NEUTRON STARS

1934: Baade and Zwicky proposed that *supernovae* represented the transition of normal stars to neutron stars

1939: Oppenheimer and Volkoff published the first *theoretical model*

1967: *discovery of pulsars* by Bell and Hewish

Maximum mass of a neutron star:

- Neutrons are *fermions: degenerate neutrons* are unable to support a neutron star with a mass above a certain value (c.f. *Chandrasekhar mass* limit for white dwarfs).
- Important differences from white dwarf case:
  - (i) *interactions between neutrons* are very important at high densities
  - ii) very *strong gravitational fields* (i.e. use *General Relativity*)

  N.B. There is a maximum mass but the calculation is very difficult; the result is very important for black-hole searches in our Galaxy
- Crude estimate– ignore interactions and General Relativity – apply same theory as for white dwarfs: $M_{max} \simeq 6\,M_\odot$
- *interactions between neutrons* increase the theoretical maximum mass
  - ▷ The interaction is attractive at distances $\sim 1.4\,\mathrm{fm}$ but repulsive at shorter distances

  → matter harder to compress at high densities
  - ▷ *but, at high densities,* degenerate *neutrons* are *energetic* enough to produce *new particles (hyperons and pions)* → the pressure is reduced because the new particles only produce a small pressure
- *Effect of gravity* (gravitational binding energy of neutron star is comparable with rest mass):
- Gravity is strengthened at very high densities and pressures. Consider the pressure gradient:

$$\textit{Newton:}\ \frac{dP}{dr} = -\frac{Gm\rho}{r^2}$$

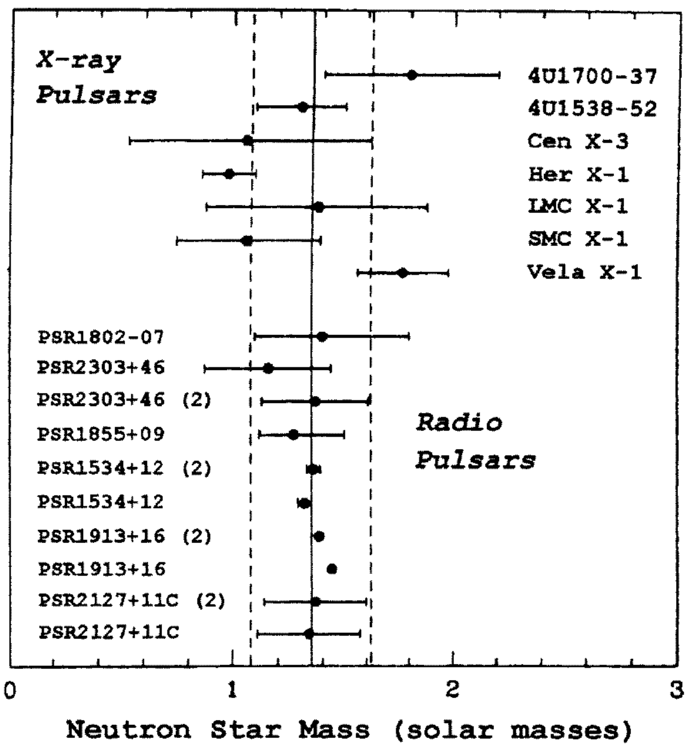
$$\textit{Einstein:}\ \frac{dP}{dr} = -\frac{Gm\rho}{r^2} \times \frac{(1 + P/(\rho c^2))(1 + 4\pi r^3 P/(mc^2))}{1 - 2Gm/(rc^2)}$$

- Pressure P occurs on RHS. *Increase of pressure,* needed to oppose gravitational collapse, leads to *strengthening of gravitational field*
- need an equation of state $P(\rho)$ that takes account of $n - n$ interactions
- Oppenheimer and Volkoff (1939) calculated M_{max} for a star composed of non-interacting neutrons.
  Result: $M_{max} = 0.7\,M_\odot$
- *Enhanced gravity leads to collapse at finite density* when *neutrons* are just becoming relativistic – *not ultrarelativistic*
- Various calculations, using different compressibilities for neutron star matter, predict

$$\boxed{M_{max} \in [1.5, 3]\,M_\odot} \qquad R_{NS} \simeq 7 - 15\,\mathrm{km}$$

- *Observed neutron star masses* (from analysis of binary systems) are mostly around $1.5\,M_\odot$

- Modern calculations suggest that M_{max} is probably less than $3M_\odot$ and definitely less than $5M_\odot$. See Phillips, "The Physics of Stars", for an example calculation: incompressible star of constant density



PULSARS

Expected properties of neutron stars:

(a) *Rotation period* (c.f. white dwarfs, e.g. 40 Eri B, $P_{WD} = 1350\,s$)

