THE EVOLUTION OF LOW-MASS STARS ($M \lesssim 8\,M_\odot$)

Pre-main-sequence phase

- observationally new-born stars appear as embedded protostars/T Tauri stars near the stellar birthline where they burn deuterium ($T_c \sim 10^6\,K$), often still accreting from their birth clouds
- after deuterium burning → star contracts → $T_c \sim (\mu m_H/k)\,(GM/R)$ increases until hydrogen burning starts ($T_c \sim 10^7 K$) → main-sequence phase

Main-sequence phase

- energy source: hydrogen burning (4 H→ ^{4}He)

  → mean molecular weight μ increases in core from 0.6 to 1.3 → R, L and T_c increase (from $T_c \propto \mu\,(GM/R)$)

- lifetime: $T_{MS} \simeq 10^{10}\,yr \left(\frac{M}{M_\odot}\right)^{-3}$

after hydrogen exhaustion:

- formation of isothermal core
- hydrogen burning in shell around inert core (shell-burning phase)

→ growth of core until $M_{core}/M \sim 0.1$ (Schönberg-Chandrasekhar limit)

  ▷ core becomes too massive to be supported by thermal pressure

  → core contraction → energy source: gravitational energy → core becomes denser and hotter

  ▷ contraction stops when the core density becomes high enough that electron degeneracy pressure can support the core

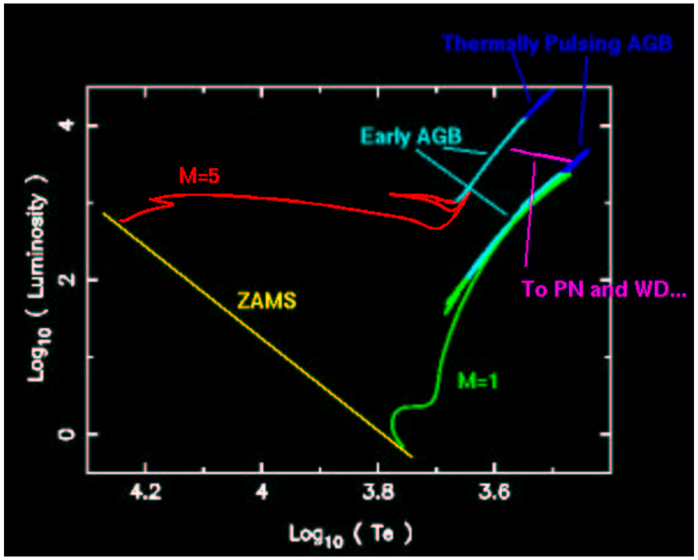
  (stars more massive than $\sim 2\,M_\odot$ ignite helium in the core before becoming degenerate)

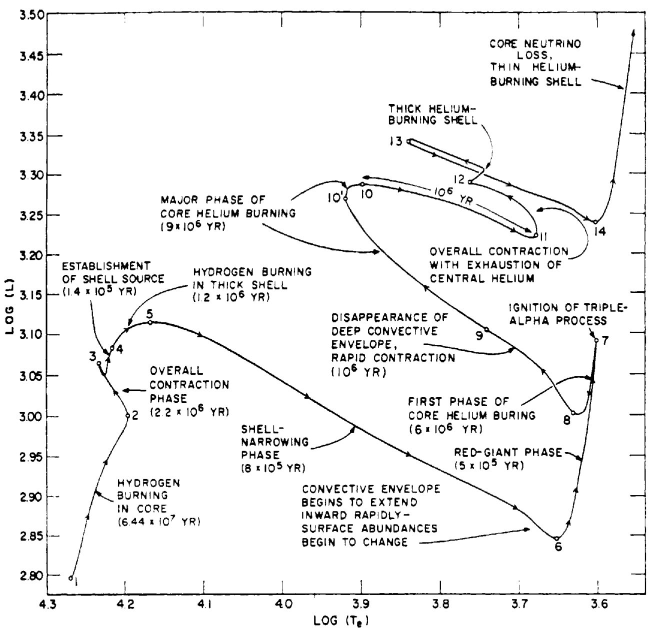
- while the core contracts and becomes degenerate, the envelope expands dramatically
  → star becomes a red giant

