THE STRUCTURE OF MAIN-SEQUENCE STARS

- main-sequence phase: *hydrogen core burning* phase
  - ▷ *zero-age main sequence (ZAMS): homogeneous* composition

Scaling relations for main-sequence stars

- use *dimensional analysis* to derive scaling relations (relations of the form $L \propto M^{\gamma}$)
- replace differential equations by characteristic quantities (e.g. $dP/dr \sim P/R$, $\rho \sim M/R^3$)
- *hydrostatic equilibrium* $\rightarrow P \sim \frac{GM^2}{R^4}$ (1)
- *radiative transfer* $\rightarrow L \propto \frac{R^4T^4}{\kappa M}$ (2)
- to derive *luminosity–mass relationship,* specify *equation of state* and *opacity law*

(1) *massive stars:* ideal-gas law, electron scattering opacity, i.e.

▷ $P = \frac{\rho}{\mu m_H} kT \sim \frac{kT}{\mu m_H}\left(\frac{M}{R^3}\right)$ and $\kappa \simeq \kappa_{Th} = \text{constant}$

$$\Rightarrow \frac{kT}{\mu m_H} \sim \frac{GM}{R} \quad (3)$$

▷ substituting (3) into (2): $L \propto \frac{\mu^4 M^3}{\kappa_{Th}}$

(2) *low-mass stars:* ideal-gas law, Kramer's opacity law, i.e. $\kappa \propto \rho T^{-3.5}$

$$\Rightarrow L \propto \frac{\mu^{7.5} M^{5.5}}{R^{0.5}}$$

- *mass–radius relationship*
  - ▷ central temperature determined by characteristic nuclear-burning temperature (hydrogen fusion: $T_c \sim 10^7$ K; helium fusion: $T_c \sim 10^8$ K)
  - ▷ from (3) $\Rightarrow R \propto M$ (in reality $R \propto M^{0.6-0.8}$)

(3) *very massive stars:* radiation pressure, electron scattering opacity, i.e.

▷ $P = \frac{1}{3}aT^4 \rightarrow T \sim \frac{M^{1/2}}{R} \Rightarrow L \propto M$

- power-law index in mass–luminosity relationship decreases from ~ 5 *(low-mass)* to 3 *(massive)* and 1 *(very massive)*
- near a solar mass: $L \simeq L_{\odot}\left(\frac{M}{M_{\odot}}\right)^4$
- *main-sequence lifetime:* $T_{MS} \propto M/L$

  typically: $T_{MS} = 10^{10}\,\text{yr}\left(\frac{M}{M_{\odot}}\right)^{-3}$
- *pressure* is inverse proportional to the *mean molecular weight* μ
  - ▷ higher μ (fewer particles) implies higher temperature to produce the same pressure, but T_c *is fixed* (hydrogen burning *(thermostat):* $T_c \sim 10^7$ K)
  - ▷ during H-burning μ increases from ~ 0.62 to ~ 1.34
  - $\rightarrow$ *radius increases* by a factor of ~ 2 (equation [3])

- *opacity* at low temperatures depends strongly on *metallicity* (for bound-free opacity: $\kappa \propto Z$)
  - ▷ *low-metallicity stars* are much *more luminous* at a given mass and have proportionately shorter lifetimes
  - ▷ mass-radius relationship only weakly dependent on metallicity
  - → low-metallicity stars are *much hotter*
  - ▷ *subdwarfs:* low-metallicity main-sequence stars lying just below the main sequence

General properties of homogeneous stars:

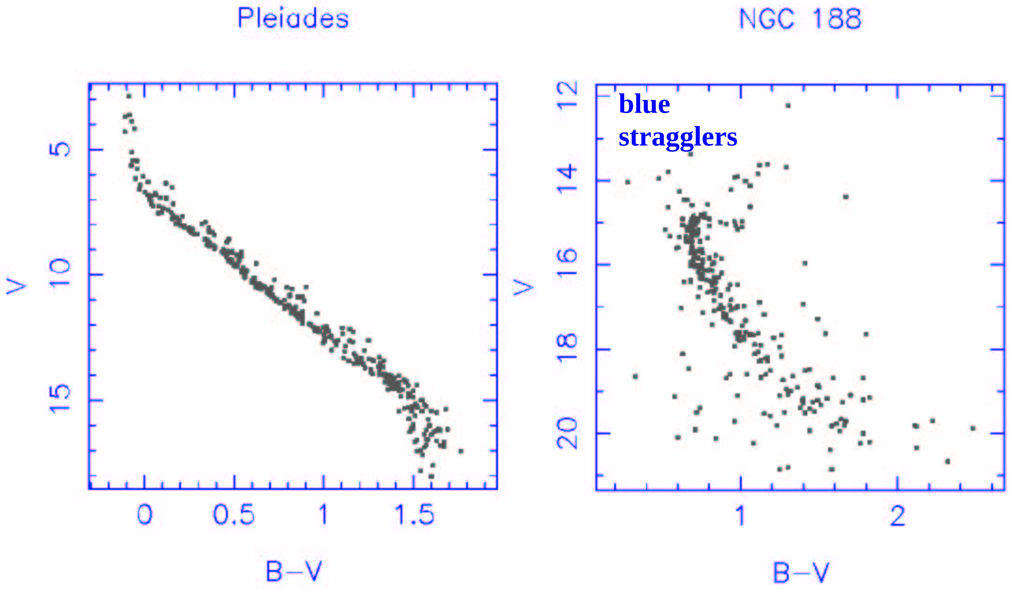
	Upper main sequence ($M_s > 1.5\,M_\odot$)	Lower main sequence ($M_s < 1.5\,M_\odot$)
core	*convective;* well mixed	*radiative*
ϵ	*CNO cycle*	*PP chain*
κ	*electron scattering*	*Kramer's opacity* $\kappa \simeq \kappa_3 \rho T^{-3.5}$
surface	*H fully ionized* energy transport by *radiation*	*H/He neutral* *convection* zone just below surface

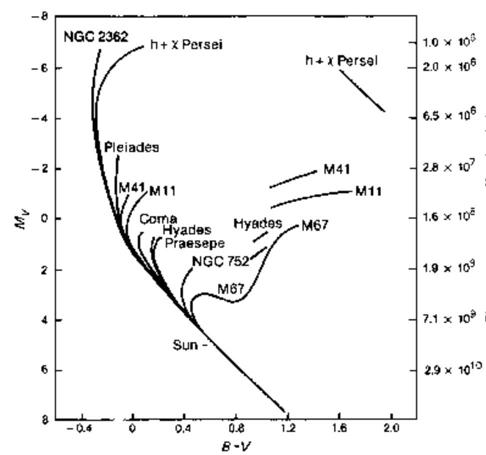
N.B. T_c is an increasing function of M_s; ρ_c decreases as M_s increases.

- *Hydrogen-burning limit:* $M_s \simeq 0.08\,M_\odot$
  - ▷ low-mass objects (brown dwarfs) do not burn hydrogen, since they are supported by *electron degeneracy*
- *Giants, supergiants and white dwarfs* cannot be chemically homogeneous stars supported by nuclear burning

Turnoff Ages in Open Clusters

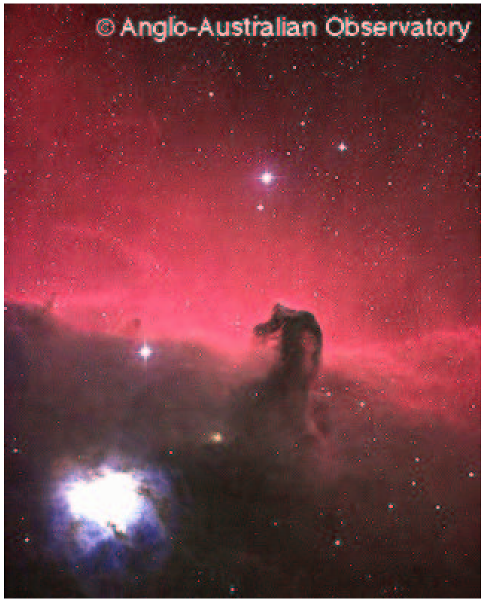
$$T = 10^{10}\,\text{yr}\left(\frac{L_{TO}}{L_\odot}\right)^{-3/4}$$





