THE EVOLUTION OF LOW-MASS STARS

($M \lesssim 8\,M_\odot$)

Pre-main-sequence phase

- observationally new-born stars appear as *embedded protostars/T Tauri stars* near the *stellar birthline* where they burn *deuterium* ($T_c \sim 10^6$ K), often still *accreting* from their birth clouds
- *after deuterium burning* → star *contracts* → $T_c \sim (\mu m_H/k)\,(GM/R)$ increases until hydrogen burning starts ($T_c \sim 10^7$K) → main-sequence phase

Main-sequence phase

- energy source: *hydrogen burning* (4 H→ ^{4}He)
  - → mean molecular weight μ increases in core from 0.6 to 1.3 → R, L and T_c increase (from $T_c \propto \mu\,(GM/R)$)
- lifetime: $T_{MS} \simeq 10^{10}\,\mathrm{yr}\left(\frac{M}{M_\odot}\right)^{-3}$

after hydrogen exhaustion:

- formation of *isothermal core*
- *hydrogen burning in shell* around inert core (shell-burning phase)

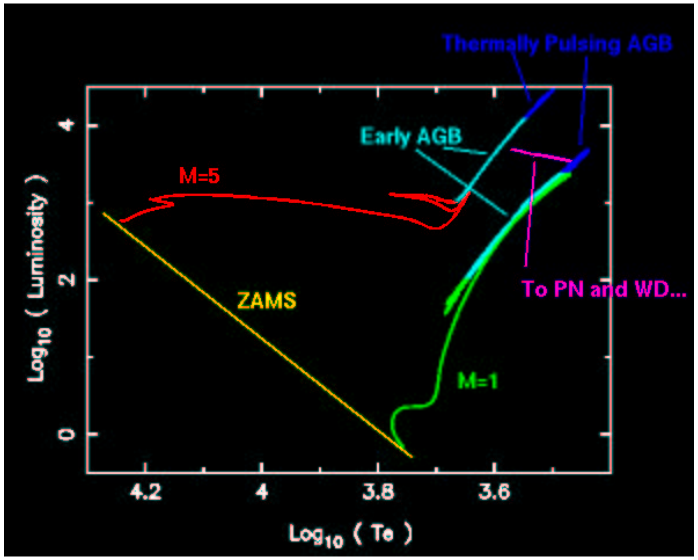
→ growth of core until $M_{core}/M \sim 0.1$ *(Schönberg-Chandrasekhar limit)*

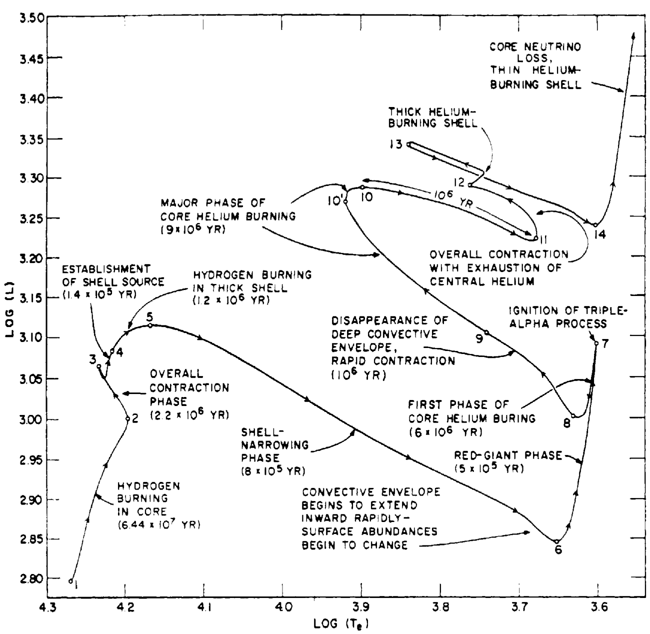
- ▷ core becomes too massive to be supported by thermal pressure
- → *core contraction* → energy source: *gravitational energy* → core becomes denser and hotter
- ▷ contraction stops when the core density becomes high enough that *electron degeneracy pressure* can support the core

  (stars more massive than $\sim 2\,M_\odot$ ignite helium in the core before becoming degenerate)

