln\left(1+q^{-1/3}\right)}\,A,$$

where $q = M_1/M_2$ is the mass ratio, A orbital separation.

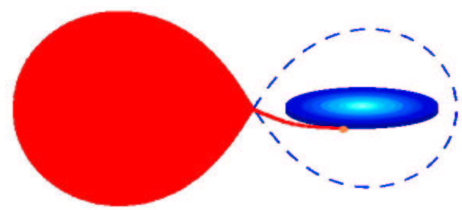
Classification of close binaries

- Detached binaries:
  - ▷ both stars underfill their Roche lobes, i.e. the photospheres of both stars lie beneath their respective Roche lobes
  - ▷ gravitational interactions only (e.g. tidal interaction, see Earth-Moon system)
- Semidetached binaries:
  - ▷ one star fills its Roche lobe
  - ▷ the Roche-lobe filling component transfers matter to the detached component
  - ▷ mass-transferring binaries
- Contact binaries:
  - ▷ both stars fill or overfill their Roche lobes
  - ▷ formation of a common photosphere surrounding both components
  - ▷ W Ursae Majoris stars

BINARY MASS TRANSFER

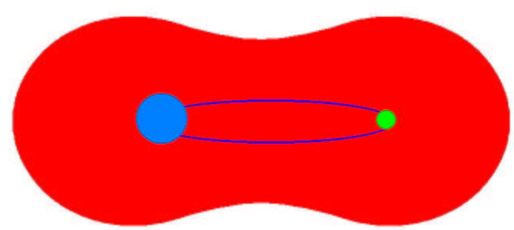
- 30 - 50 % of all stars experience mass transfer by Roche-lobe overflow during their lifetimes (generally in late evolutionary phases)