  ▷ the transition to the red-giant branch is not well understood (in intuitive terms)

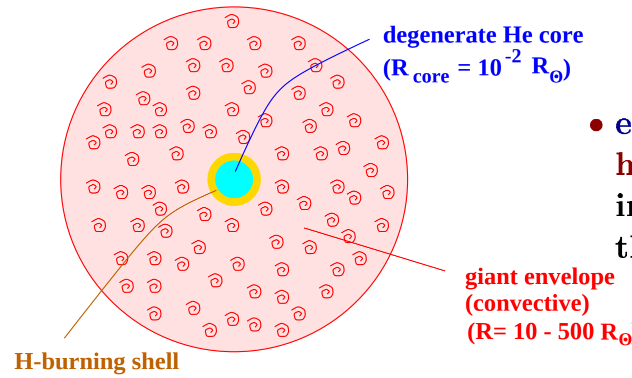
  ▷ for stars with $M \gtrsim 1.5\,M_\odot$, the transition occurs very fast, i.e. on a thermal timescale of the envelope → few stars observed in transition region (Hertzsprung gap)

Evolutionary Tracks (1 to 5$M_\odot$)





THE RED-GIANT PHASE

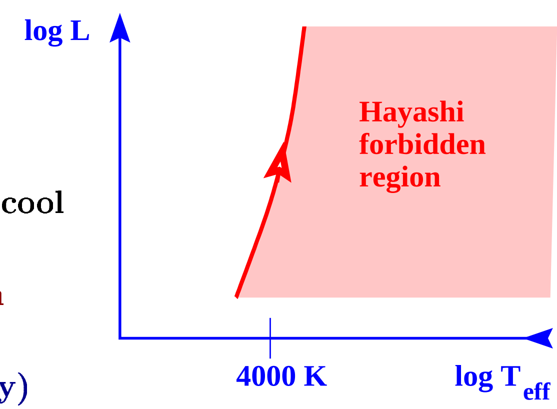


- energy source: hydrogen burning in thin shell above the degenerate core

- core mass grows → temperature in shell increases → luminosity increases → star ascends red-giant branch

- Hayashi track: vertical track in H-R diagram
  - ▷ no hydrostatic solutions for very cool giants
  - ▷ Hayashi forbidden region

    (due to H^- opacity)



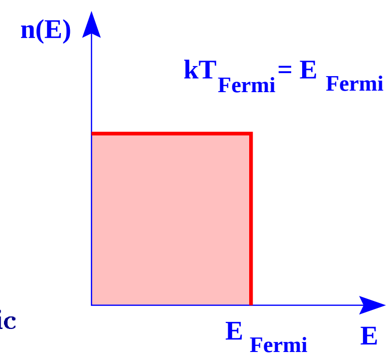
- when the core mass reaches $M_c \simeq 0.48\,M_\odot$ → ignition of helium → helium flash

HELIUM FLASH

- ignition of He under degenerate conditions (for $M \lesssim 2\,M_\odot$; core mass $\sim 0.48\,M_\odot$)
  - ▷ i.e. P is independent of T → no self-regulation
    [in normal stars: increase in T → decrease in ρ (expansion) → decrease in T (virial theorem)]
  - ▷ in degenerate case: nuclear burning → increase in T → more nuclear burning → further increase in T

→ thermonuclear runaway

- runaway stops when matter becomes non-degenerate (i.e. $T \sim T_{Fermi}$)
- disruption of star?
  - ▷ energy generated in runaway:
  - ▷ $\Delta E = \underbrace{\frac{M_{burned}}{\mu m_H}}_{\text{number of particles}} \underbrace{kT_{Fermi}}_{\text{characteristic energy}}$



→ $\Delta E \sim 2 \times 10^{42}\,J \left(\frac{M_{burned}}{0.1\,M_\odot}\right) \left(\frac{\rho}{10^9\,kg\,m^{-3}}\right)^{2/3} \quad (\mu \simeq 2)$