From simple theory : $\dfrac{R_{WD}}{R_{NS}} \simeq \dfrac{m_n}{2^{5/3} m_e} \simeq 600$

conservation of angular momentum: with $M_{WD} \sim M_{NS}$ and $I = (2/5)MR^2$ (uniform sphere)

$$I_i \omega_i = I_f \omega_f$$

$$\omega_f = \omega_i (R_i/R_f)^2$$

$$P_{NS} \simeq 3 \times 10^{-6}\, P_{WD} \simeq 4\,\mathrm{ms}$$

$\rightarrow$ neutron stars *rotate rapidly* when they form

- ▷ but angular momentum is probably lost in the supernova explosion
- ▷ rotation is likely to slow down rapidly

(b) *magnetic field*

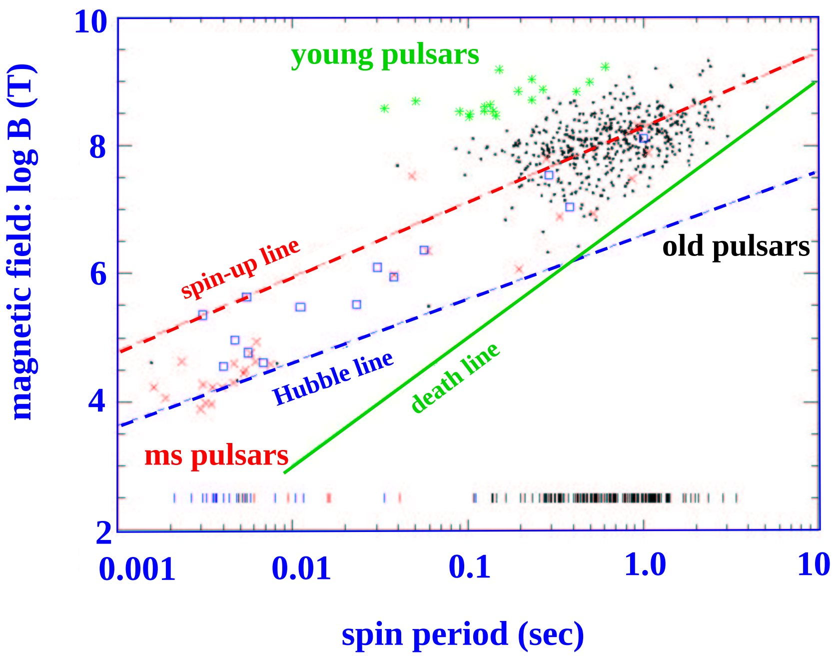
- ▷ *Flux conservation* requires $\int B\, dS = $ constant

$$B_i\, 4\pi R_i^2 = B_f\, 4\pi R_f^2$$

- ▷ Take largest observed white dwarf field, $B_{WD} \simeq 5 \times 10^4$ T

$$B_{NS} \simeq B_{WD}(R_{WD}/R_{NS})^2 \simeq 10^{10}\,\mathrm{T} \qquad \text{(upper limit)}$$

Radio Pulsars: the P-B Diagram



(c) *Luminosity*

- ▷ neutron star forms at $T \sim 10^{11}$ K but T drops to $\sim 10^{9}$ K within 1 day
- ▷ main *cooling process: neutrino emission* (first $\sim 10^{3}$ yr), then *radiation*
- ▷ after a few hundred years, $T_{\rm internal} \sim 10^{8}$ K, $T_{\rm surface} \sim$ a few $\times 10^{6}$ K.
- ▷ star cools at constant R for $\sim 10^{4}$ yr with $T_{\rm surface} \sim 10^{6}$ K

$$L \sim 4\pi R^2 \sigma T_s^4 \sim 10^{26}\,\mathrm{W}\ (\text{mostly X}-\text{rays},\ \lambda_{\max} \sim 3\ \text{nm})$$

Discovery of neutron stars

- The *first pulsar* was discovered by Bell and Hewish at Cambridge in 1967
  - ▷ A radio interferometer (2048 dipole antennae) had been set up to study the scintillation which was observed when radio waves from distant point sources passed through the solar wind. Bell discovered a signal, regularly spaced radio pulses 1.337 sec apart, coming from the same point in the sky every night.
- Today, about *1500 pulsars* are known
  - ▷ Most have periods between *0.25 s and 2 s* (average 0.8 s)
  - ▷ *Extremely well defined pulse periods* that challenge the best atomic clocks
  - ▷ Periods increase very gradually
  - ▷ *spin-down timescale* for young pulsars $\sim P/\dot{P} \sim 10^{6} - 10^{7}$ yr

