NUCLEAR REACTIONS

- *Binding energy* of nucleus with Z protons and N neutrons is:

$$Q(Z,N) = [ZM_p + NM_n - M(Z,N)]c^2.$$

- *Energy release:*

$4\,H \rightarrow {}^4He$ $\quad 6.3 \times 10^{14}\,J\,kg^{-1} = 0.007\,c^2 \;\; (\varepsilon = 0.007)$

$56\,H \rightarrow {}^{56}Fe$ $\quad 7.6 \times 10^{14}\,J\,kg^{-1} = 0.0084\,c^2 \;\; (\varepsilon = 0.0084)$

- *H burning* already releases most of the available nuclear binding energy.

Nuclear reaction rates:

$$\boxed{1} + \boxed{2} \rightarrow \boxed{1,2} + \text{Energy}$$

Reaction rate is proportional to:

1. *number density* n_1 of particles 1
2. *number density* n_2 of particles 2
3. *frequency of collisions* depends on *relative velocity* v of colliding particles $r_{1+2} = n_1 n_2 \langle \sigma(v) v \rangle$
4. *probability* $P_p(v)$ *for penetrating Coulomb barrier* (Gamow factor)

$$P_p(v) \propto \exp[-(4\pi^2 Z_1 Z_2 e^2/hv)]$$

Nuclear Binding Energy

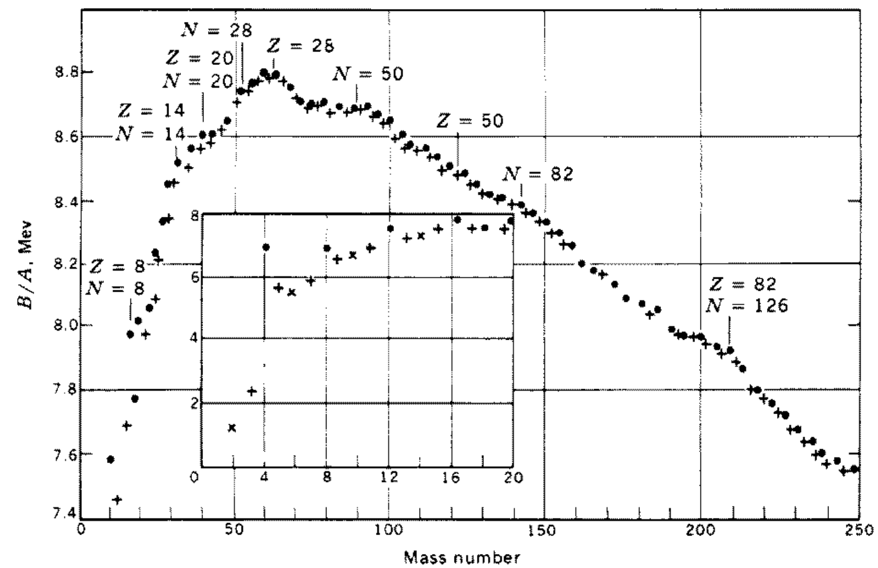
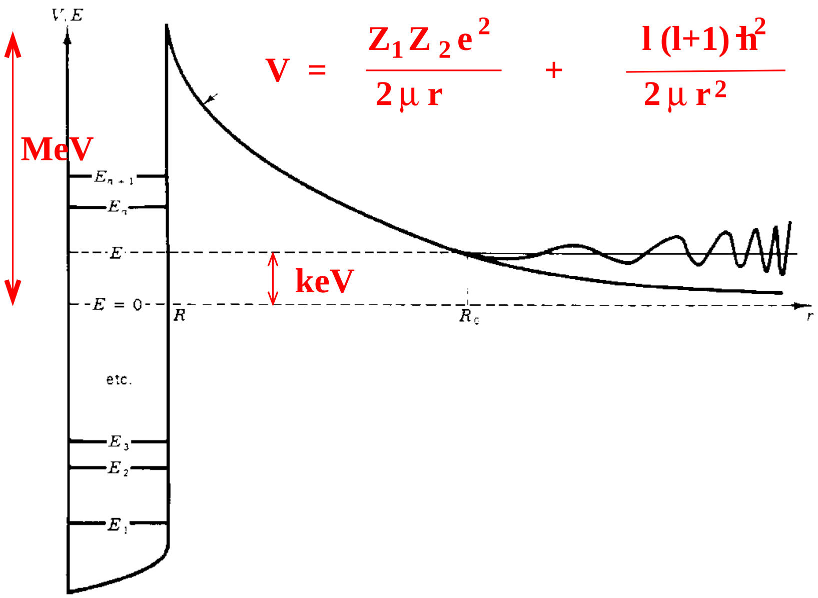
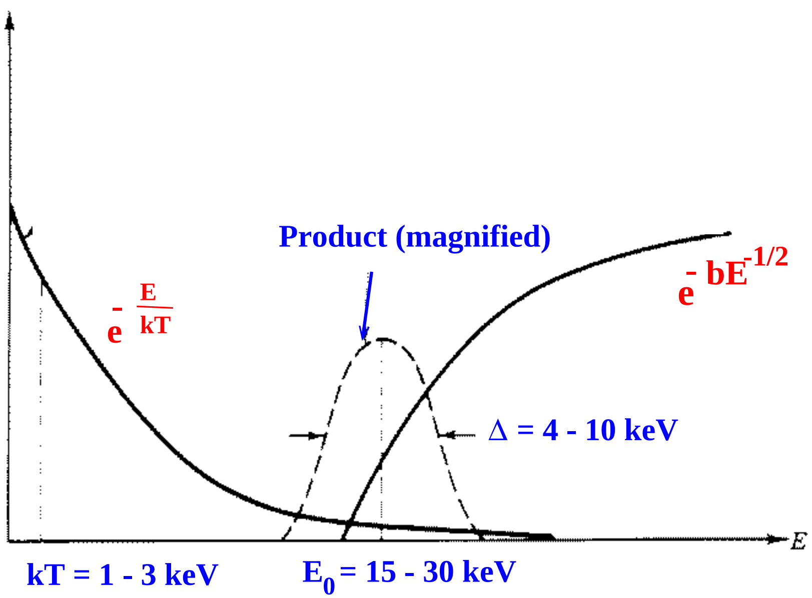