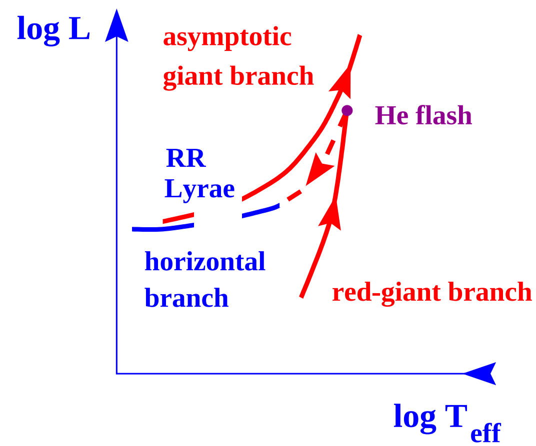
▷ compare ΔE to the binding energy of the core
$E_{bind} \simeq GM_c^2/R_c \sim 10^{43}\,J$ ($M_c = 0.5\,M_\odot$; $R_c = 10^{-2}\,R_\odot$)

→ expect significant dynamical expansion, but no disruption ($t_{dyn} \sim$ sec)

→ core expands → weakening of H shell source → rapid decrease in luminosity

→ star settles on horizontal branch

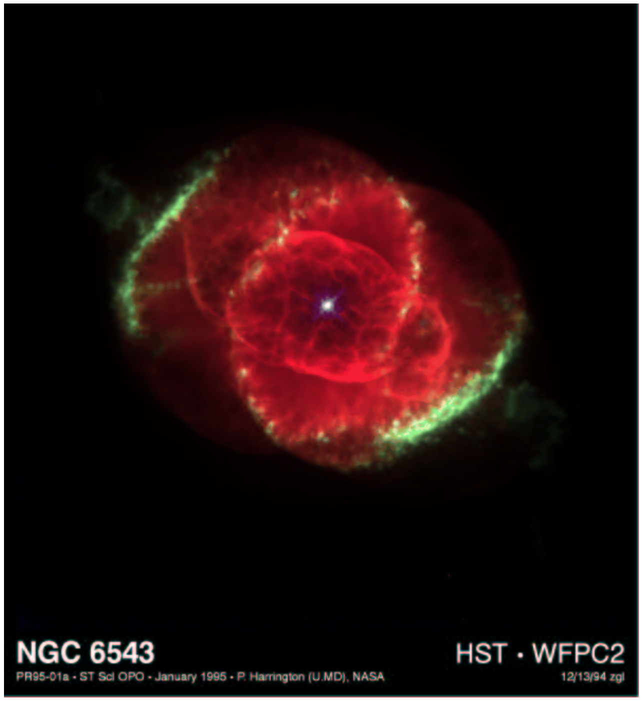
THE HORIZONTAL BRANCH (HB)



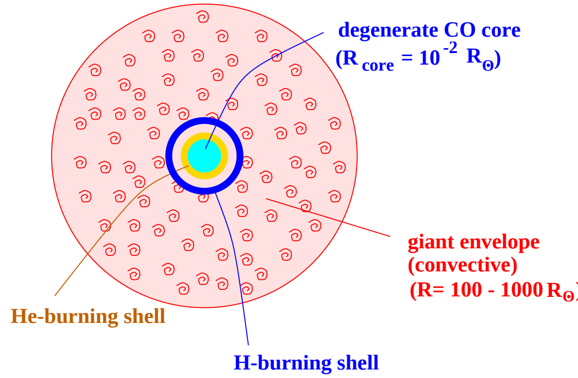
- He burning in center: conversion of He to mainly C and O ($^{12}C + \alpha \rightarrow ^{16}O$)
- H burning in shell (usually the dominant energy source)
- lifetime: $\sim 10\,\%$ of main-sequence lifetime (lower efficiency of He burning, higher luminosity)
- RR Lyrae stars are pulsationally unstable (L, $B - V$ change with periods $\lesssim 1$ d)
  easy to detect → popular distance indicators
- after exhaustion of central He

  → core contraction (as before) → degenerate core → asymptotic giant branch

Planetary Nebulae with the HST



THE ASYMPTOTIC GIANT BRANCH (AGB)