SUPERNOVA REMNANTS

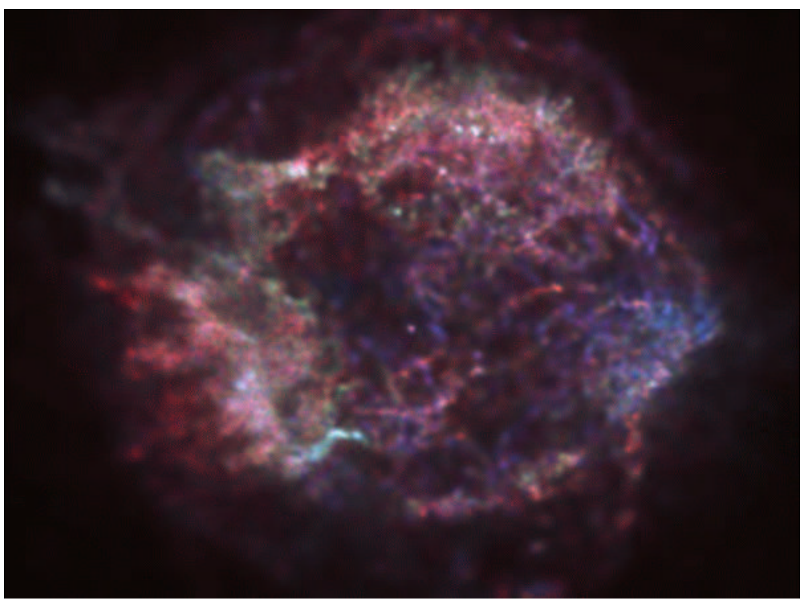
The Crab Nebula (plerionic/filled)

VLT

Chandra

(X-rays)

Cassiopeia A (shell-like)



The Crab Pulsar

- The *Crab nebula* is the *remnant* of a *supernova* explosion observed optically in 1054 AD.
- The *Crab pulsar* is at the centre of the nebula, emitting X-ray, optical and radio pulses with $\mathrm{P} = 0.033\,\mathrm{s}$.
- The Crab nebula is morphologically different from two other recent supernova remnants, *Cas A* and *Tycho* (both $\sim 400\,\mathrm{yr}$ old) which are *shell-like.*
- The present *rate of expansion* of the nebula can be measured: uniform expansion extrapolates back to 90 years *after* the explosion, i.e. the expansion must be accelerating
- The observed *spectrum* is a *power law* from $\sim 10^{14}\,\mathrm{Hz}$ (IR) to $\sim 1\,\mathrm{MeV}$ (hard X-rays); also, in the extended nebulosity, the *X-rays* are 10-20 % *polarised* $\rightarrow$ signature of *synchrotron radiation* (relativistic electrons spiralling around magnetic field lines with $\mathrm{B} \sim 10^{-7}\,\mathrm{T}$).
- Synchrotron radiation today requires *(i) replenishment of magnetic field* and *(i) continuous injection of energetic electrons.*
- *Total power* needed $\sim 5 \times 10^{31}\,\mathrm{W}$
- *Energy source* is a *rotating neutron star* $(\mathrm{M} \simeq 1.4\,\mathrm{M}_\odot,\ \mathrm{R} \simeq 10\,\mathrm{km})$

$$\mathrm{U} = (1/2)\,\mathrm{I}\omega^2 = 2\pi^2\,\mathrm{I}/\mathrm{P}^2$$

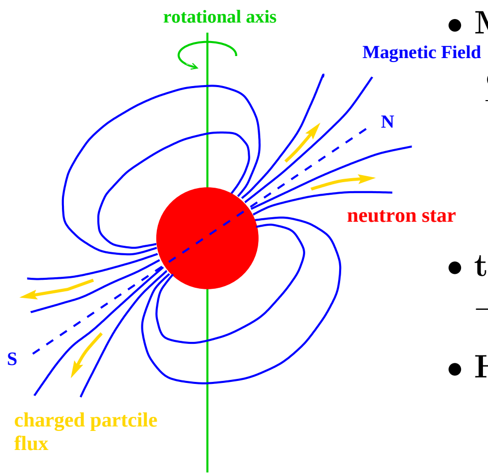
$$\frac{\mathrm{dU}}{\mathrm{dt}} = -\frac{4\pi^2\mathrm{I}\dot{\mathrm{P}}}{\mathrm{P}^3}$$

Taking $\mathrm{I} = (2/5)\,\mathrm{MR}^2 \sim 1 \times 10^{38}\,\mathrm{kg\,m}^2;\ \ \mathrm{P} = 0.033\,\mathrm{s};$

$$\dot{\mathrm{P}} = 4.2 \times 10^{-13} \quad \rightarrow \quad \mathrm{dU/dt} \simeq 5 \times 10^{31}\,\mathrm{W}$$

A simple pulsar model

- A pulsar can be modelled as a *rapidly rotating neutron star* with a strong *dipole magnetic field* inclined to the rotation axis at angle θ.
- Pulsar emission is *beamed* (like a lighthouse beam) $\rightarrow$ observer has to be in the beam to see *pulsed emission.*