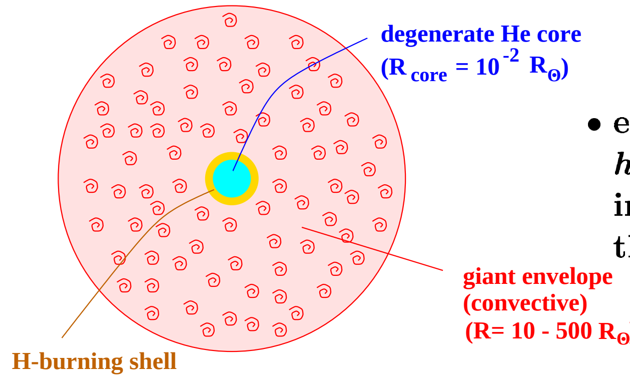
- while the core contracts and becomes degenerate, the *envelope expands* dramatically
  → star becomes a *red giant*
  - ▷ the transition to the red-giant branch is not well understood (in intuitive terms)
  - ▷ for stars with $M \gtrsim 1.5\,M_\odot$, the transition occurs very fast, i.e. on a thermal timescale of the envelope → few stars observed in transition region *(Hertzsprung gap)*

Evolutionary Tracks (1 to 5$M_\odot$)





THE RED-GIANT PHASE

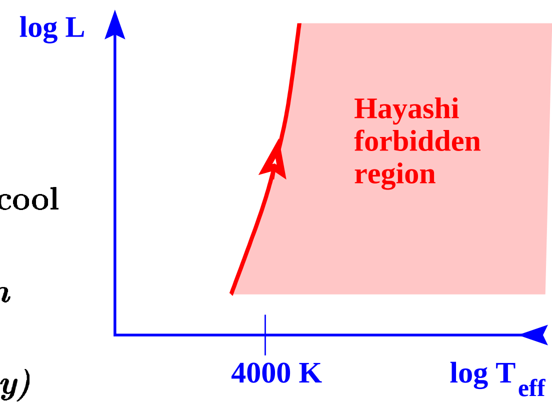


- energy source: *hydrogen burning* in thin *shell* above the degenerate core

- core mass grows → temperature in shell increases → luminosity increases → star *ascends red-giant branch*

- ***Hayashi track:*** vertical track in H-R diagram
  - ▷ no hydrostatic solutions for very cool giants
  - ▷ ***Hayashi forbidden region***

    (due to H^- opacity)



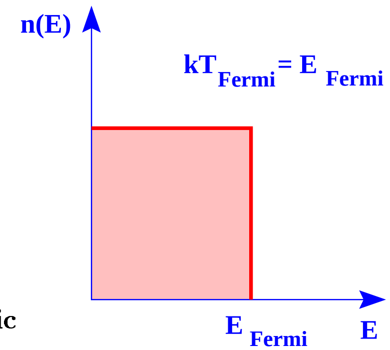
- when the core mass reaches $M_c \simeq 0.48\,M_\odot$ → ignition of helium → *helium flash*

HELIUM FLASH

- *ignition of He under degenerate conditions* (for $M \lesssim 2\,M_\odot$; core mass $\sim 0.48\,M_\odot$)
  - ▷ i.e. P is independent of T → *no self-regulation*
    [in normal stars: increase in T → decrease in ρ (expansion) → decrease in T (virial theorem)]
  - ▷ in degenerate case: nuclear burning → increase in T → more nuclear burning → further increase in T

→ *thermonuclear runaway*

- runaway *stops* when matter becomes *non-degenerate* (i.e. $T \sim T_{Fermi}$)
- disruption of star?
  - ▷ energy generated in runaway:
  - ▷ $\Delta E = \underbrace{\frac{M_{burned}}{\mu m_H}}_{\text{number of particles}} \underbrace{kT_{Fermi}}_{\text{characteristic energy}}$



→ $\Delta E \sim 2 \times 10^{42}\,J \left(\frac{M_{burned}}{0.1\,M_\odot}\right) \left(\frac{\rho}{10^9\,kg\,m^{-3}}\right)^{2/3}$ $(\mu \simeq 2)$

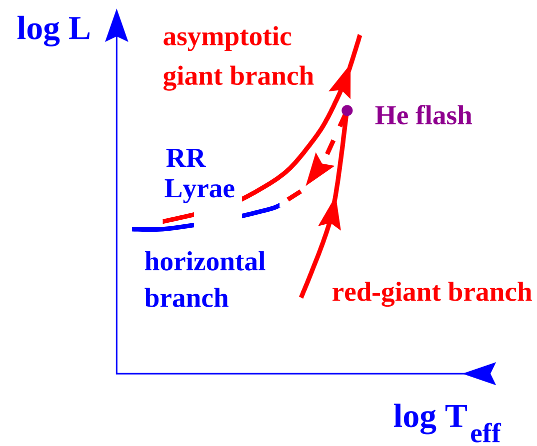
▷ compare ΔE to the binding energy of the core
$E_{bind} \simeq GM_c^2/R_c \sim 10^{43}\,J$ ($M_c = 0.5\,M_\odot$; $R_c = 10^{-2}\,R_\odot$)