Fig. 7-1 The binding energy per nucleon of the most stable isobar of atomic weight A. The solid circles represent nuclei having an even number of protons and an even number of neutrons, whereas the crosses represent odd-A nuclei. (*M. A. Preston, "Physics of the Nucleus," Addison-Wesley Publishing Company, Inc., Reading, Mass.*, 1962.)

Coulomb Barrier



Reaction Rate



5. define *cross-section factor* S(E): $\sigma = [S(E)/E]\, P_p(E)$

   ▷ depends on the details of the nuclear interactions

   ▷ insensitive to particle energy or velocity (non-resonant case)

   ▷ S(E) is typically a slowly varying function

   ▷ evaluation requires *laboratory* data except in p-p case (theoretical cross section)

6. *particle velocity distribution* (Maxwellian).

$$D(T, v) \propto (v^2/T^{3/2})\, \exp[-(m_H A' v^2/2kT)]$$

where $A' = A_1 A_2 (A_1 + A_2)^{-1}$ is the reduced mass.

The *overall reaction rate* per unit volume is:

$$\boxed{R_{12} = \int_0^\infty n_1 n_2\, v [S(E)/E\, P_p(v)]\, D(T, v)\, dv}$$

(for details of evaluating the integral see Clayton p303.)

- Setting $n_1 = (\rho/m_1)\, X_1$, $n_2 = (\rho/m_2)\, X_2$ and

$$\tau = 3E_0/kT = 3\{2\pi^4 e^4 m_H Z_1^2 Z_2^2 A'/(h^2 kT)\}^{1/3}$$

$$R_{12} = B\rho^2\, (X_1 X_2/A_1 A_2)\tau^2\, \exp(-\tau)/(A' Z_1 Z_2)$$

where B is a constant depending on the details of the nuclear interaction (from the S(E) factor)

▷ Low temperature: τ is large; exponential term leads to small reaction rate.

▷ Increasing temperature: reaction rate increases rapidly through exponential term.

▷ High temperature: τ^2 starts to dominate and *rate falls again.*
(In practice, we are mainly concerned with temperatures at which there is a rising trend in the reaction rate.)

(1) Reaction rate decreases as Z_1 and Z_2 increase. Hence, ***at low temperatures,*** reactions involving ***low Z nuclei*** are favoured.

(2) Reaction rates need only be significant over times $\sim 10^9$ years.