- Magnetic dipole radiation:

$$\frac{dU_{mag}}{dt} = -\frac{2}{3c^3}\left(\frac{\mu_0}{4\pi}\right) m^2\omega^4 \sin^2\theta$$
$$= -\frac{32\pi^4}{3c^3}\left(\frac{\mu_0}{4\pi}\right)\frac{m^2\sin^2\theta}{P^4}$$

- taking $dU/dt = -5\times10^{31}$ W $\rightarrow \quad m\sin\theta \simeq 4\times10^{27}\,\mathrm{A\,m^2}$.
- Hence, *surface magnetic field*

$$B \simeq \frac{\mu_0 m}{4\pi R^3} \simeq 10^8\,\mathrm{T}.$$

- Further argument for magnetic dipole radiation:

$$-\frac{dU_{rot}}{dt} = -I\omega\frac{d\omega}{dt} \propto \omega^4$$
$$\rightarrow \quad \frac{d\omega}{dt} = -C\omega^3.$$
$$\text{Integrate} \quad t = \frac{1}{2C}\left(\frac{1}{\omega^2} - \frac{1}{\omega_0^2}\right)$$

So $t < \dfrac{1}{2C\omega^2}$ with $C = 3.5\times10^{-16}$ s and $\omega = 190\,\mathrm{s}^{-1}$

$\rightarrow t < 1250$ yr (comparable to the known age $\simeq 950$ yr)

N.B. The physics underlying pulsar emission mechanisms is very complicated and not well understood

Pulsar dispersion measure

- Consider an electromagnetic wave of the form $E = E_0\cos(kx \pm wt)$ propagating through an *ionised medium* where the number density of electrons is n_e.
- The *dispersion relation* is

$$\omega^2 = k^2c^2 + \omega_p^2, \quad \omega_p = (n_e e^2/(m\epsilon_0))^{1/2} \text{ (plasma frequency)}.$$

- If $\omega < \omega_p \rightarrow$ *no wave propagation* (e.g. $\sim$ few MHz cut-off to radio waves through the ionosphere). If $\omega > \omega_p$, *propagation* occurs with *group velocity* v_g given by:

$$v_g = \frac{d\omega}{dk} = c\left(1 + \frac{w_p^2}{c^2k^2}\right)^{-1/2} = c\left(1 + \frac{\omega_p^2}{\omega^2 - \omega_p^2}\right)^{-1/2}$$

- i.e. *frequency dependent:* longer wavelength has lower velocity.
- A pulse of radiation travels a distance l in time $t = l/v_g$.
- Frequency dependence of arrival time is given by:

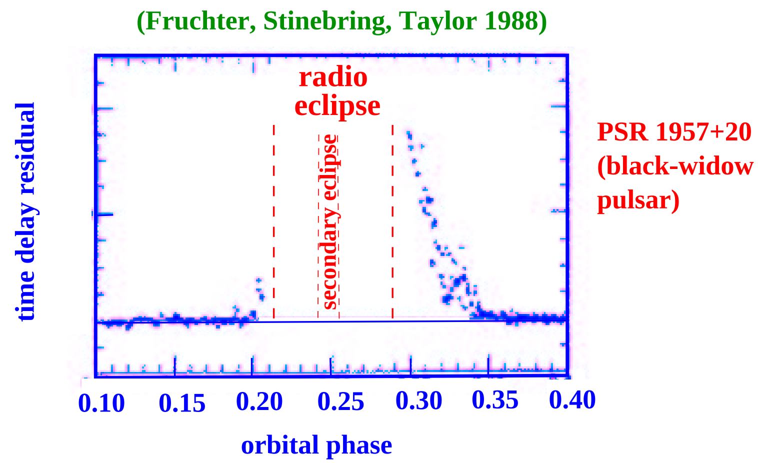
$$\frac{\Delta t}{\Delta\omega} = -\frac{l}{v_g^2}\frac{\Delta v_g}{\Delta\omega} \simeq -\frac{l}{c^2}\frac{\Delta v_g}{\Delta\omega} \quad \text{since } v_g \simeq c \text{ for } \omega >> \omega_p.$$