→ expect significant *dynamical expansion,* but no disruption ($t_{dyn} \sim$ sec)

→ core expands → *weakening of H shell source* → rapid decrease in luminosity

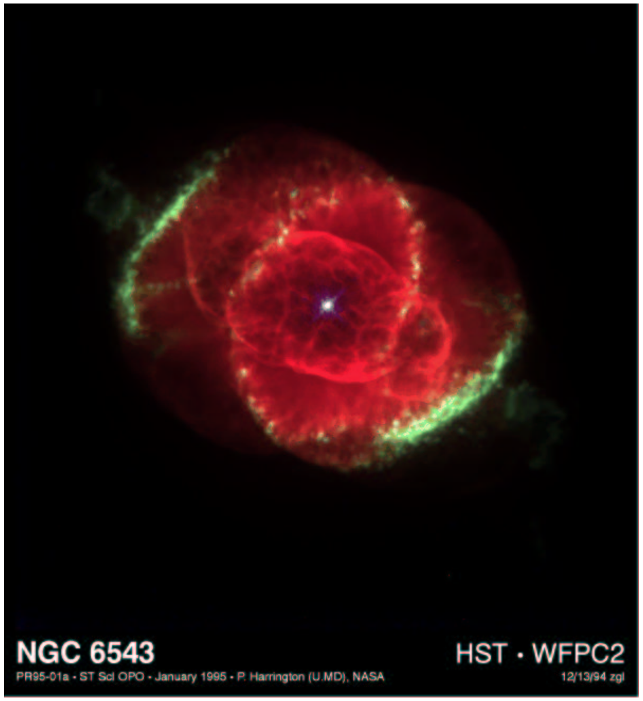
→ star settles on *horizontal branch*

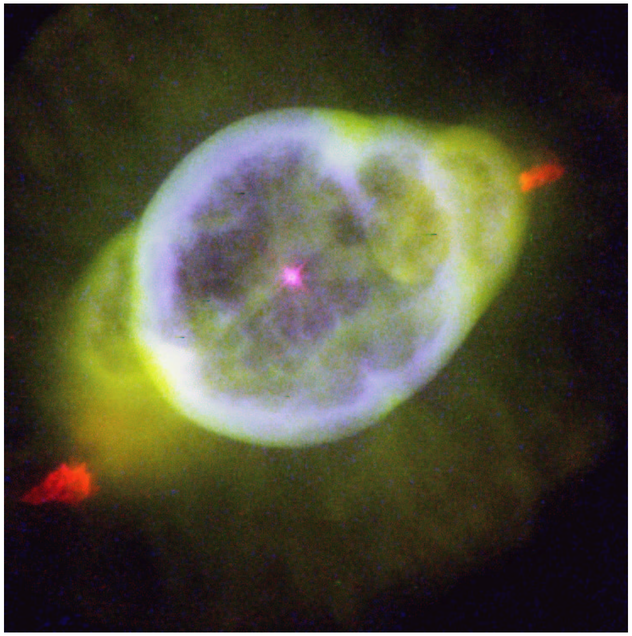
THE HORIZONTAL BRANCH (HB)



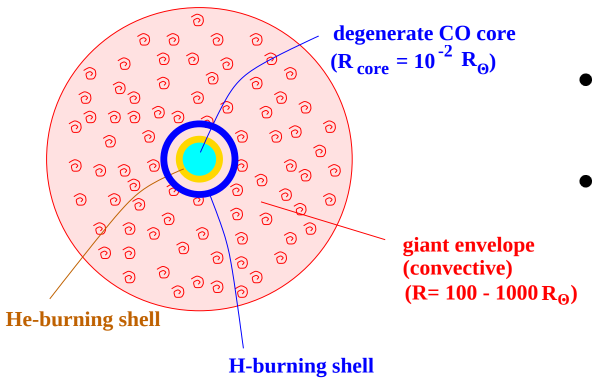
- *He burning* in center: conversion of He to mainly C and O ($^{12}C + \alpha \rightarrow {}^{16}O$)
- *H burning* in shell (usually the dominant energy source)
- *lifetime:* $\sim 10\,\%$ of main-sequence lifetime (lower efficiency of He burning, higher luminosity)
- *RR Lyrae stars* are *pulsationally unstable* (L, B − V change with periods $\lesssim 1$ d)
  easy to detect → popular *distance* indicators
- after *exhaustion of central He*
  → core *contraction* (as before) → *degenerate core* → *asymptotic giant branch*