HYDROGEN BURNING

PPI chain:

$$1)\quad {}^1H + {}^1H \rightarrow {}^2D + e^+ + \nu \qquad +1.44\,\mathrm{MeV}$$

$$2)\quad {}^2D + {}^1H \rightarrow {}^3He + \gamma \qquad +5.49\,\mathrm{MeV}$$

$$3)\quad {}^3He + {}^3He \rightarrow {}^4He + {}^1H + {}^1H \qquad +12.85\,\mathrm{MeV}$$

- for each conversion of ${}^4H \rightarrow {}^4He$, reactions (1) and (2) have to occur twice, reaction (3) once
- the ***neutrino*** in (1) carries away ***0.26 MeV*** leaving ***26.2 MeV*** to contribute to the luminosity
- reaction (1) is a ***weak interaction*** $\rightarrow$ ***bottleneck*** of the reaction chain
- ***Typical reaction times*** for $T = 3 \times 10^7\,\mathrm{K}$ are

  (1) $14 \times 10^9\,\mathrm{yr}$

  (2) 6 s

  (3) $10^6\,\mathrm{yr}$

  ▷ (these depend also on ρ, X_1 and X_2).

  ▷ ***Deuterium*** is burned up very rapidly.

If ^{4}***He is sufficiently abundant,*** two further chains can occur:

PPII chain:

$$3a)\quad {}^3He + {}^4He \rightarrow {}^7Be + \gamma \qquad +1.59\,\mathrm{MeV}$$

$$4a)\quad {}^7Be + e^- \rightarrow {}^7Li + \nu \qquad +0.86\,\mathrm{MeV}$$

$$5a)\quad {}^7Li + {}^1H \rightarrow {}^4He + {}^4He \qquad +17.35\,\mathrm{MeV}$$

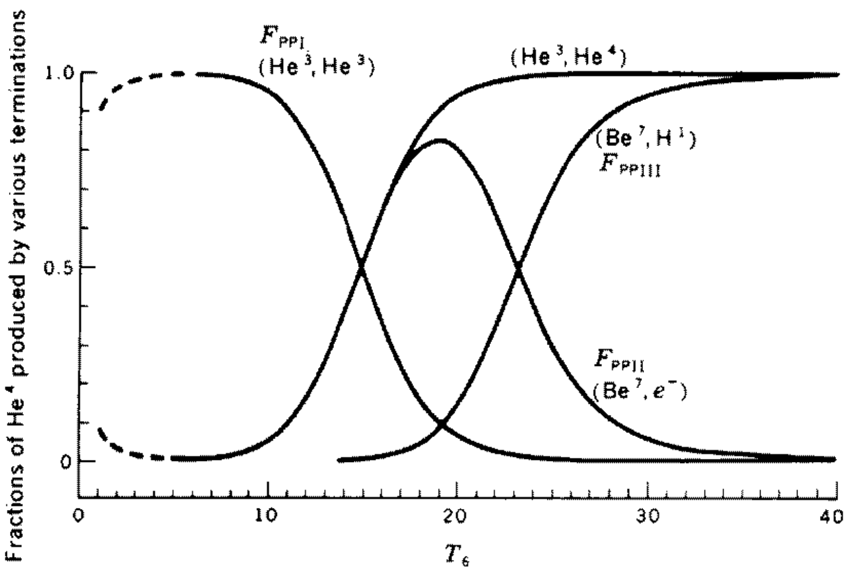
PPIII chain:

$$4b)\quad {}^7Be + {}^1H \rightarrow {}^8B + \gamma \qquad +0.14\,\mathrm{MeV}$$

$$5b)\quad {}^8B \rightarrow {}^8Be + e^+ + \nu$$

$$6b)\quad {}^8Be \rightarrow {}^4He + {}^4He \qquad +18.07\,\mathrm{MeV}$$

- In both ***PPII and PPIII,*** a ^{4}***He atom*** acts as a ***catalyst*** to the conversion of ${}^3He + {}^1H \rightarrow {}^4He + \nu$.
- E_{total} is the same in each case but the energy carried away by the neutrino is different.
- All three PP chains ***operate simultaneously*** in a H burning star containing significant ${}^4He$: details of the cycle depend on density, temperature and composition.