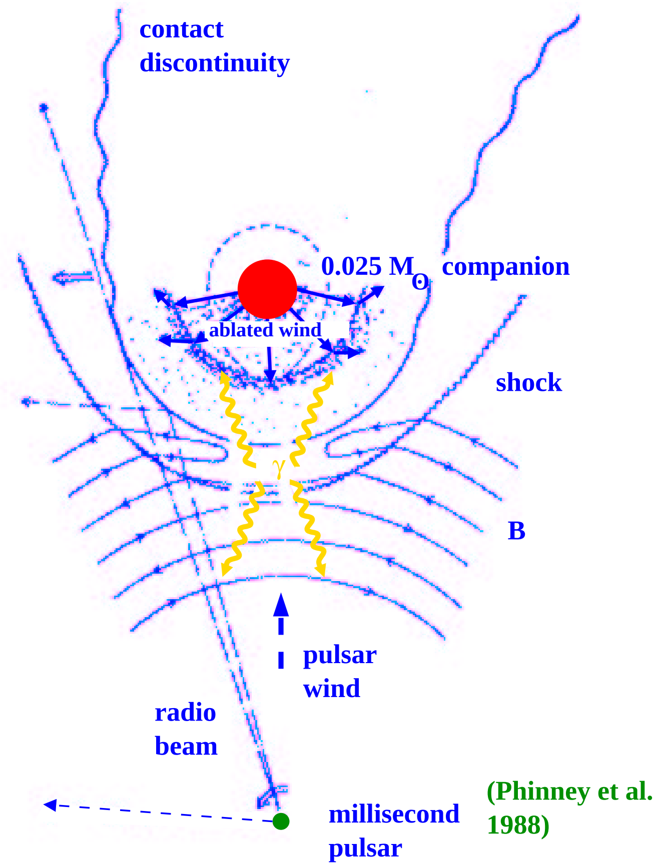
$$\text{But} \quad \frac{\Delta v_g}{\Delta\omega} = \frac{\omega\omega_p^2 c}{(\omega^2-\omega_p^2)^2}\left(1 + \frac{\omega_p^2}{\omega^2-\omega_p^2}\right)^{-3/2} \simeq \frac{\omega_p^2 c}{\omega^3}.$$

$$\text{Therefore} \quad \frac{\Delta\nu}{\Delta t} = \frac{1}{2\pi}\frac{\Delta\omega}{\Delta t} = -\frac{4\pi^2 mc\ \epsilon_0}{e^2}\frac{\nu^3}{n_e l}$$

$$\text{Strictly} \quad \frac{\Delta\nu}{\Delta t} = -\frac{4\pi^2 mc\ \epsilon_0}{e^2}\frac{\nu^3}{\int n_e dl} \quad \text{since } n_e \text{ varies with } l.$$

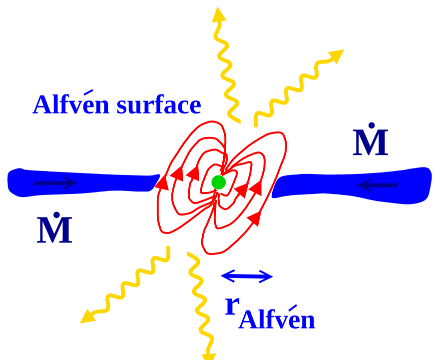
- $\int n_e dl$ is known as the ***DISPERSION MEASURE.*** It is a useful distance indicator if n_e is uniform (typical value for the Galactic plane: $1-3\times10^5\,\mathrm{m^{-3}}$)





MILLISECOND (RECYCLED) PULSARS

- a group of ~ 100 radio pulsars with very *short spin periods* (shortest: 1.6 ms) and *relatively weak magnetic fields* ($B \lesssim 10^6\,\mathrm{T}$)
- they are preferentially members of *binary systems,*
- they have *spin-down timescales* comparable or longer than the Hubble time (age of the Universe)
- *standard model*
  - ▷ these pulsars are neutron stars in *binary systems* that *spin-down* first, *lose their strong magnetic field* (due to accretion?)
  - ▷ and are *spun-up* by *accretion* from a companion



  - ▷ *magnetospheric accretion:* magnetic field becomes dominant when magnetic pressure > ram pressure in flow → flow follows *magnetic field lines* (below r_A)
  - ▷ spin-up due to accretion of *angular momentum*
- *equilibrium spin period:* $v_{rot}(r_A) = v_{Kepler}(r_A)$

$$\rightarrow \boxed{P_{eq} = \simeq 2\,\mathrm{ms}\,\left(B/10^5\,\mathrm{T}\right)^{6/7}\left(\dot{M}/\dot{M}_{Edd}\right)^{-3/7}}$$

- a significant fraction of millisecond pulsars are *single*

→ *pulsar radiation* has *evaporated the companion*

  - ▷ example: *PSR 1957+20 (the black-widow pulsar):* companion mass: only $0.025\,M_\odot$
  - ▷ direct evidence for an *evaporative wind* from the *radio eclipse* (much larger than the secondary)