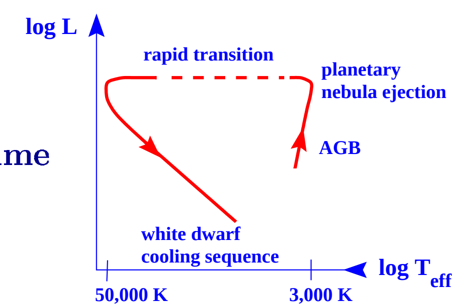
- H burning and He burning (in thin shells)
- H/He burning do not occur simultaneous, but alternate → thermal pulsations

- low-/intermediate-mass stars ($M \lesssim 8\,M_\odot$) do not experience nuclear burning beyond helium burning
- evolution ends when the envelope has been lost by stellar winds
  - ▷ superwind phase: very rapid mass loss ($\dot{M} \sim 10^{-4}\,M_\odot\,\mathrm{yr}^{-1}$)
  - ▷ probably because envelope attains positive binding energy (due to energy reservoir in ionization energy)
  - → envelopes can be dispersed to infinity without requiring energy source
  - ▷ complication: radiative losses

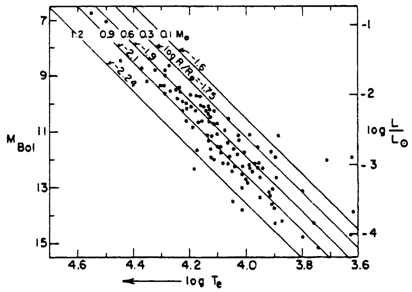
- after ejection:
  hot CO core is exposed
  and ionizes the ejected shell

→ planetary nebula phase (lifetime $\sim 10^4$ yr)

- CO core cools, becomes degenerate → white dwarf



WHITE DWARFS



	Mass ($M_\odot$)	Radius ($R_\odot$)
Sirius B	1.053 ± 0.028	0.0074 ± 0.0006
40 Eri B	0.48 ± 0.02	0.0124 ± 0.0005
Stein 2051	0.50 ± 0.05	0.0115 ± 0.0012

- first white dwarf discovered: Sirius B (companion of Sirius A)
  - ▷ mass (from orbit): $M \sim 1\,M_\odot$
  - ▷ radius (from $L = 4\pi R^2 \sigma T_{\rm eff}^4$) $R \sim 10^{-2}\,R_\odot \sim R_\oplus$

  → $\rho \sim 10^9\,{\rm kg\,m^{-3}}$
- Chandrasekhar (Cambridge 1930)
  - ▷ white dwarfs are supported by electron degeneracy pressure
  - ▷ white dwarfs have a maximum mass of $1.4\,M_\odot$
- most white dwarfs have a mass very close to $M \sim 0.6\,M_\odot$: $M_{\rm WD} = 0.58 \pm 0.02\,M_\odot$
- most are made of carbon and oxygen (CO white dwarfs)
- some are made of He or O-Ne-Mg

Mass-Radius Relations for White Dwarfs

Non-relativistic degeneracy

- $P \sim P_e \propto (\rho/\mu_e m_H)^{5/3} \sim GM^2/R^4$

$$\rightarrow \boxed{R \propto \frac{1}{m_e} (\mu_e m_H)^{5/3} M^{-1/3}}$$

- note the negative exponent

  → R decreases with increasing mass

  → ρ increases with M

Relativistic degeneracy (when $p_{\rm Fe} \sim m_e c$)

- $P \sim P_e \propto (\rho/\mu_e m_H)^{4/3} \sim GM^2/R^4$

→ M independent of R

→ existence of a maximum mass

THE CHANDRASEKHAR MASS

- consider a star of radius R containing N Fermions (electrons or neutrons) of mass m_f
- the mass per Fermion is $\mu_f m_H$ (μ_f = mean molecular weight per Fermion) → number density $n \sim N/R^3$ → volume/Fermion $1/n$
- Heisenberg uncertainty principle $[\Delta x\, \Delta p \sim \hbar]^3$ → typical momentum: $p \sim \hbar\, n^{1/3}$