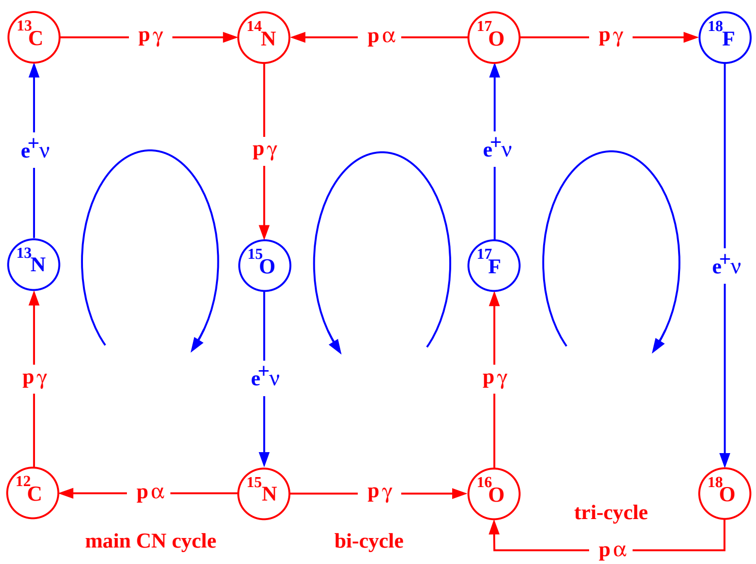
THE CNO CYCLE $(\mathrm{T} < 10^8\,\mathrm{K})$

- ***Carbon, nitrogen and oxygen*** **serve as** ***catalysts*** **for the** ***conversion of H to He***

$$
\begin{aligned}
{}^{12}\mathrm{C} + {}^{1}\mathrm{H} &\rightarrow {}^{13}\mathrm{N} + \gamma \\
{}^{13}\mathrm{N} &\rightarrow {}^{13}\mathrm{C} + \mathrm{e}^{+} + \nu \\
{}^{13}\mathrm{C} + {}^{1}\mathrm{H} &\rightarrow {}^{14}\mathrm{N} + \gamma \\
{}^{14}\mathrm{N} + {}^{1}\mathrm{H} &\rightarrow {}^{15}\mathrm{O} + \gamma \\
{}^{15}\mathrm{O} &\rightarrow {}^{15}\mathrm{N} + \mathrm{e}^{+} + \nu \\
{}^{15}\mathrm{N} + {}^{1}\mathrm{H} &\rightarrow {}^{12}\mathrm{C} + {}^{4}\mathrm{He}
\end{aligned}
$$

- The *seed nuclei* are believed to be predominantly ^{12}C and ^{16}O*:* these are the main *products of He burning,* a later stage of nucleosynthesis.
- *cycle timescale:* is determined by the *slowest reaction* $({}^{14}\mathrm{N} + {}^{1}\mathrm{H})$
- *Approach to equilibrium* in the CNO cycle is determined by the second slowest reaction $({}^{12}\mathrm{C} + {}^{1}\mathrm{H})$
- in equilibrium $\lambda_{^{12}\mathrm{C}}\,{}^{12}\mathrm{C} = \lambda_{^{13}\mathrm{C}}\,{}^{13}\mathrm{C} = \lambda_{^{14}\mathrm{N}}\,{}^{14}\mathrm{N} = \lambda_{^{15}\mathrm{N}}\,{}^{15}\mathrm{N}$
- most of the *CNO seed elements* are converted into $^{14}\mathrm{N}$
- *Observational* evidence for CNO cycle:
  1. In *some red giants* $^{13}\mathrm{C}/^{12}\mathrm{C} \sim 1/5$ (terrestrial ratio $\sim 1/90$)
  2. *Some stars* with *extremely nitrogen-rich* compositions have been discovered

CNO Tri-Cycle



- once every ~ 2500 times the reaction $^{15}\mathrm{N} + \mathrm{H}$ produces $^{16}\mathrm{O} + \gamma$

→ break-out from the main CN cycle

→ *bi-cycle* and of equal importance *tri-cycle* ($^{17}\mathrm{O} + \mathrm{H}$ produces $^{14}\mathrm{N} + \alpha$ and $^{18}\mathrm{F} + \gamma$ in comparable numbers)

- $^{16}\mathrm{O}$ (another seed element) is added to the main CN cycle

- *equilibration timescale for CN cycle:* $\sim 10^6\,\mathrm{yr}$ ($\mathrm{T} \sim 15 \times 10^6\,\mathrm{K}$)

- *equilibration timescale for all cycles:* $\sim 10^{11}\,\mathrm{yr}$ ($\mathrm{T} \sim 15 \times 10^6\,\mathrm{K}$)