→ *comet-like* evaporative tail

SCHWARZSCHILD BLACK HOLES

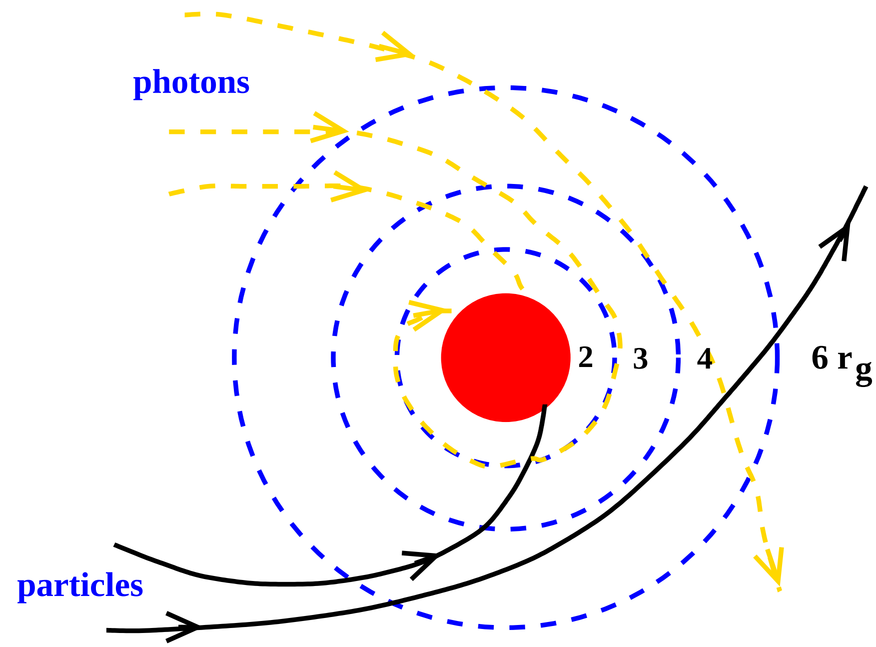
- *event horizon:* (after Michell 1784)
  - ▷ the *escape velocity* for a particle of mass m from an object of mass M and radius R is $v_{esc} = \sqrt{\frac{2GM}{R}}$ ($11\,km\,s^{-1}$ for Earth, $600\,km\,s^{-1}$ for Sun)
  - ▷ assume *photons* have *mass:* $m \propto E$ (Newton's corpuscular theory of light)
  - ▷ photons travel with the *speed of light c*

  → photons cannot escape, if $v_{esc} > c$

  → $\boxed{R < R_s \equiv \frac{2GM}{c^2}}$ *(Schwarzschild radius)*

  - ▷ $R_s = 3\,km\,(M/M_\odot)$
    Note: for neutron stars $R_s \simeq 5\,km$; only a factor of 2 smaller than R_{NS} → GR important
- *particle orbits* near a black hole
  - ▷ the *most tightly bound circular orbit* has a radius $R_{min} = 3\,R_s = (6GM)/c^2$ (defines inner edge of accretion disk)
  - ▷ for a black hole *accreting* from a thin disk, the *efficiency* of energy generation is (usually) determined by the binding energy of the inner most stable orbit ($\sim 6\,\%$ for a Schwarzschild black hole)
- *no hair theorem:* the structure of a black hole is completely determined by its mass M, angular momentum L and electric charge Q

Orbits near Schwarzschild Black Holes



gravitational radius: $r_g = \frac{GM}{c^2}$

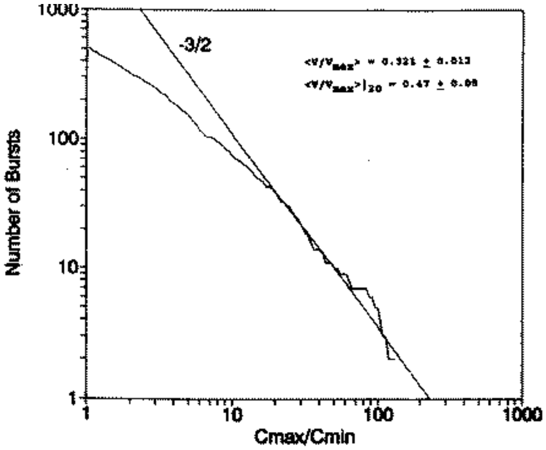


Figure 4. Intensity distribution for BATSE bursts. The measure of intensity is the maximum count rate divided by the threshold count rate.

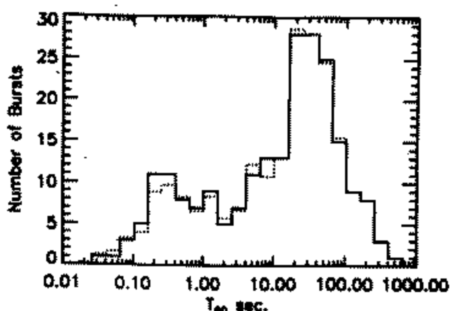


Figure 7. The duration distribution for 222 BATSE bursts, as measured by T_{90}. The solid histogram represents the raw data; the dashed histogram represents the data convolved with measurement errors.