Planetary Nebulae with the HST





THE ASYMPTOTIC GIANT BRANCH (AGB)



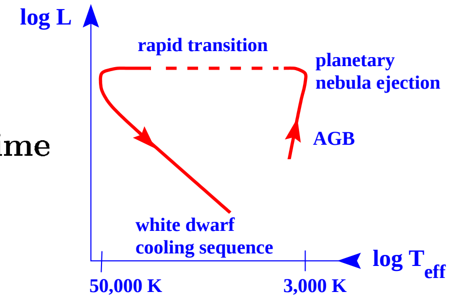
- *H burning* and *He burning* (in thin shells)
- H/He burning do not occur simultaneous, but alternate → *thermal pulsations*

- low-/intermediate-mass stars ($M \lesssim 8\,M_\odot$) do not experience nuclear burning beyond helium burning
- *evolution ends* when the *envelope* has been *lost* by stellar winds
  - ▷ *superwind phase:* very rapid mass loss ($\dot{M} \sim 10^{-4}\,M_\odot\,yr^{-1}$)
  - ▷ probably because envelope attains *positive binding energy* (due to energy reservoir in ionization energy)
  - → envelopes can be dispersed to infinity without requiring energy source
  - ▷ complication: radiative losses

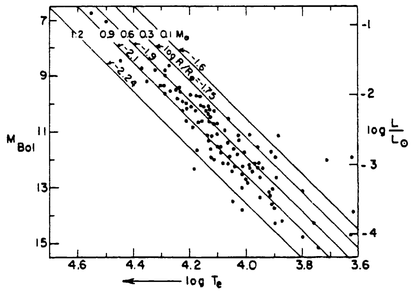
- after ejection: hot CO core is exposed and *ionizes* the ejected shell

→ *planetary nebula phase* (lifetime $\sim 10^4$ yr)

- CO core cools, becomes *degenerate* → *white dwarf*



WHITE DWARFS



	Mass ($M_\odot$)	Radius ($R_\odot$)
Sirius B	1.053 ± 0.028	0.0074 ± 0.0006
40 Eri B	0.48 ± 0.02	0.0124 ± 0.0005
Stein 2051	0.50 ± 0.05	0.0115 ± 0.0012

- first white dwarf discovered: *Sirius B* (companion of Sirius A)
  - ▷ mass (from orbit): $M \sim 1\,M_\odot$
  - ▷ radius (from $L = 4\pi R^2 \sigma T_{\rm eff}^4$) $R \sim 10^{-2}\,R_\odot \sim R_\oplus$

  $\rightarrow$ $\rho \sim 10^9\,{\rm kg\,m^{-3}}$
- Chandrasekhar (Cambridge 1930)
  - ▷ white dwarfs are supported by *electron degeneracy pressure*
  - ▷ white dwarfs have a *maximum mass* of $1.4\,M_\odot$
- most white dwarfs have a *mass* very close to $M \sim 0.6\,M_\odot$: $M_{\rm WD} = 0.58 \pm 0.02\,M_\odot$
- most are made of carbon and oxygen *(CO white dwarfs)*
- some are made of He or O-Ne-Mg

Mass-Radius Relations for White Dwarfs

Non-relativistic degeneracy

- $P \sim P_e \propto (\rho/\mu_e m_H)^{5/3} \sim GM^2/R^4$

$$\rightarrow \boxed{R \propto \frac{1}{m_e} (\mu_e m_H)^{5/3} M^{-1/3}}$$

- note the *negative exponent*

  $\rightarrow$ R decreases with increasing mass

  $\rightarrow$ ρ increases with M

Relativistic degeneracy (when $p_{\rm Fe} \sim m_e c$)