→ CN cycle is usually in equilibrium, CNO cycle may not be

The CNO Tri-Cycle

		rate
CN:	$^{12}\mathrm{C}\,(\mathrm{p},\gamma)\,^{13}\mathrm{N}\,(\mathrm{e}^+\,\nu)\,^{13}\mathrm{C}$	λ_{12}
	$^{13}\mathrm{C}\,(\mathrm{p},\gamma)\,^{14}\mathrm{N}$	λ_{13}
	$^{14}\mathrm{N}\,(\mathrm{p},\gamma)\,^{15}\mathrm{O}\,(\mathrm{e}^+\,\nu)\,^{15}\mathrm{N}$	λ_{14}
	$^{15}\mathrm{N}\,(\mathrm{p},\alpha)\,^{12}\mathrm{C}$	λ_{15}
bi:	$^{15}\mathrm{N}\,(\mathrm{p},\gamma)\,^{16}\mathrm{O}$	$\lambda_{15}^{\mathrm{bi}}$
	$^{16}\mathrm{O}\,(\mathrm{p},\gamma)\,^{17}\mathrm{F}\,(\mathrm{e}^+\,\nu)\,^{17}\mathrm{O}$	λ_{16}
	$^{17}\mathrm{O}\,(\mathrm{p},\alpha)\,^{14}\mathrm{N}$	λ_{17}
tri:	$^{17}\mathrm{O}\,(\mathrm{p},\gamma)\,^{18}\mathrm{F}\,(\mathrm{e}^+\,\nu)\,^{18}\mathrm{O}$	$\lambda_{17}^{\mathrm{tri}}$
	$^{18}\mathrm{O}\,(\mathrm{p},\alpha)\,^{15}\mathrm{N}$	λ_{18}

$$\frac{d\,^{12}\mathrm{C}}{dt} = \lambda_{15}\,^{15}\mathrm{N\,H} - \lambda_{12}\,^{12}\mathrm{C\,H}$$

$$\frac{d\,^{13}\mathrm{C}}{dt} = \lambda_{12}\,^{12}\mathrm{C\,H} - \lambda_{13}\,^{13}\mathrm{C\,H}$$

$$\frac{d\,^{14}\mathrm{N}}{dt} = \lambda_{13}\,^{13}\mathrm{C\,H} - \lambda_{14}\,^{14}\mathrm{N\,H} + \lambda_{17}\,^{17}\mathrm{O\,H}$$

$$\frac{d\,^{15}\mathrm{N}}{dt} = \lambda_{14}\,^{14}\mathrm{N\,H} - \left(\lambda_{15} + \lambda_{15}^{\mathrm{bi}}\right)\,^{15}\mathrm{N\,H} + \lambda_{18}\,^{18}\mathrm{O\,H}$$

$$\frac{d\,^{16}\mathrm{O}}{dt} = \lambda_{15}^{\mathrm{bi}}\,^{15}\mathrm{N\,H} - \lambda_{16}\,^{16}\mathrm{O\,H}$$

$$\frac{d\,^{17}\mathrm{O}}{dt} = \lambda_{16}\,^{16}\mathrm{O\,H} - \left(\lambda_{17} + \lambda_{17}^{\mathrm{tri}}\right)\,^{17}\mathrm{O\,H}$$

$$\frac{d\,^{18}\mathrm{O}}{dt} = \lambda_{17}^{\mathrm{tri}}\,^{17}\mathrm{O\,H} - \lambda_{18}\,^{18}\mathrm{O\,H}$$