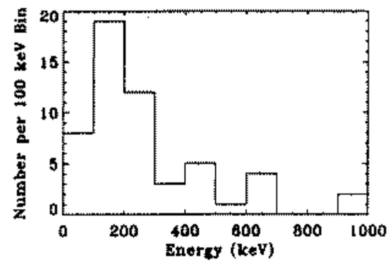


Figure 8. The distribution of the energy of the peak emission per ı logarithmic energy interval.

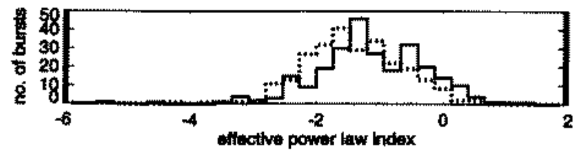
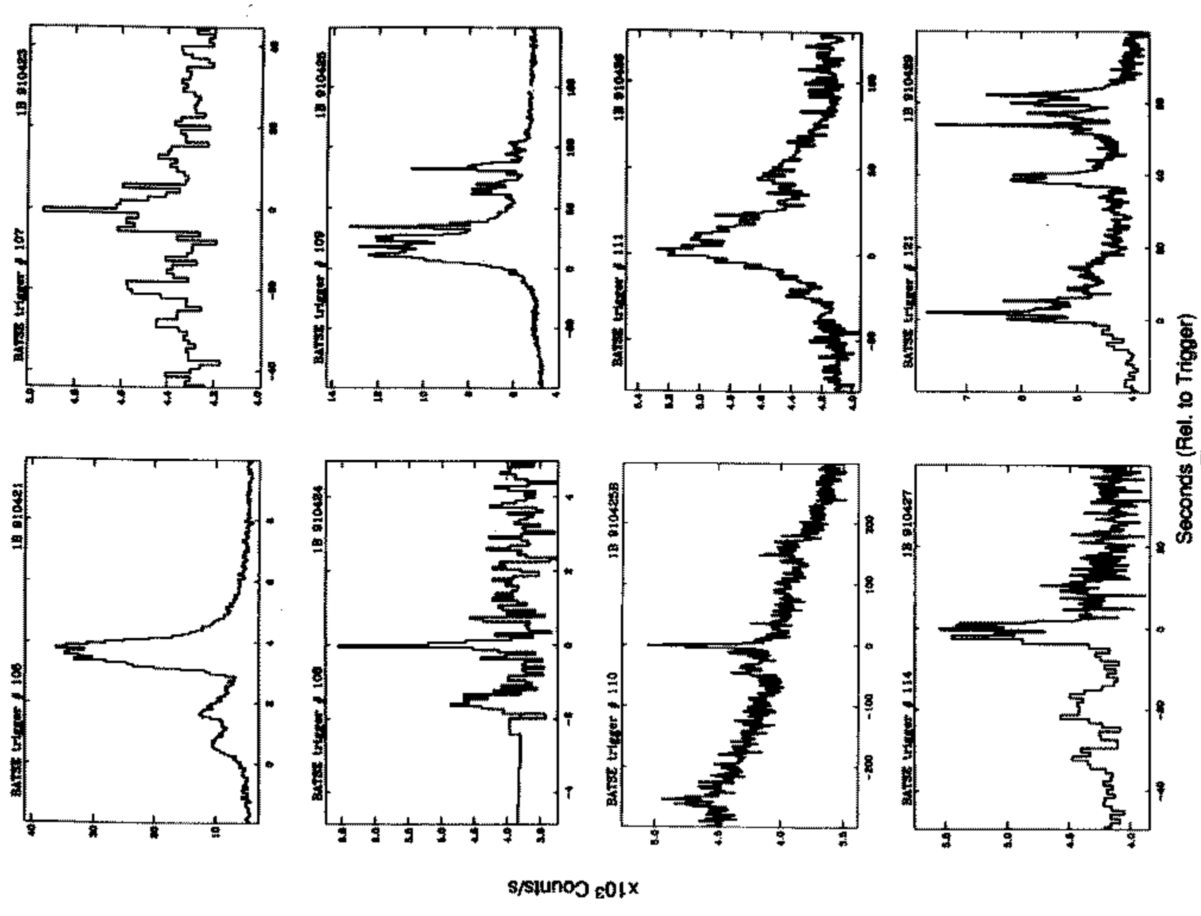
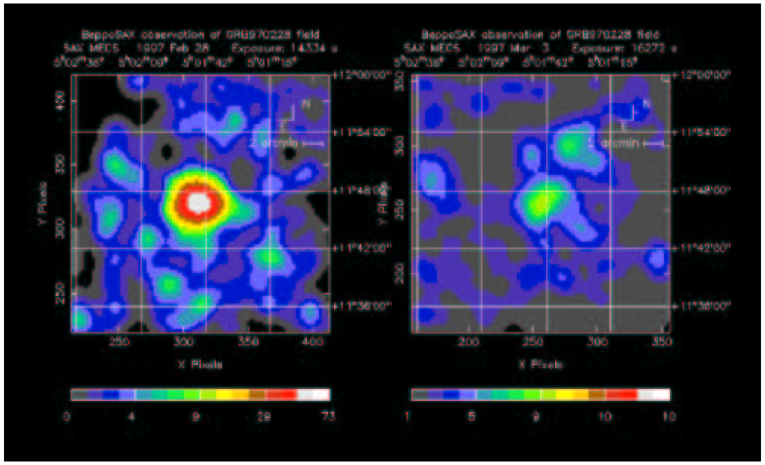


