C1: Astrophysics Major Option

Problem Set 4: Supernovae, Pulsars

(Ph. Podsiadlowski, Weeks 1 and 2, HT07)

1 Core-Collapse Supernovae [20 points]

Consider the final iron core of a massive star with a mass $M_{\rm Fe} \simeq 1.5\,M_\odot$ and radius $R_{\rm Fe} \simeq 3 \times 10^6\,$m, spinning with a spin period $P \simeq 500\,$s and having a magnetic field at its outer edge $B_{\rm Fe} \simeq 2 \times 10^3\,$T.

a) Stating your assumptions, estimate the final spin and the strength of the magnetic field of the neutron star that forms from the collapse of such a core. Compare the spin period to the maximum spin period for a neutron star.

During the collapse phase, the initial collapse stops when the central core with a mass $M_{\rm core} \simeq 0.7\,M_\odot$ reaches a mass density $\rho \simeq 3 \times 10^{16}\,{\rm kg\,m^{-3}}$. At this density the core bounces driving a shock with an energy $E_{\rm bounce} \sim 10^{44}$J into the infalling outer core.

b) Estimate the energy that is required to photodissociate $0.1\,M_\odot$ of Fe into neutrons and protons. Compare this energy to the bounce shock energy and comment on the fate of the shock. [Remember that $\sim 0.8\,\%$ of the rest mass energy of protons is released in the conversion $56\,^1{\rm H} \rightarrow {}^{56}{\rm Fe}$.]

c) In the proto-neutron star (with an initial radius $\sim 30\,$km), the mean free path of neutrinos is $l_\nu \sim 0.3\,$m. Estimate the diffusion time for neutrinos to escape from the proto-neutron star and hence estimate the neutrino luminosity during the initial neutron-star cooling phase. [Hint: assume that all the gravitational potential energy escapes in the form of neutrinos and use a standard random-walk argument to estimate the neutrino diffusion time.]

d) Assuming that 5 to 10 % of the neutrino luminosity is absorbed by the infalling outer core, estimate how long it takes to absorb enough neutrino energy to reverse the infall of the outer core and drive a successful supernova explosion (with a typical explosion energy of $10^{44}\,$J). Compare this time to the dynamical timescale of the proto-neutron star.

2 The Binary Pulsar PSR J0737-3039: Supernova Kicks [40 points]

Recently, the first binary pulsar was discovered (Lyne, A.G. et al. 2004, Science, 303, 1153), which provides a rare laboratory for relativistic physics. The system consists of two pulsars

(A and B) in a mildly eccentric orbit with an orbital period $P_{\rm orb} \simeq 2.4\,{\rm hr}$ and eccentricity $e \simeq 0.088$. The spin periods and spin-down rates of the two pulsars have been measured to be $P_{\rm A} \simeq 22.7\,{\rm ms}$, $P_{\rm B} \simeq 2.77\,{\rm s}$, $\dot{P}_{\rm A} \simeq 1.7 \times 10^{-18}\,{\rm s\,s^{-1}}$ and $\dot{P}_{\rm B} \simeq 0.88 \times 10^{-15}\,{\rm s\,s^{-1}}$ and the masses have been determined to be $M_{\rm A} \simeq 1.34\,M_\odot$ and $M_{\rm B} \simeq 1.25\,M_\odot$, respectively.

a) Making reasonable assumptions about the pulsar properties, estimate the spin-down luminosities and the spin-down ages (i.e. $P/2\dot{P}$) for both pulsars. Considering the evolutionary history of the system, explain why the spin-down ages should roughly agree.

b) Assuming that the spin-down is caused entirely by magnetic dipole radiation, show that the magnetic field of the pulsars can be estimated from

$$B \simeq \frac{1}{R^3 \sin\theta} \sqrt{\left(\frac{3c^3\mu_0}{32\pi^3}\right) P\dot{P}I},$$

where R is the radius of the pulsar, θ the (generally unknown) inclination of the magnetic axis with respect to the rotation axis and I is the moment of inertia of the pulsar (μ_0 is the magnetic permeability and c the speed of light in vacuo). Estimate the magnetic fields of the two pulsars.

c) It is believed that Pulsar A was spun up by accretion of matter from the progenitor of Pulsar B. Neglecting magnetic fields during the accretion phase, estimate how much mass Pulsar A would have had to accrete from an accretion disc to be spun-up to the observed spin period. How does the actual magnetic field of Pulsar A affect this estimate?

It is reasonable to assume that before the second supernova, in which Pulsar B was formed, the immediate pre-supernova binary system was circular and had an orbital separation $a_0 \simeq 1.4\,R_\odot$.

d) Assuming that in the second supernova an amount of mass ΔM was instantaneously ejected and that Pulsar B did not receive a recoil in its own frame, show that the post-supernova eccentricity e, post-supernova semimajor axis $a_{\rm PSN}$ and post-supernova system velocity $v_{\rm sys}$ (i.e. the velocity of the new centre-of-mass (CM) frame defined by the two pulsars relative to the pre-supernova CM frame) are given by

$$e = \frac{\Delta M}{M_{\rm A} + M_{\rm B}},$$

$$a_{\rm PSN} = \frac{a_0}{1-e},$$

$$v_{\rm sys} = v_{\rm orb}^0 \,\frac{\Delta M}{M_{\rm A} + M_{\rm B}}\, \frac{M_{\rm A}}{M_{\rm He} + M_{\rm A}},$$

where $M_{\rm He}$ is the mass of the progenitor of Pulsar B just before the supernova (i.e. $M_{\rm B} + \Delta M$) and $v_{\rm orb}^0$ is the pre-supernova orbital velocity. Determine ΔM assuming that the post-supernova eccentricity was $e \simeq 0.1$ and estimate $v_{\rm sys}$.

[Hint: You need to compare the energies and momenta of the system before and after the supernova. The eccentricity e and semi-major axis a of an eccentric orbit are related to the distance of closest approach, the periastron separation, r_{p} by $r_{\mathrm{p}} = (1 - e)\, a$, and the total energy of an eccentric binary is

$$E_{\mathrm{binary}} = -\frac{GM_1 M_2}{2a} = -\frac{GM_1 M_2}{r} + \frac{1}{2}\frac{M_1 M_2}{M_1 + M_2} v^2,$$

where r is the separation and v the relative orbital velocity at a particular binary phase, and M_1 and M_2 are the masses of the two components. See, e.g., Carroll & Ostlie, Chapter 2.3.]

e*) Show that in the limit, where there is no mass loss associated with the second supernova but where Pulsar B received an asymmetric supernova kick of magnitude v_{kick}, the post-supernova system velocity is given by

$$v_{\mathrm{sys}} = \frac{M_{\mathrm{B}}}{M_{\mathrm{A}} + M_{\mathrm{B}}} v_{\mathrm{kick}}.$$

What is v_{sys} in this case for a typical $v_{\mathrm{kick}} \simeq 250\,\mathrm{km\,s^{-1}}$?

f*) Discuss how the observed eccentrities and system velocities of systems like the double pulsar may be used to constrain supernova kicks.