a) (quasi-)conservative mass transfer

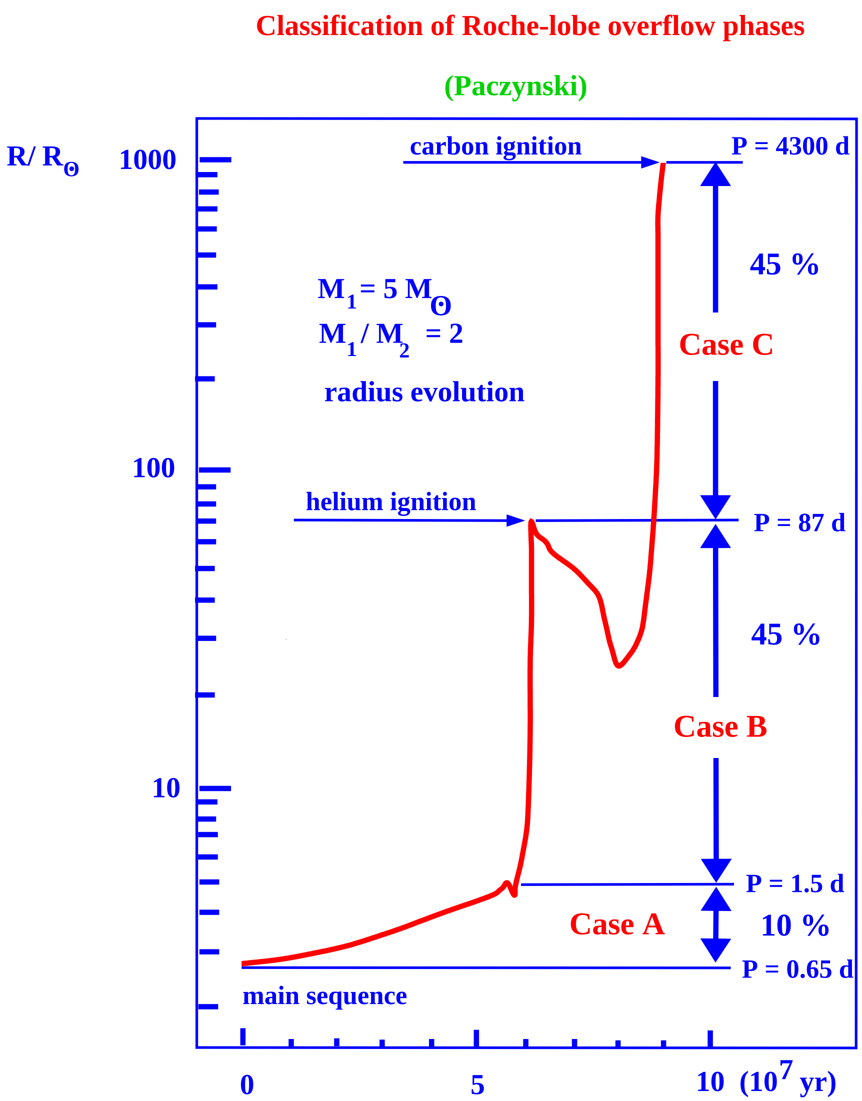


- ▷ mass loss + mass accretion
- ▷ the mass loser tends to lose most of its envelope → formation of helium stars
- ▷ the accretor tends to be rejuvenated (i.e. behaves like a more massive star with the evolutionary clock reset)
- ▷ orbit generally widens

b) dynamical mass transfer → common-envelope and spiral-in phase (mass loser is usually a red giant)



- ▷ accreting component also fills its Roche lobe
- ▷ mass donor (primary) engulfs secondary
- ▷ spiral-in of the core of the primary and the secondary immersed in a common envelope
- ▷ if envelope ejected → very close binary (compact core + secondary)
- ▷ otherwise: complete merger of the binary components → formation of a single, rapidly rotating star



INTERACTING BINARIES (SELECTION)

Algols and the Algol paradox

- Algol is an eclipsing binary with orbital period 69 hr, consisting of a B8 dwarf ($M = 3.7\,M_\odot$) and a K0 subgiant ($M = 0.8\,M_\odot$)
- the eclipse of the B0 star is very deep → B8 star more luminous than the more evolved K0 subgiant
- the less massive star is more evolved
- inconsistent with stellar evolution → Algol paradox
- explanation:
  - ▷ the K star was initially the more massive star and evolved more rapidly
  - ▷ mass transfer changed the mass ratio
  - ▷ because of the added mass the B stars becomes the more luminous component

Formation of Low-Mass X-Ray Binaries (I)

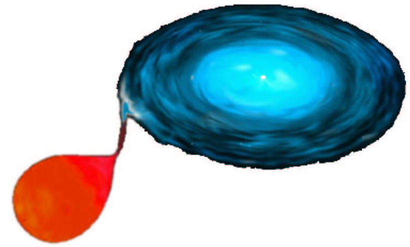
wide binary with large mass ratio

dynamical mass transfer

common-envelope and spiral-in phase

ejection of common envelope and subsequent supernova

Interacting binaries containing compact objects



- short orbital periods (11 min to typically 10s of days) → requires common-envelope and spiral-in phase

Cataclysmic Variables (CV)

- main-sequence star (usually) transferring mass to a white dwarf through an accretion disk
- nova outbursts: thermonuclear explosions on the surface of the white dwarf
- dwarf novae: accretion-disk instabilities
- orbit shrinks because of angular-momentum loss due to gravitational radiation and magnetic braking

X-Ray Binaries

- compact component: neutron star, black hole
- mass donor can be of low, intermediate or high mass
- very luminous X-ray sources (accretion luminosity)
- neutron-star systems: luminosity distribution peaked near the Eddington limit, (accretion luminosity for which radiation pressure balances gravity)
  $$L_{\rm Edd} = \frac{4\pi c G M}{\kappa} \simeq 2\times 10^{31}\,{\rm W}\left(\frac{M}{1.4\,M_\odot}\right)$$
- accretion of mass and angular momentum → spin-up of neutron star → formation of millisecond pulsar
- soft X-ray transients: best black-hole candidates (if $M_X >$ max. neutron-star mass $\sim 2-3\,M_\odot$ → likely black hole [but no proof of event horizon yet])