Figure 9. The distribution of burst spectral indices in the 50 to 300 keV ergy band. The solid line represents the distribution for the peak rate spectr and the dotted line represents the distribution for the total fluence spectrun

Meegan et al. (1994)



Gamma-Ray Bursts



Beppo-Sax X-ray detection

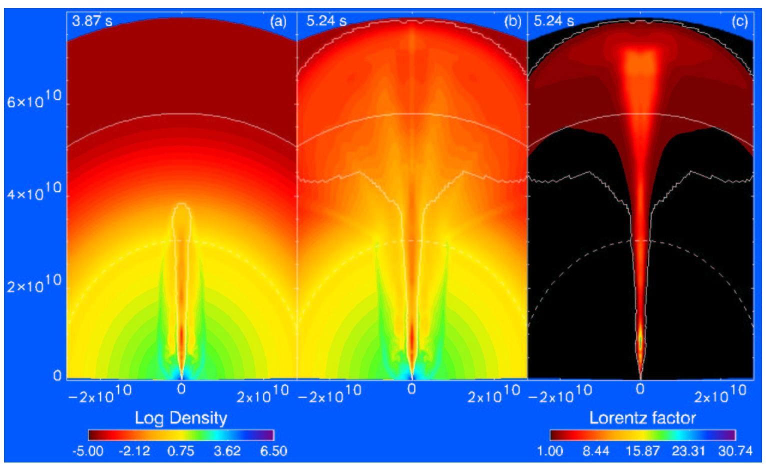


FIG. 1.— Contour maps of the logarithm of the rest–mass density after 3.87 s and 5.24 s (left two panels), and of the Lorentz factor (right panel) after 5.24 s. X and Y axis measure distance in centimeters. Dashed and solid arcs mark the stellar surface and the outer edge of the exponential atmosphere, respectively. The other solid line encloses matter whose radial velocity $> 0.3c$, and whose specific internal energy density $> 5 \times 10^{19}\,\mathrm{erg\,g^{-1}}$.

Collapsar Model for GRBs

GAMMA-RAY BURSTS

- *discovered* by U.S. *spy satellites* (1967; secret till 1973)
- have remained one of the biggest mysteries in astronomy until 1998 (*isotropic* sky distribution; location: solar system, Galactic halo, distant Universe?)
- *discovery of afterglows* in 1998 (X-ray, optical, etc.) with *redshifted absorption lines* has resolved the puzzle of the location of GRBs $\rightarrow$ *GRBs are the some of the most energetic events in the Universe*
- *duration:* 10^{-3} to $10^{3}\,$s (large variety of burst shapes)
- *bimodal* distribution of *durations:* 0.3 s (short-hard), 20 s (long-soft) (different classes/viewing angles?)
- GRBs are *no* standard candles! (isotropic) energies range from 5×10^{44} to $2 \times 10^{47}\,$J
- *highly relativistic outflows (fireballs):* ($\gamma \gtrsim 100$,) possibly highly *collimated/beamed*
- GRBs are produced far from the source ($10^{11} - 10^{12}\,$m): interaction of outflow with surrounding medium (external or internal shocks) $\rightarrow$ *fireball model*
- *relativistic energy* $\sim 10^{46} - 10^{47}\,\mathrm{J}\,\epsilon^{-1}\,\mathrm{f}_\Omega$ (ϵ: efficiency, f_Ω: beaming factor; typical energy $10^{45}\,$J?)
- *event rate/Galaxy:* $\sim 10^{-7}\,\mathrm{yr}^{-1}\,(3 \times 10^{45}\,\mathrm{J}/\epsilon\,\mathrm{E})$

Popular Models

- *merging compact objects* (two NS's, BH+NS) $\rightarrow$ can explain short-duration bursts
- *hypernova* (very energetic supernova associated with formation of a rapidly rotating *black hole*) $\rightarrow$ *jet penetrates stellar envelope* $\rightarrow$ GRB along jet axis (large beaming)