→ Fermi energy of relativistic particle ($E = pc$)

$$E_f \sim \hbar\, n^{1/3}\, c \sim \frac{\hbar c\, N^{1/3}}{R}$$

- gravitational energy per Fermion

$$E_g \sim -\frac{GM(\mu_f m_H)}{R}, \text{ where } M = N\, \mu_f m_H$$

→ total energy (per particle)

$$E = E_f + E_g = \frac{\hbar c N^{1/3}}{R} - \frac{GN(\mu_f m_H)^2}{R}$$

- stable configuration has minimum of total energy
- if $E < 0$, E can be decreased without bound by decreasing R → no equilibrium → gravitational collapse
- maximum N, if $E = 0$

$$\rightarrow \quad N_{max} \sim \left(\frac{\hbar c}{G(\mu_f m_H)^2}\right)^{3/2} \sim 2 \times 10^{57}$$

$$M_{max} \sim N_{max}\, (\mu_e m_H) \sim 1.5\, M_\odot$$

Chandrasekhar mass for white dwarfs

$$\boxed{M_{Ch} = 1.457 \left(\frac{2}{\mu_e}\right)^2 M_\odot}$$

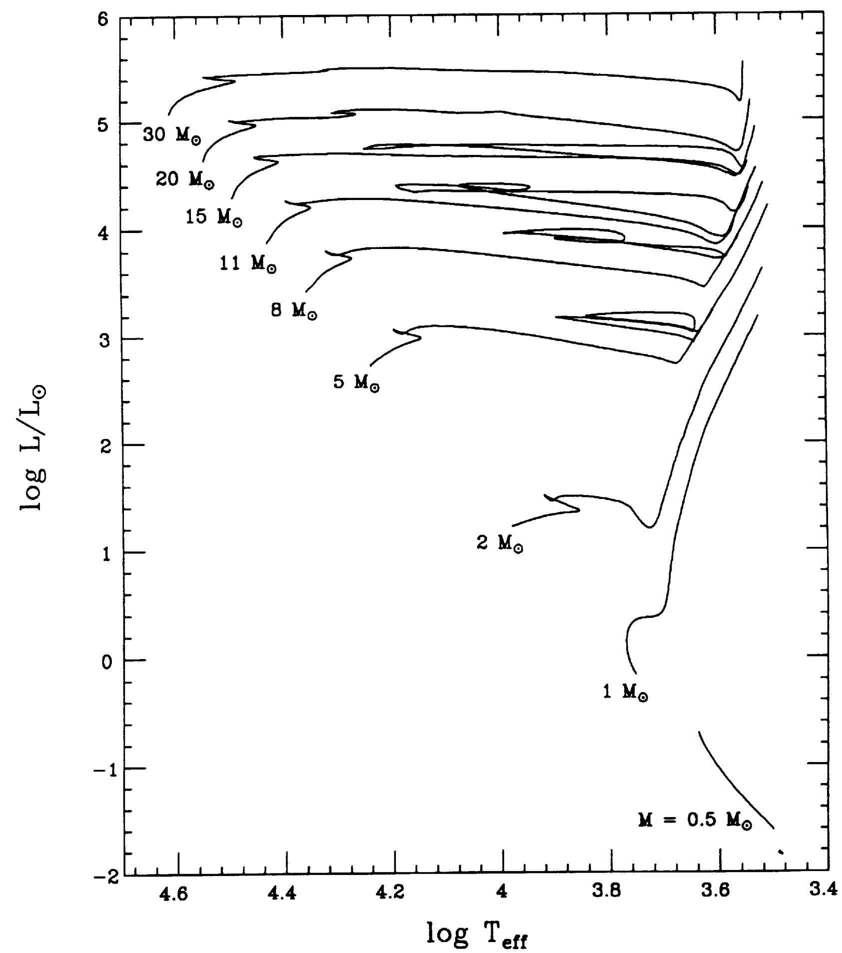


Figure B.1: Composite H-R diagram presenting the evolutionary tracks for stars between $0.5\, M_\odot$ and $30\, M_\odot$. The calculations assume an initially solar composition ($Y = 0.28$, $Z = 0.02$) and a mixing length parameter $\alpha = 1.5$.