Star Formation (I)

Orion Nebula



STAR FORMATION

Star-Forming Regions

a) *Massive stars*

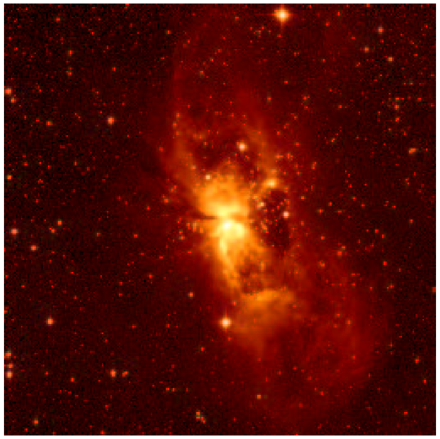
- born in *OB associations* in warm molecular clouds
- produce *brilliant HII regions*
- shape their environment
  - ▷ photoionization
  - ▷ stellar winds
  - ▷ supernovae
  - → induce further (low-mass) star formation?

b) *Low-mass stars*

- born in *cold, dark molecular clouds* ($T \simeq 10\,K$)
- Bok globules
- near massive stars?
- *recent:* most low-mass stars appear to be born in *cluster-like environments*
- *but:* most low-mass stars are not found in clusters → embedded *clusters do not survive*

  Relationship between massive and low-mass star formation?
  - ▷ massive stars trigger low-mass star formation?
  - ▷ massive stars terminate low-mass star formation?

Star Formation (II)



massive star +
cluster of low-mas stars

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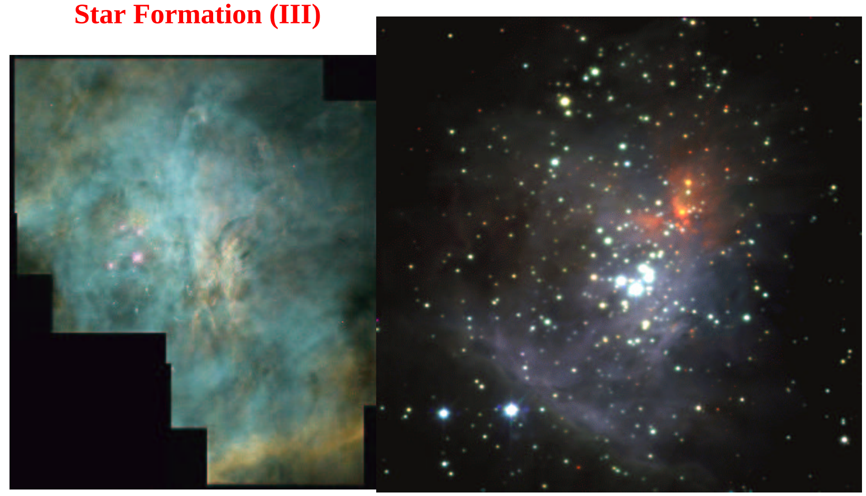
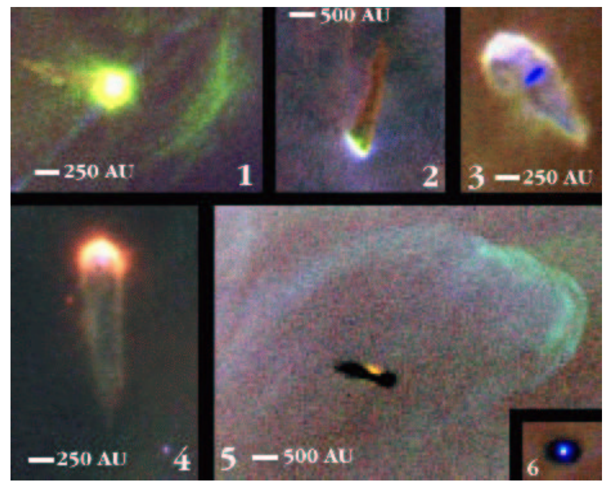
Bok globules