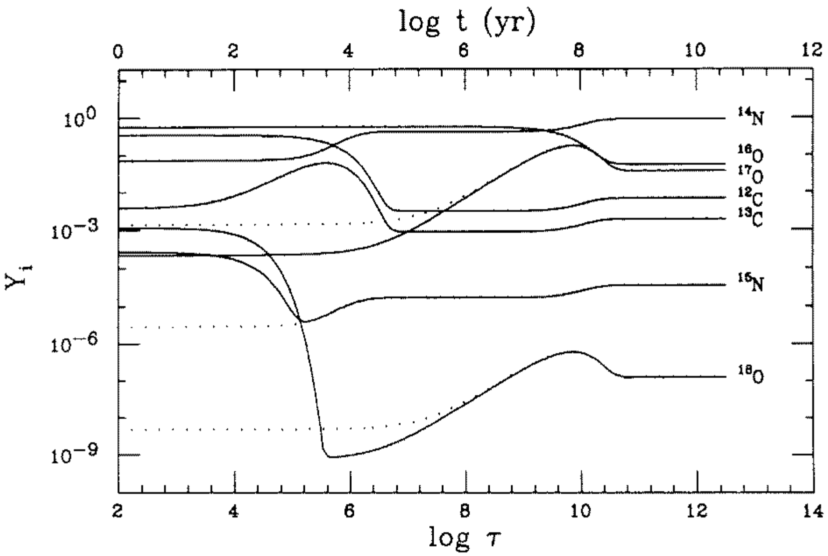


Figure A.4: Same as figure A.3 for $T = 20 \times 10^6$ K.

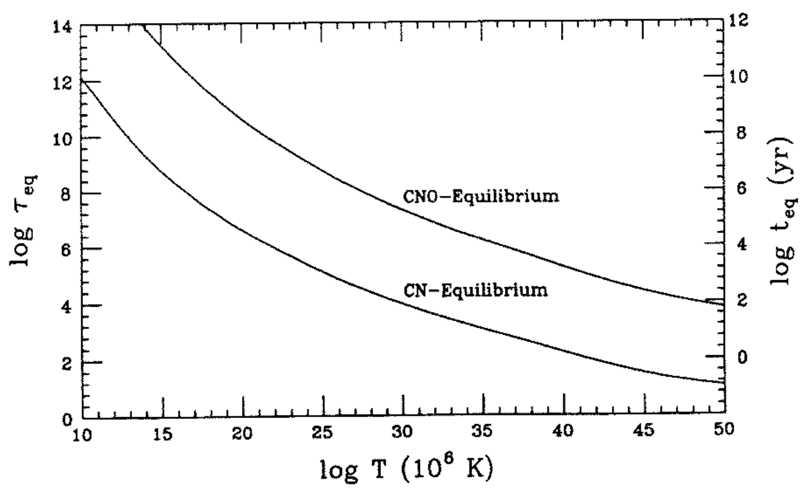


Figure A.8: Equilibration times for the CN main-cycle and the CNO tri-cycle as a function of temperature.

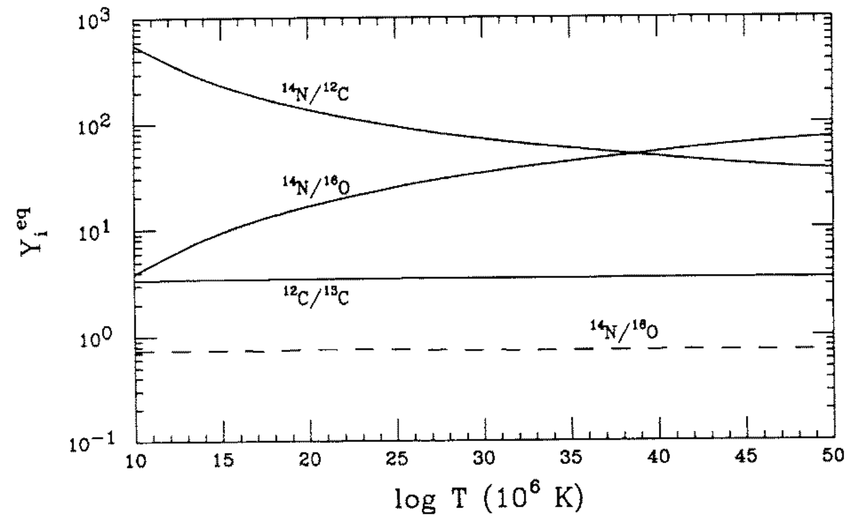


Figure A.7: Selected CNO equilibrium abundance ratios as a function of temperature. (The dashed curve refers to CN-equilibrium.)