- $P \sim P_e \propto (\rho/\mu_e m_H)^{4/3} \sim GM^2/R^4$

$\rightarrow$ *M independent of R*

$\rightarrow$ existence of a *maximum mass*

THE CHANDRASEKHAR MASS

- consider a star of radius R containing N Fermions (electrons or neutrons) of mass m_f
- the mass per Fermion is $\mu_f m_H$ (μ_f = mean molecular weight per Fermion) $\to$ number density $n \sim N/R^3 \to$ volume/Fermion 1/n
- ***Heisenberg uncertainty principle***
  $[\Delta x\, \Delta p \sim \hbar]^3 \to$ typical momentum: $p \sim \hbar\, n^{1/3}$

$\to$ ***Fermi energy*** of relativistic particle ($E = pc$)

$$E_f \sim \hbar\, n^{1/3}\, c \sim \frac{\hbar c\, N^{1/3}}{R}$$

- ***gravitational energy*** per Fermion

$$E_g \sim -\frac{GM(\mu_f m_H)}{R}, \text{ where } M = N\, \mu_f m_H$$

$\to$ total energy (per particle)

$$E = E_f + E_g = \frac{\hbar c N^{1/3}}{R} - \frac{GN(\mu_f m_H)^2}{R}$$

- stable configuration has minimum of total energy
- if $E < 0$, E can be decreased without bound by decreasing R $\to$ no equilibrium $\to$ *gravitational collapse*
- maximum N, if $E = 0$

$$\to N_{max} \sim \left(\frac{\hbar c}{G(\mu_f m_H)^2}\right)^{3/2} \sim 2 \times 10^{57}$$

$$M_{max} \sim N_{max}\, (\mu_e m_H) \sim 1.5\, M_\odot$$

Chandrasekhar mass for white dwarfs

$$\boxed{M_{Ch} = 1.457 \left(\frac{2}{\mu_e}\right)^2 M_\odot}$$

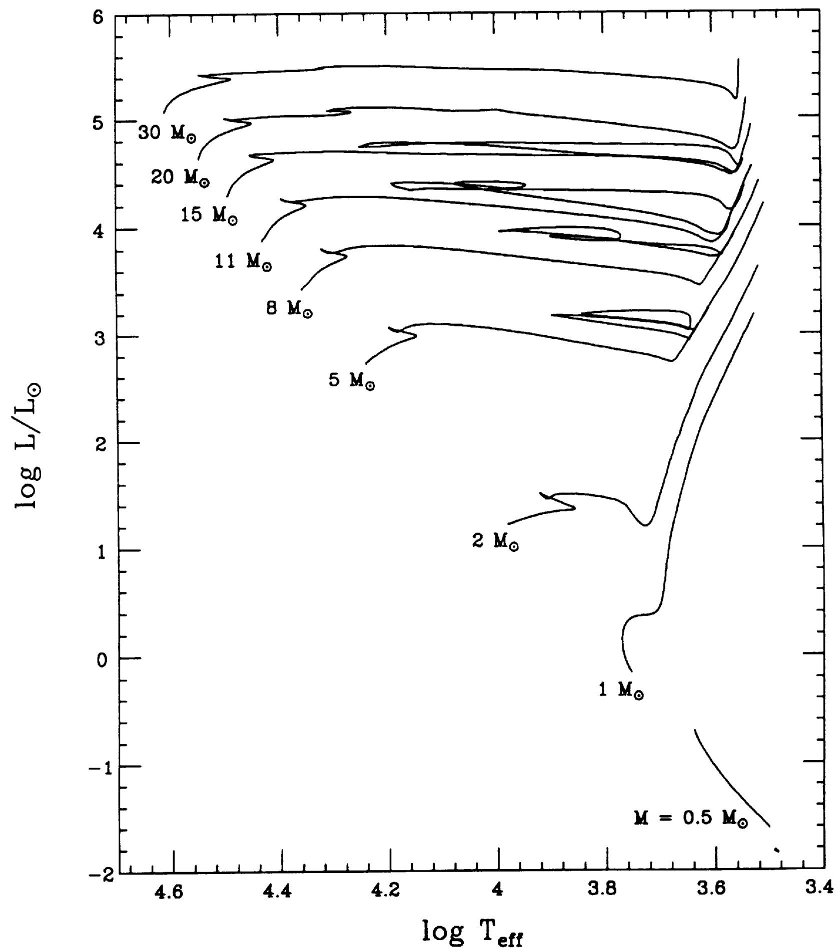


Figure B.1: Composite H-R diagram presenting the evolutionary tracks for stars between $0.5\, M_\odot$ and $30\, M_\odot$. The calculations assume an initially solar composition ($Y = 0.28$, $Z = 0.02$) and a mixing length parameter $\alpha = 1.5$.