Star Formation (III)

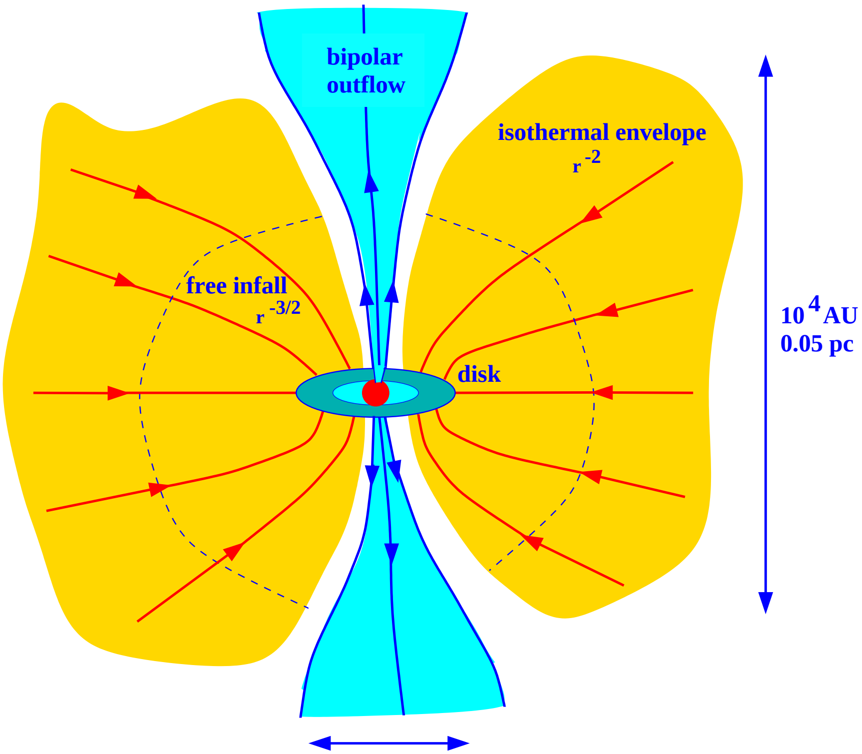


The Trapezium Cluster (IR)

HST



Dusty Disks in Orion (seen as dark silhouettes)