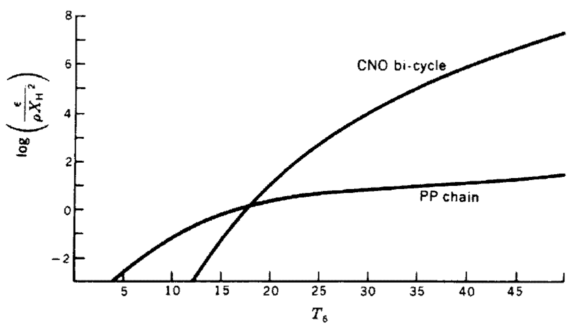
Energy generation from H burning

- Using experimental or extrapolated reaction rates, it is possible to calculate $\varepsilon(\mathrm{T})$ for the various chains.

$$\varepsilon_{\mathrm{PP}} \propto \rho \mathrm{X}_{\mathrm{H}}^2 \qquad \varepsilon_{\mathrm{CNO}} \propto \rho \mathrm{X}_{\mathrm{H}} \mathrm{X}_{\mathrm{CNO}}$$

- Energy generation occurs by *PP chain* at $\mathrm{T} \sim 5 \times 10^6\,\mathrm{K}$.
- ***High-mass stars*** have higher $\mathrm{T_c}$ (CNO cycle dominant) than low-mass stars (pp chain)
- ***Analytical fits to the energy generation rate:***

$$\varepsilon_{\mathrm{PP}} \simeq \varepsilon_1 \mathrm{X}_{\mathrm{H}}^2\, \rho \mathrm{T}^4; \qquad \varepsilon_{\mathrm{CNO}} \simeq \varepsilon_2 \mathrm{X}_{\mathrm{H}} \mathrm{X}_{\mathrm{CNO}}\, \rho \mathrm{T}^{17}.$$



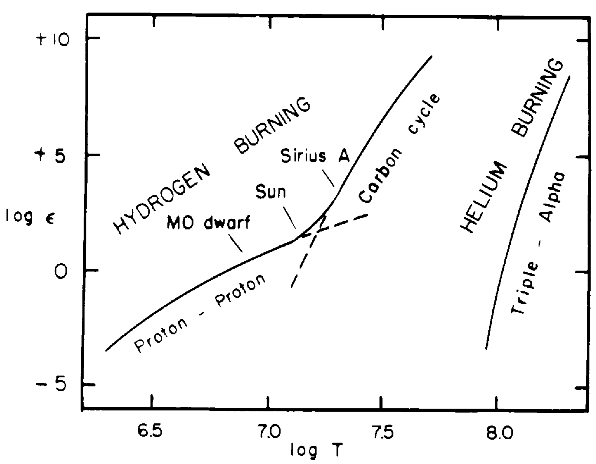


Fig. 10.1. Nuclear energy generation as a function of temperature (with $\rho X^2 = 100$ and $X_{CN} = 0.005X$ for the proton-proton reaction and the carbon cycle, but $\rho^2 Y^3 = 10^8$ for the triple-alpha process).

Other Reactions Involving Light Elements

- Both the ***PP chain*** and the ***CNO cycle*** involve ***weak interactions.*** First reaction of PP chain involves two steps

$$^1\mathrm{H} + {}^1\mathrm{H} \rightarrow {}^2\mathrm{He} \begin{cases} \rightarrow {}^2\mathrm{D} + \mathrm{e}^+ + \nu \\ \rightarrow {}^1\mathrm{H} + {}^1\mathrm{H} \end{cases}$$

- In the ***CNO cycle, high nuclear charges slow the reaction rate.*** D, Li, Be and B burn at lower temperatures than H, because all can burn without β-decays and with $\mathrm{Z} < 6$.

$$\begin{aligned}
{}^2\mathrm{D} + {}^1\mathrm{H} &\rightarrow {}^3\mathrm{He} + \gamma \\
{}^6\mathrm{Li} + {}^1\mathrm{H} &\rightarrow {}^4\mathrm{He} + {}^3\mathrm{He} \\
{}^7\mathrm{Li} + {}^1\mathrm{H} &\rightarrow {}^8\mathrm{Be} + \gamma \rightarrow {}^4\mathrm{He} + {}^4\mathrm{He} + \gamma \\
{}^9\mathrm{Be} + {}^1\mathrm{H} &\rightarrow {}^6\mathrm{Li} + {}^4\mathrm{He} \\
{}^{10}\mathrm{B} + {}^1\mathrm{H} &\rightarrow {}^7\mathrm{Be} + {}^4\mathrm{He} \\
{}^{11}\mathrm{B} + {}^1\mathrm{H} &\rightarrow {}^4\mathrm{He} + {}^4\mathrm{He} + {}^4\mathrm{He} + \gamma
\end{aligned}$$