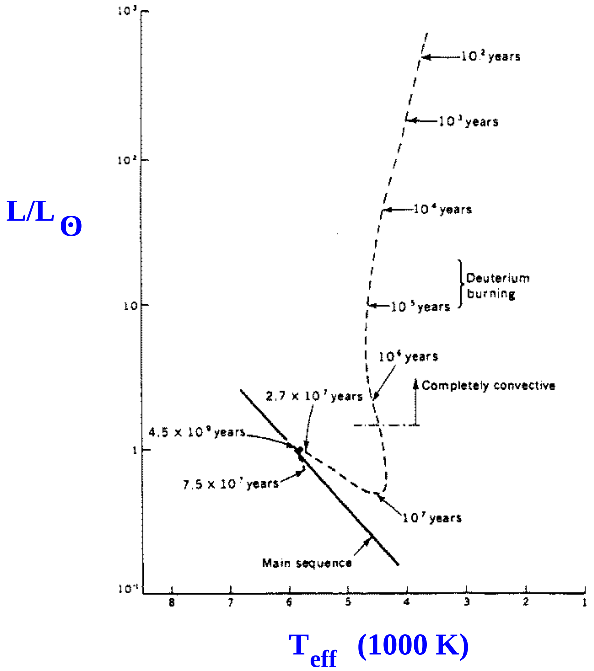


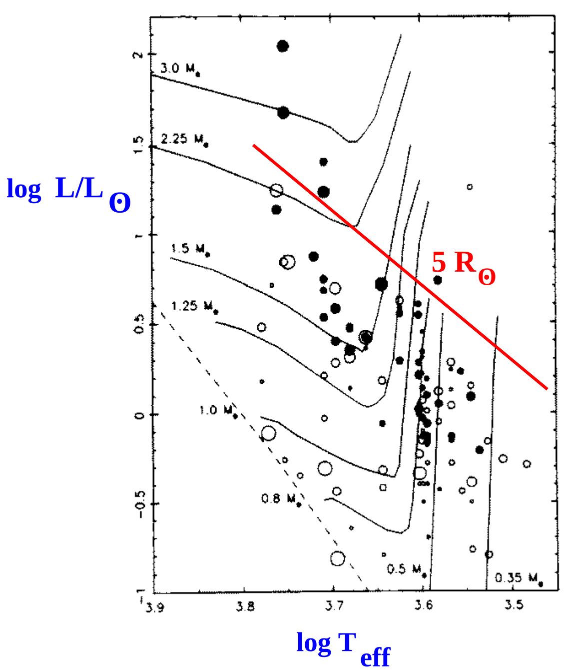
Protostar Structure

Stellar Collapse (Low-mass)

- cool, *molecular cores* (H_2) collapse when their mass exceeds the *Jeans Mass*
  - ▷ no thermal pressure support if $P_c = \rho/(\mu m_H) kT < GM^2/(4\pi R^4)$
  - ▷ or $M > M_J \simeq 6\,M_\odot \left(\frac{T}{10\,K}\right)^{3/2} \left(\frac{n_{H_2}}{10^{10}\,m^{-3}}\right)^{-1/2}$
- collapse triggered:
  - ▷ by *loss of magnetic support*
  - ▷ by *collision* with other cores
  - ▷ by *compression* caused by nearby supernovae
- *inside-out isothermal collapse* (i.e. efficient radiation of energy) from $\sim 10^6\,R_\odot$ to $\sim 5\,R_\odot$
- *timescale:* $t_{dyn} \sim 1/\sqrt{4\,G\rho} \sim 10^5 - 10^6$ yr
- collapse *stops* when material becomes *optically thick* and can no longer remain isothermal *(protostar)*
- *the angular-momentum problem*
  - ▷ each molecular core has a small amount of angular momentum (due to the velocity shear caused by the Galactic rotation)
  - ▷ characteristic $\Delta v/\Delta R \sim 0.3$km/s/ly
  - → characteristic, specific angular momentum $j \sim (\Delta v/\Delta R\, R_{cloud})\, R_{cloud} \sim 3 \times 10^{16}\,m^2\,s^{-1}$
  - ▷ cores cannot collapse directly
  - → formation of an *accretion disk*

Pre-Main-Sequence Evolution





- ▷ characteristic disk size from angular-momentum conservation $j = rv_\perp = rv_{Kepler} = \sqrt{GMr}$

  → $r_{min} = j^2/GM \sim 10^4\,R_\odot \simeq 50\mathrm{AU}$

- *Solution:* Formation of *binary systems and planetary systems* which store the angular momentum (Jupiter: 99 % of angular momentum in solar system)

  → *most stars should have planetary systems and/or stellar companions*

  → stars are initially *rotating rapidly* (spin-down for stars like the Sun by magnetic braking)

- inflow/outflow: ∼ 1/3 of material accreted is ejected from the accreting protostar → bipolar jets

Pre-main-sequence evolution

- Old picture: stars are born with *large radii* ($\sim 100\,R_\odot$) and slowly contract to the main sequence

  ▷ energy source: *gravitational energy*

  ▷ contraction stops when the central temperature reaches $10^7\,\mathrm{K}$ and H-burning starts (main sequence)

  ▷ note: D already burns at $T_c \sim 10^6\,\mathrm{K}$ → temporarily halts contraction

- *Modern picture:* stars are born with *small radii* ($\sim 5\,R_\odot$) and small masses

  → first appearance in the H-R diagram on the *stellar birthline* (where accretion timescale is comparable to Kelvin-Helmholtz timescale: $t_{\dot{M}} \equiv M/\dot{M} \sim t_{KH} = GM^2/(2RL)$)

  ▷ continued accretion as *embedded protostars/T Tauri stars* until the mass is exhausted or accretion stops because of dynamical interactions with other cores/stars