- $^7\mathrm{Be}$ is destroyed as in the PP chain
- These elements always have ***low abundances*** and play no major role for nuclear burning
- they take place at $\mathrm{T} \sim 10^6 - 10^7\,\mathrm{K}$
- they ***burn up everywhere,*** including surface layers, because convection occurs during ***pre-main-sequence contraction.***

HELIUM BURNING

- When ***H is exhausted*** in central regions, further ***gravitational contraction*** will occur leading to a ***rise in*** T_c, (provided material remains perfect gas)
- ***Problem with He burning: no stable nuclei at*** $A = 8$***;*** no chains of light particle reactions bridging gap between ^{4}He and ^{12}C (next most abundant nucleus).
  - ▷ Yet ^{12}C and ^{16}O are equivalent to 3 and 4 α-particles.
  - ▷ Perhaps many body interactions might be involved? These would only occur fast enough if ***resonant.***
  - ▷ ***Triple*** α ***reaction:*** $^4He + {}^4He + {}^4He \rightarrow {}^{12}C + \gamma$
  - ▷ Ground state of ^{8}Be has $\gamma = 2.5\,\mathrm{eV}$ $\rightarrow \tau = 2.6 \times 10^{-16}\,\mathrm{s}$
  - ▷ Time for two α's to scatter off each other: $t_{scatt} \sim 2d/v \sim 2 \times 10^{-15}/2 \times 10^5 \sim 10^{-20}$ sec
  - ▷ A small concentration of ^{8}Be builds up in ^{4}He gas until rate of break-up = rate of formation.
  - ▷ At $T = 10^8$ K and $\rho = 10^8\,\mathrm{kg\,m^{-3}}$, $n(^8\mathrm{Be})/n(^4\mathrm{He}) \sim 10^{-9}$.
  - ▷ This is sufficient to allow: $^8\mathrm{Be} + {}^4\mathrm{He} \rightarrow {}^{12}\mathrm{C} + \gamma$
- The overall reaction rate would still not be fast enough unless this reaction were ***also resonant at stellar temperatures.***
  - ▷ An s-wave resonance requires ^{12}C to have a 0^+ state with energy $E_0 + 2\Delta E_0$ where $E_0 = 146(T \times 10^{-8})^{2/3}\,\mathrm{keV}$ and $2\Delta E_0 = 164(T \times 10^{-8})^{5/6}\,\mathrm{keV}$.
  - ▷ Such an excited state is found to lie at a resonance energy $E_{res} = 278\,\mathrm{keV}$ above the combined mass of ^{8}Be + ^{4}He .
  - ▷ Best available estimates of partial widths are: $\gamma_\alpha \simeq \gamma = 8.3$ eV; $\gamma_\gamma = (2.8 \pm 0.5)10^{-3}$ eV.
  - ▷ Thus resonant state breaks up ***almost every time.***
  - ▷ ***Equilibrium concentration of*** ^{12}C and the energy generation rate can be calculated.
  - ▷ ***At*** $T \sim 10^8$ ***K*** $\quad \varepsilon_{3\alpha} \simeq \varepsilon_3 X_{He}^3\, \rho^2\, T^{30}$.
- ***energy generation in He core*** strongly concentrated towards regions of highest T
- other important ***He-burning reactions:***

$$
\begin{aligned}
{}^{12}\mathrm{C} + \alpha &\rightarrow {}^{16}\mathrm{O} + \gamma \\
{}^{13}\mathrm{C} + \alpha &\rightarrow {}^{16}\mathrm{O} + \mathrm{n} \\
{}^{14}\mathrm{N} + \alpha &\rightarrow {}^{18}\mathrm{O} + \mathrm{e}^+ + \nu \\
{}^{16}\mathrm{O} + \alpha &\rightarrow {}^{20}\mathrm{Ne} + \gamma \\
{}^{18}\mathrm{O} + \alpha &\rightarrow {}^{22}\mathrm{Ne} + \gamma \\
{}^{20}\mathrm{Ne} + \alpha &\rightarrow {}^{24}\mathrm{Mg} + \gamma
\end{aligned}
$$

  in some phases of stellar evolution and outside the core, these can be the dominant He-burning reactions
- in a stellar core supported by ***electron degeneracy,*** the onset of He burning is believed to be accompanied by an explosive reaction – ***THE HELIUM FLASH***
- once He is used up in the central regions, further contraction and heating may occur, leading to additional nuclear reactions e.g. ***carbon burning***
- by the time that H and He have been burnt most of the possible energy release from fusion reactions has occurred